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Research article

Title: A Study On Quasi-Interior Hyperideals In Hypersemigroups

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Abstract: The concept of hypersemigroups generalizes semigroups and has several real-world applications. Hyperideals play a key role in hypersemigroups. This paper introduces quasi-interior hyperideals, which combine quasi hyperideals and interior hyperideals in hypersemigroups. We investigate the properties of quasi-interior hyperideals in intra-regular and simple hypersemigroups. Additionally, we prove that in regular hypersemigroups, left (right) hyperideals, interior hyperideals, and left (right) quasi-interior hyperideals are equivalent.

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Keywords: Hypersemigroup; Regular hypersemigroup; hyperideal; Interior hyperideal; Bi-hyperideal; Bi-quasi hyperideal; Quasi-interior hyperideal

1 Introduction

The theory of algebraic hyperstructures, which generalizes classical algebraic structures, was first introduced by Marty [14]. Since then, extensive research has focused on hyperstructures, exploring both their theoretical aspects and their diverse applications in pure and applied mathematics. A key characteristic that distinguishes hyperstructures from traditional algebraic systems is that, the composition of two elements in classical structures yields a single element, in hyperstructures this operation produces a set. Several applications of hyperstructure theory, particularly those developed during the past fifteen years [1–6]. Hyperstructures find use in fields such as artificial intelligence, cryptography, coding theory, biology, chemistry, and applied mathematics.

In semigroup theory, the notion of ideals has played a crucial role in studying structural properties and classifications of algebraic systems. These ideas have been extended to hypersemigroups, where hyperideals and their generalizations have provided a framework for examining algebraic regularity, simplicity, and decomposition. This adaptation has opened new directions for abstraction and generalization.

In ring theory, one-and two-sided ideals are fundamental concepts. The notion of a one-sided ideal in any algebraic structure generalizes the concept of an ideal. Quasi-ideals extend left and right ideals, while bi-ideals generalize quasi-ideals. The concept of bi-ideals in semigroups was introduced by Lajos [12]. The notion of a bi-ideal in semirings is a special case of the (m, n)-ideal introduced by Lajos. As a further generalization, Steinfeld [22] introduced the concept of quasi-ideals first for semigroups and later for rings. Good and Hughes [7] also studied bi-ideals in semigroups. M. Murali Krishna Rao [15–18,20] developed and investigated several related structures, including bi-quasi ideals, bi-interior ideals, quasi-interior ideals, tri-ideals, weak-interior ideals, and tri-quasi ideals in Γ-semirings, Γ- semigroups, semirings, and semigroups, as a generalization of ideals, left(right)ideals, bi-ideals, quasi-ideals, and interior ideals. The concept of quasi-interior ideals in semigroups was first introduced by M. Murali Krishna Rao [19]. The concept of quasi-interior hyperideals naturally arises as a generalization that combines the ideas of quasi-hyperideals and interior hyperideals in hypersemigroups. A quasi-interior hyperideal reflects structural features associated with intersections of left and right hyperideals, while preserving certain absorption properties characteristic of interior ideals. This extension enriches the study of hypersemigroups and deepens understanding of the lattice structure of hyperideals, their minimality conditions, and their connections to regularity. The objective of this study is to introduce and analyze the properties of quasi-interior hyperideals in hypersemigroups. We investigate their formal characterizations, explore their relationships with other hyperideal types, and establish conditions under which quasi-interior hyperideals coincide with or differ from classical notions. These results extend existing theories on quasi-hyperideals and interior hyperideals, contributing to the broader framework of algebraic hyperstructure theory.

2 Preliminaries

In this section, we recall some of the fundamental concepts and definitions which are necessary for this paper.

A map $\circ : \mathcal{H} \times \mathcal{H} \to \mathcal{P}(\mathcal{H})$, is called a hyper operation on a non empty set $\mathcal{H}$, where $\mathcal{P}(\mathcal{H})$ is the set of all nonempty subsets of $\mathcal{H}$. The algebraic structure $(\mathcal{H}, \circ)$ is called a hyper groupoid.
Let X be a nonempty set and $A, B \in \mathcal{P}(\mathcal{H})$ and $x \in \mathcal{H}$. Then we denote
$A \circ B = \bigcup_{a \in A, b \in B}(a \circ b),\ A \circ x = A \circ \{x\}$ and $x \circ B = \{x\} \circ B$.

2.1 Definition

[9,10] A hyper groupoid $(\mathcal{H}, \circ)$ is said to be a hypersemigroup if for every $x, y, z \in \mathcal{H}$, $(x \circ y) \circ z = x \circ (y \circ z)$ $\Rightarrow \bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$.
For simplicity a hypersemigroup as $\mathcal{H}$ instead of $(\mathcal{H}, \circ)$ and AB represents $A \circ B$, for all non-empty subsets A and B of $\mathcal{H}$ and xy represents $x \circ y$, for all $x, y \in \mathcal{H}$.

