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Research article

On 2-Movable Total Domination in the Join and Corona of Graphs

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Abstract: Let G be a connected graph. A non-empty set $T \subseteq V(G)$ is a 2-*movable total dominating set* of G if T is a total dominating set and for every pair $x, y \in T$, $T\backslash\{x, y\}$ is a total dominating set in G, or there exist $u, v \in V(G)\backslash T$ such that u and v are adjacent to x and y, respectively, and $(T\backslash\{x, y\}) \cup \{u, v\}$ is a total dominating set in G. The 2-*movable total domination number* of G, denoted by $\gamma^2_{mt}(G)$, is the minimum cardinality of a 2-movable total dominating set of G. A 2-movable total dominating set with cardinality equal to $\gamma^2_{mt}(G)$ is called γ^2_{mt}-*set* of G. This paper present the 2-movable total domination in the join and corona of graphs.

Mathematics Subject Classification: 54C05, 54C08, 54C10
Keywords: Total Domination; 2-Movable Total Domination; 2-Movable Domination

1 Introduction

All graphs considered in this paper are all connected, finite, simple and undirected. Let $G = (V, E)$ be a connected, finite, simple and undirected graph. The graph G has a vertex set $V = V(G)$ and an edge set $E = E(G)$. Further, let the order of the graph G be p, that is, $|V| = |V(G)| = p$ and the size of the graph G be q, that is, $|E| = |E(G)| = q$.

One of the interesting fields of Graph Theory is graph domination. Several variants of domination has been introduced and explored such as total domination [2], 1-movable domination in graphs [7], neighbourhoud transversal domination of some graphs [3] and others [4–6]. Inspired by the study of Blair et. al [1] about 1-movable domination, Pedrano and Paluga [8] defined a new variant of domination and called it 2-movable domination. A non-empty $S \subseteq V(G)$ is a 2-*movable dominating set* of G if S is a dominating set and for every pair $x, y \in S$, $S\backslash\{x, y\}$ is a dominating set in G, or there exist $u, v \in V(G)\backslash S$ such that u and v are adjacent to x and y, respectively, and $(S\backslash\{x, y\}) \cup \{u, v\}$ is a dominating set in G. The 2-*movable domination number* of G, denoted by $\gamma^2_m(G)$, is the minimum cardinality of a 2-movable dominating set of G. A 2-movable dominating set with cardinality equal to $\gamma^2_m(G)$ is called γ^2_m-*set* of G.

In this study, the concept of 2-movable total domination in a graph will be introduced and initially investigated. In particular, the 2-movable total domination number will be given for the join and corona of graphs.

2 Preliminaries

Definition 1: *[7] A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G)\backslash S$, there exists $u \in S$ such that $uv \in E(G)$, that is, $N_G[S] = V(G)$. The domination number of G is denoted by $\gamma(G)$ which refers to the smallest cardinality of a dominating set of G. A dominating set of G with cardinality equal to $\gamma(G)$ is called a $\gamma-$set of G.*

Definition 2: *[2] Let G be a graph without isolated vertices. A subset S of $V(G)$ is a total dominating set of G if $N_G(S) = V(G)$. That is, every vertex in $V(G)$ is adjacent to some vertex in S.*

Definition 3: *[8] Let G be a connected graph. A non-empty set $T \subseteq V(G)$ is a 2-movable total dominating set of G if T is a total dominating set and for every pair of distinct vertices $x, y \in T$, $T\backslash\{x, y\}$ is a total dominating set in G, or there exist distinct vertices $u, v \in V(G)\backslash T$ such that u and v are adjacent to distinct vertices x and y, respectively, and $(T\backslash\{x, y\}) \cup \{u, v\}$ is a total dominating set in G. The 2-movable total domination number of G, denoted by $\gamma^2_{mt}(G)$, is the minimum cardinality of a 2-movable total dominating set of G. A 2-movable total dominating set with cardinality equal to $\gamma^2_{mt}(G)$ is called γ^2_{mt}-set of G.*

3 THE 2-MOVABLE TOTAL DOMINATION ON GRAPHS

This section presents the results of 2-movable total domination in the join and corona of graphs.

Theorem 1: If S is a 2-movable total dominating set of G, then S is a 2-movable dominating set of G. Furthermore,

$$\gamma^2_m(G) \leq \gamma^2_{mt}(G).$$

Proof: Suppose S is a 2-movable total dominating set of G. It follows that S is a total dominating set of G by Definition 3. Thus, S is a dominating set of G. Since S is a 2-movable total dominating set of G, then for every $a_1, a_2 \in S$, there exists $b_1, b_2 \in V(G)\backslash S$ such that $a_1b_1, a_2b_2 \in E(G)$ and $(S\backslash\{a_1, a_2\}) \cup \{b_1, b_2\}$ is a total dominating set of G. It follows that $(S\backslash\{a_1, a_2\}) \cup \{b_1, b_2\}$ is a dominating set of G. Hence, S is 2-movable dominating set of G. Therefore,

$$\gamma^2_m(G) \leq \gamma^2_{mt}(G).$$

□

The next remark follows directly from Definition 3:

Remark 1: For any connected graph G of order $n \geq 4$, $\gamma^2_{mt}(G) \geq 2$.