2.2 Definition

Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a non-empty subset of $\mathcal{H}$. Then $\mathfrak{B}$ is called a hypersubsemigroup of $\mathcal{H}$ if $\mathfrak{B} \circ \mathfrak{B} \subseteq \mathfrak{B}$.

2.3 Definition

Let $\mathcal{H}$ be a hypersemigroup. Then the non-empty subset $\mathfrak{B}$ of $\mathcal{H}$ is said to be:
a left hyperideal of $\mathcal{H}$ if $\mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$
a right hyperideal of $\mathcal{H}$ if $\mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$
a hyperideal of $\mathcal{H}$ if $\mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$ and $\mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$
a quasi hyperideal of $\mathcal{H}$ if $\mathfrak{B} \circ \mathcal{H} \cap \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$
a bi-hyperideal of $\mathcal{H}$ if $\mathfrak{B} \circ \mathfrak{B} \subseteq \mathfrak{B}$ and $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$
an interior hyperideal of $\mathcal{H}$ if $\mathfrak{B} \circ \mathfrak{B} \subseteq \mathfrak{B}$ and $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$.

2.4 Theorem

[10] Let $\mathcal{H}$ be a hypersemigroup, $\mathfrak{L}$ be a left hyperideal and $\mathfrak{R}$ be a right hyperideal. Then $\mathfrak{R} \circ \mathfrak{L} = \mathfrak{R} \cap \mathfrak{L}$.

2.5 Definition

[10] A hypersemigroup $\mathcal{H}$ is said to be regular if for every $\mathfrak{B} \in P^*(\mathcal{H}), \mathfrak{B} \subseteq \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B}$

2.6 Definition

[10] A hypersemigroup $\mathcal{H}$ is said to be intra-regular if for every $\mathfrak{B} \in P^*(\mathcal{H}), \mathfrak{B} \subseteq \mathcal{H} \circ \mathfrak{B}^2 \circ \mathcal{H}$

3 Quasi-interior hyperideals of hypersemigroups

3.1 Definition

Let $\mathcal{H}$ be a hypersemigroup. The non-empty subset $\mathfrak{B}$ of $\mathcal{H}$ is said to be left(right) quasi-interior hyperideal of $\mathcal{H}$ if it is a subhypersemigroup of $\mathcal{H}$ and $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}(\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B})$

3.2 Definition

Let $\mathcal{H}$ be a hypersemigroup. The non-empty subset $\mathfrak{B}$ of $\mathcal{H}$ is called a quasi-interior hyperideal of $\mathcal{H}$ if it is both left quasi-interior hyperideal and a right quasi-interior hyperideal of $\mathcal{H}$.

3.3 Example

Let $\mathcal{H} = \{a, b\}$ be a hypersemigroup with a hyper operation $\circ$ on $\mathcal{H}$ defined as

o	a	b
a	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{b\}$

(i)Then $\mathcal{H}$ is a hypersemigroup.
(ii)Let A={a }. Then A is a left hyperideal.

3.4 Example

Let $\mathcal{H} = \{a, b, c, d, e\}$. Define a hyper operation o on $\mathcal{H}$ by the following table.

o	a	b	c	d	e
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
d	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a, b\}$
e	$\{a\}$	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a\}$

We can verify that $\mathcal{H} = (\mathcal{H}, \circ)$ is a hypersemigroup.
Let $Q = \{a, e\}$. Q is a hypersubsemigroup of $\mathcal{H}$.
But $Q \circ \mathcal{H} = \{a, b, c\}$ *and* $\mathcal{H} \circ Q = \{a, b\}$. Therefore, Q is niether right hyperideal nor left hyperideal of $\mathcal{H}$.
Q is a left quasi-interior hyperideal and right quasi-interior hyperideal of $\mathcal{H}$.

3.5 Definition

A nonempty subset $\mathfrak{B}$ of $\mathcal{H}$ is called a left bi-quasi hyperideal (right bi-quasi hyperideal) of $\mathcal{H}$ if $\mathfrak{B}$ is a hypersubsemigroup and
$\mathcal{H} \circ \mathfrak{B} \cap \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$ ($\mathfrak{B} \circ \mathcal{H} \cap \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$).

3.6 Definition

A nonempty subset $\mathfrak{B}$ of $\mathcal{H}$ is said to be bi-interior hyperideal if $\mathfrak{B}$ is a hypersubsemigroup and $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \cap \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$.