Theorem 2: Let G and H be graphs of order at least 2. Then

$$\gamma_{mt}^2(G+H)=2.$$

Proof: Let G and H be graphs of order at least 2, where $V(G+H)=V(G)\cup V(H)$ and $|V(G+H)|\geq 4$. Let $S=\{u,v\}$ where $u\in V(G)$ and $v\in V(H)$. We want to show that S is a total dominating set of $G+H$. Let $x\in V(G+H)$. If $x\in V(G)$, then $xv\in E(G+H)$. If $x\in V(H)$, then $xu\in E(G+H)$. Therefore, S is a total dominating set of $G+H$.

Now, since $|V(G)|\geq 2$ and $|V(H)|\geq 2$, there exists $u'\in V(G)$ and $v'\in V(H)$ such that $u\neq u'$ and $v\neq v'$. Moreover, $uu',vv'\in E(G+H)$. Thus, $S'=(S\backslash\{u,v\})\cup\{u',v'\}=\{u',v'\}$. By following the same arguments above, S' is a total dominating set of $G+H$. Hence, S is a 2-movable total dominating set of $G+H$.

Therefore, $\gamma_{mt}^2(G+H)\leq 2$. By Remark 1, $\gamma_{mt}^2(G+H)=2$. $\square$

Theorem 3: Let G be a graph of order at least 3. Then

$$\gamma_{mt}^2(G+K_1)=\gamma_t(G).$$

Proof: Let S be a γ_t-set of G. Let $x\in V(G+K_1)$. Suppose $x\in V(G)$. Since S is a total dominating set of G, there exists $y\in S$ such that $xy\in E(G)$. Note that $E(G)\subseteq E(G+K_1)$. Thus, $xy\in E(G+K_1)$. Therefore, S is a total dominating set of $G+K_1$.

Now, let $x_1,x_2\in S$. We want to show that S is a 2-movable total dominating set of $G+K_1$. Since S is a γ_t-set of G, at least one of the following conditions hold:

(i) $S\backslash\{x_1,x_2\}$ is a total dominating set of G or
(ii) for each i, there exists $y_i\in(V(G)\backslash S)\cap N(x_i)$ such that $(S\backslash\{x_1,x_2\})\cup\{y_1,y_2\}$ is a total dominating set of G.

Suppose $S\backslash\{x_1,x_2\}$ is a total dominating set of G. Let $x\in V(G+K_1)$. If $x\in V(G)$, there exists $y\in S\backslash\{x_1,x_2\}$ such that $xy\in E(G)\subseteq E(G+K_1)$. Suppose $x\in V(K_1)$. Let $x_3\in S\backslash\{x_1,x_2\}$. Then $xx_3\in E(G+K_1)$. Therefore, condition (i) holds.

Suppose condition (ii) holds. Then for each i, there exists $y_i\in(V(G)\backslash S)\cap N_G(x_i)$ such that $S'=(S\backslash\{x_1,x_2\})\cup\{y_1,y_2\}$ is a total dominating set of G. Since $V(G)\subseteq V(G+K_1)$ and $N_G(x_i)\subseteq N_{G+K_1}(x_i)$, then $y_i\in(V(G+K_1)\backslash S)\cap N_{G+K_1}(x_i)$. Let $x\in V(G+K_1)$. If $x\in V(G)$, then there exists $y\in S'$ such that $xy\in E(G)\subseteq E(G+K_1)$ since S' is a total dominating set of G. Suppose $x\in V(K_1)$. Let $x_3\in S'$. Then $xx_3\in E(G+K_1)$.

Hence, S is a 2-movable total dominating set of $G+K_1$. Therefore, $\gamma_{mt}(G+K_1)\leq|S|=\gamma_t(G)$.

Now, suppose $\gamma_{mt}(G+K_1)\leq\gamma_t G$. Then there exists a minimum 2-movable total dominating set T of $G+K_1$ such that $|T|<\gamma_t(G)$.

Let $x_1,x_2\in T$ such that $x_i\in V(K_1)$. Since T is a 2-movable total dominating set of $G+K_1$, at least one of the following conditions holds:

(i) $T^*=T\backslash\{x_1,x_2\}$ is a total dominating set of $G+K_1$ or
(ii) for each i, there exists $y_i\in(V(G+K_1)\backslash T)\cap N_{G+K_1}(x_i)$ such that $T'=(T\backslash\{x_1,x_2\})\cup\{y_1,y_2\}$ is a total dominating set of $G+K_1$.

Suppose (i) holds. Let $x \in V(G) \subseteq V(G+K_1)$. Then, there exists $y \in T^*$ such that $xy \in E(G+K_1)$. Since $x, y \in V(G)$, $xy \in E(G)$. Thus, T^* is a total dominating set of G. But this means that $|T^*| < |T| < \gamma_t(G)$. This is a contradiction since $\gamma_t(G)$ is the minimum cardinality of a total dominating set in G.

Suppose (ii) holds. Let $x \in V(G) \subseteq V(G+K_1)$. Then, there exists $y \in T'$ such that $xy \in E(G+K_1)$. Since $x, y \in V(G)$, $xy \in E(G)$. Thus, T' is a total dominating set of G. But this means that $|T'| = |T| < \gamma_t G$. This is a contradiction since $\gamma_t(G)$ is the minimum cardinality of a total dominating set in G. Therefore,

$$\gamma_{mt}^2(G+K_1) = \gamma_t G.$$

□

Lemma 1: For any corona graph $G \circ H$ and $a \in V(G)$, if T is 2-movable total dominating set of $G \circ H$, $a \in V(G) \cap T$, and $T_a = T \cap V(H^a)$, then one of the following holds:

(i) $T_a \backslash \{a, u\}$ where $u \in T_a$ is a total dominating set of H^a, or

(ii) there exists x_a and x_u such that $ax_a, ux_u \in E(H^a)$ and $(T_a \backslash \{a, u\}) \cup \{x_a, x_u\}$ is a total dominating set of H^a

Proof: Suppose $a \in V(G)$ and T is a 2-movable total dominating set of $G \circ H$. Let $u \in T_a$. Since T is a 2-movable total dominating set of $G \circ H$, then one of the following conditions holds:

(1) $T \backslash \{a, u\}$ is a total dominating set of $G \circ H$, or

(2) there exists b and v such that $ab, uv \in E(G \circ H)$ and $(T \backslash \{a, u\}) \cup \{b, v\}$ is a total dominating set of $G \circ H$

Suppose (1) holds. Let $x \in V(H^a) \subseteq V(G \circ H)$. Since $T \backslash \{a, u\}$ is a total dominating set of $G \circ H$, there exists $y \in T \backslash \{a, u\}$ such that $xy \in E(G \circ H)$. Note that $y \in V(H^a)$. It follows that $y \in T_a \backslash \{a, u\}$. Thus, $T_a \backslash \{a, u\}$ where $u \in T_a$, is a total dominating set of H^a.

Suppose (2) holds. Let $b \in V(H^a)$. Further, suppose $x_a = b$ and $x_u = v$. Let $x \in V(H^a) \subseteq V(G \circ H)$. Since $(T \backslash \{a, u\}) \cup \{b, v\}$ is a total dominating set of $G \circ H$, there exists $y \in (T \backslash \{a, u\}) \cup \{b, v\}$ such that $xy \in E(G \circ H)$. Note that $y \in V(H^a)$ since $x \in V(H^a)$ and $y \neq a$. Thus, $y \in (T_a \backslash \{a, u\}) \cup \{b, v\}$ since $[(T \cap V(H^a)) \backslash \{a, u\}] \cup \{b, v\} = (T_a \backslash \{a, u\}) \cup \{b, v\}$. Since $y \in V(H^a)$ and $b \notin V(a + H^a)$, $y \neq b$. It follows that $y \in (T_a \backslash \{a, u\}) \cup \{v\} \subseteq (T_a \backslash \{a, u\}) \cup \{x_a, v\}$. Moreover, $xy \in E(H^a)$. Hence, $(T_a \backslash \{a, u\}) \cup \{x_a, v\}$ is a total dominating set of H^a. □

Lemma 2: Let T be a total dominating set of $G \circ H$ and $a \in V(G)$. If $a \notin T$, then $T \cap V(H^a)$ is a total dominating set of H^a.

Proof: Suppose T is a total dominating set of $G \circ H$ and $a \in V(G) \backslash T$. Let $x \in V(H^a)$. Since T is a total dominating set of $G \circ H$, there exists $y \in T$ such that $xy \in E(G \circ H)$. Note that $y \in V(H^a)$ since $xy \in E(G \circ H)$ and $x \in V(H^a)$. Moreover, $y \in T \cap V(H^a)$. Thus, $T \cap V(H^a)$ is a total dominating set of H^a. □

Theorem 4: Let G and H be connected graphs such that $|V(G \circ H)| \geq 3$ and $\gamma_t(H) < |V(G)|$. Then

$$\gamma_{mt}^2(G \circ H) = |V(G)|\gamma_t(H).$$

Proof: For each $x \in V(G)$, let S_x be a γ_t-set of H^x. Further, let $S = \bigcup_{x \in V(G)} S_x$ and $a \in V(G \circ H)$. If $a \in V(G)$, then there exists $b \in S_a \subseteq S$ such that $ab \in E(G \circ H)$. Suppose $a \in V(H^y)$ for some $y \in V(G)$. Since S_y is a γ_t-set of H^y, then there exists $b \in S_y \subseteq S$ such that $ab \in E(H^y) \subseteq E(G \circ H)$. Thus, S is a total dominating set of $G \circ H$.