3.7 Definition

(i) A hypersemigroup $\mathcal{H}$ is said to be simple hypersemigroup if $\mathcal{H}$ has no proper hyperideals.
(ii) A hypersemigroup $\mathcal{H}$ is said to be left(right) simple if $\mathcal{H}$ has no proper left(right) hyperideals.
(iii) A hypersemigroup $\mathcal{H}$ is said to be simple left(right) quasi-interior hyperideal if $\mathcal{H}$ has no proper left(right) quasi-interior hyperideals.
A hypersemigroup $\mathcal{H}$ is called quasi-interior simple if $\mathcal{H}$ is both left quasi-interior simple and right quasi-interior simple hyperideal.
(iv)A hypersemigroup $\mathcal{H}$ is said to be simple interior hypersemigroup, if $\mathcal{H}$ has no proper interior hyperideals.

3.8 Theorem

Let $\mathcal{H}$ be a hypersemigroup. The intersection of left(right) quasi-interior hyperideals $\{\mathfrak{B}_\lambda | \lambda \in \Lambda\}$ of $\mathcal{H}$, is a left(right) quasi-interior hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B} = \cap_{\lambda \in \Lambda} \mathfrak{B}_\lambda$.
Then $\mathfrak{B}$ is a hypersubsemigroup of $\mathcal{H}$.

Since, $\mathfrak{B}_\lambda$ is a left quasi-interior hyperideal of $\mathcal{H}$. We have
$\mathcal{H} \circ \mathfrak{B}_\lambda \circ \mathcal{H} \circ \mathfrak{B}_\lambda \subseteq \mathfrak{B}_\lambda$, for all $\lambda \in \Lambda$, and
$\mathcal{H} \circ (\cap \mathfrak{B}_\lambda) \circ \mathcal{H} \circ (\cap \mathfrak{B}_\lambda) \subseteq \cap \mathfrak{B}_\lambda$. Thus $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Hence, $\mathfrak{B}$ is a left quasi-interior hyperideal of $\mathcal{H}$.
Similarly, we can prove the intersection of right quasi-interior hyperideals $\{\mathfrak{B}_\lambda | \lambda \in \Lambda\}$ of $\mathcal{H}$, is a right quasi-interior hyperideal of $\mathcal{H}$. □

3.9 Theorem

Every quasi-hyperideal is a left(right) quasi-interior hyperideal.

Proof: Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a quasi-hyper ideal. Then $\mathfrak{B} \circ \mathcal{H} \cap \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Obviously, $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$, and $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$.
Then $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathcal{H} \circ \mathfrak{B}$, and $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B} \circ \mathcal{H}$.
Therefore,

$$\begin{aligned}
\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \cap \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \\
&\subseteq \mathrm{H} \circ \mathfrak{B} \cap \mathfrak{B} \circ \mathrm{H} \\
&\subseteq \mathfrak{B}.
\end{aligned}$$

Hence, the theorem. □

3.10 Theorem

Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a hypersubsemigroup. $\mathfrak{B}$ is a quasi-interior hyperideal of $\mathcal{H}$ if and only if $\mathfrak{L} \circ \mathfrak{R} \subseteq \mathfrak{B} \subseteq \mathfrak{L} \cap \mathfrak{R}$, where $\mathfrak{L}$ is a left hyperideal and $\mathfrak{R}$ is a right hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a quasi-interior hyperideal of $\mathcal{H}$. Then $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$ and $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Let $\mathfrak{R} = \mathfrak{B} \circ \mathcal{H}$ and $\mathfrak{L} = \mathcal{H} \circ \mathfrak{B}$. Then $\mathfrak{R}$ is a right hyperideal and $\mathfrak{L}$ is a left hyperideal of $\mathcal{H}$. Now,

$$\mathfrak{L} \circ \mathfrak{L} \circ \mathfrak{R} \circ \mathfrak{R} \circ \mathfrak{L} \circ \mathfrak{R} \subseteq \mathfrak{B} \subseteq \mathfrak{R} \cap \mathfrak{L}.$$

$$\Rightarrow \mathfrak{L} \circ \mathfrak{R} \subseteq \mathfrak{B} \subseteq \mathfrak{R} \cap \mathfrak{L}.$$

Conversely, suppose that $\mathfrak{L} \circ \mathfrak{R} \subseteq \mathfrak{B} \subseteq \mathfrak{R} \cap \mathfrak{L}$, where $\mathfrak{L}$ is a left hyperideal and $\mathfrak{R}$ is a right hyperideal of $\mathcal{H}$. Hence,

$$\begin{aligned}
\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \cap \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \\
&\subseteq \mathcal{H} \circ (\mathfrak{L} \cap \mathfrak{R}) \circ \mathcal{H} \circ (\mathfrak{L} \cap \mathfrak{R}) \cap (\mathfrak{L} \cap \mathfrak{R}) \circ \mathcal{H} \circ (\mathfrak{L} \cap \mathfrak{R}) \circ \mathcal{H} \\
&\subseteq \mathcal{H} \circ \mathfrak{L} \circ \mathcal{H} \circ \mathfrak{R} \cap \mathfrak{L} \circ \mathcal{H} \circ \mathfrak{R} \circ \mathcal{H} \\
&\subseteq \mathcal{H} \circ \mathfrak{L} \circ \mathfrak{R} \cap \mathfrak{L} \circ \mathfrak{R} \circ \mathcal{H} \\
&\subseteq B.
\end{aligned}$$

Hence, $\mathfrak{B}$ is a quasi-interior hyperideal of $\mathcal{H}$. □

3.11 Theorem

Every left(right) hyperideal is a left(right) quasi-interior hyperideal.