Let $u, v \in S$. Then there exists $a, b \in V(G)$ such that $u \in S_a$ and $v \in S_b$. To show that S is a 2-movable total dominating set of $G \circ H$, we consider the following cases:

Case 1: $a = b$

Since $\gamma_t(H) < |V(H)|$, there exists $w \in S_a$ such that $u \neq w$ and $v \neq w$. Let $S' = (S \backslash \{u, v\}) \cup \{a, w\}$. Since S is a total dominating set of $G \circ H$ and $(S_a \backslash \{u, v\}) \cup \{a, w\}$ is a total dominating set of H^a, it follows that S' is a total dominating set of $G \circ H$.

Case 2: $a \neq b$

Since S is a total dominating set of $G \circ H$, $(S_a \backslash \{u\}) \cup \{a\}$ is a total dominating set of H^a and $(S_b \backslash \{v\}) \cup \{b\}$ is a total dominating set of H^b, then $S' = (S \backslash \{u, v\}) \cup \{a, b\}$ is a total dominating set of $G \circ H$.

Hence, S is a 2-movable total dominating set of $G \circ H$. Therefore,

$$\begin{aligned}
\gamma_{mt}(G \circ H) \leq |S| &= \sum_{x \in V(G)} |S_x| = \sum_{x \in V(G)} \gamma_t(H^x) \\
&= \sum_{x \in V(G)} \gamma_t(H) = |V(G)| \cdot \gamma_t(H)
\end{aligned}$$

Suppose $\gamma_{mt}(G \circ H) < |V(G)|\gamma_t(H)$. Then there exists a γ_{mt}^2-set T of $G \circ H$ such that $|T| < |V(G)|\gamma_t(H)$. For each $x \in V(G)$, let $T_x = T \cap V(x + H^x)$. Since $|T| < |V(G)|\gamma_t(H)$, there exists $a \in V(G)$ such that $|T_a| < \gamma_t(H) = \gamma_t(H^a)$. Suppose $a \in T$. Then $T_a = S_a \cup \{a\}$, that is, $|T_a| = |S_a| + 1$. By Lemma 1, one of the following conditions holds:

(i) $T_a \backslash \{a, u\}$ where $u \in T_a$ is a total dominating set of H^a, or

(ii) there exists x_a and x_u such that $ax_a, ux_u \in E(H^a)$ and $(T_a \backslash \{a, u\}) \cup \{x_a, x_u\}$ is a total dominating set of H^a

Suppose (i) holds. Then $S_a \backslash \{a, u\}$ is a total dominating set of H^a. Now, $|S_a \backslash \{a, u\}| < |S_a| < |T_a| < \gamma_t(H^a)$. This is a contradiction since $\gamma_t(H^a)$ is the minimum cardinality of total dominating set of H^a. Suppose (ii) holds. Then $(S_a \backslash \{a, u\}) \cup \{x_a, x_u\}$ is a total dominating set of H^a. Now, observe that $|(S_a \backslash \{a, u\}) \cup \{x_a, x_u\}| = |S_a| < |T_a| < \gamma_t(H^a)$. This is a contradiction since $\gamma_t(H^a)$ is the minimum cardinality of total dominating set of H^a.

Suppose $a \in T$. Then $T_a = S_a$, that is, $|T_a| = |S_a|$. By Lemma 2, S_a is a total dominating set of H^a. Now, $|S_a| = |T_a| < \gamma_t(H^a)$. This is a contradiction since $\gamma_t(H^a)$ is the minimum cardinality of total dominating set of H^a. Therefore, $\gamma_{mt}(G \circ H) = |V(G)|\gamma_t(H)$. □

4 Conclusion

In conclusion, the study on 2-movable total domination in the join of graphs and corona of graphs establishes how structural graph operations influence domination properties under mobility constraints. The results show that the 2-movable total domination number can be explicitly determined or bounded for these graph classes by analyzing how vertices interact after the join and corona constructions.

Moreover, the study highlights that the added edges in the join operation and the pendant structures in the corona significantly affect the flexibility of dominating sets, often leading to predictable patterns or formula-based characterizations. Overall, the work contributes to the broader development of domination theory by extending movable domination concepts to more complex graph constructions and provides a foundation for future investigations on other graph operations and variations of domination.

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