Proof: Let $\mathcal{H}$ be a left hyperideal. Then $\mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Now, $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Similarly, $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Hence, the theorem. □

3.12 Theorem

Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a left(right) hyperideal of $\mathcal{H}$. Then $\mathfrak{B}$ is a bi-interior hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a left hyperideal of $\mathcal{H}$. Then $\mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$. Now,

$$\begin{aligned}
\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \cap \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \\
&\subseteq \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\
&\subseteq \mathfrak{B} \circ \mathcal{B} \\
&\subseteq \mathfrak{B}.
\end{aligned}$$

Hence, $\mathfrak{B}$ is a bi-interior ideal of $\mathcal{H}$.
Similarly, if $\mathfrak{B}$ be a right hyperideal of $\mathcal{H}$. Then $\mathfrak{B}$ is a bi-interior hyperideal of $\mathcal{H}$. □

3.13 Theorem

If $\mathfrak{B}$ is a left(right) quasi-interior hyperideal $\mathcal{H}$. Then $\mathcal{B}$ is a bi-hyperideal of $\mathcal{H}$.

Proof: Let $\mathfrak{B}$ be a left(right) quasi-interior hyperideal of $\mathcal{H}$. Then
$\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$, and $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$, Now

$$\begin{aligned}\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} &\subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B}(\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H}) \\ &\subseteq \mathfrak{B}.\end{aligned}$$

Therefore, $\mathfrak{B}$ is a bi-hyperideal of $\mathcal{H}$. □

3.14 Theorem

Every interior hyperideal of a hypersemigroup $\mathcal{H}$ is a left(right) quasi-interior hyperideal.

Proof: Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be an interior hyperideal of $\mathcal{H}$.
Then $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$. This gives $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Hence, $\mathfrak{B}$ is a left quasi-interior hyperideal of $\mathcal{H}$.
Similarly, every interior hyperideal of $\mathcal{H}$ is a right quasi-interior hyperideal. □

3.15 Theorem

Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a left(right) hyperideal of $\mathcal{H}$. Then $\mathfrak{B}$ is an interior hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a left hyperideal of $\mathcal{H}$. Then $\mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}(\mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B})$.
Now, $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$,
And $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Hence, $\mathfrak{B}$ is an interior hyperideal of $\mathcal{H}$. □

3.16 Theorem

Let $\mathcal{H}$ be a hypersemigroup. If $\mathfrak{B}$ is a left(right) quasi-interior hyperideal, then $\mathfrak{B}$ is a left(right) bi-quasi hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{B}$ be a left quasi-interior hyperideal of $\mathcal{H}$. Then $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq B$.
Thus $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B}$. Therefore,

$$\begin{aligned}&\mathcal{H} \circ \mathfrak{B} \cap \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\ &\quad \subseteq \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\ &\quad \subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\ &\quad \subseteq \mathfrak{B}.\end{aligned}$$

Hence, $\mathfrak{B}$ is a left bi-quasi hyperideal of $\mathcal{H}$.
Similarly, If $\mathfrak{B}$ is a right quasi-interior hyperideal, then $\mathfrak{B}$ is a right bi-quasi hyperideal of $\mathcal{H}$. □

3.17 Theorem

Let $\mathcal{H}$ be a hypersemigroup. If $\mathfrak{B}$ is a left(right) quasi-interior hyperideal, then $\mathfrak{B}$ is a bi-interior hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be a hypersemigroup, and $\mathfrak{B}$ be a left quasi-interior hyperideal of $\mathcal{H}$. Then $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B}(\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H}) \subseteq \mathfrak{B}$. Thus

$$\begin{aligned}
&\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \cap \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \\
&\quad \subseteq \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\
&\quad \subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B}(\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H}) \\
&\quad \subseteq \mathfrak{B}.
\end{aligned}$$

Therefore, $\mathfrak{B}$ is a bi-interior hyperideal of $\mathcal{H}$.
□

3.18 Theorem

Let $\mathcal{H}$ be a hypersemigroup. If $\mathfrak{R}$ is a right hyperideal and $\mathfrak{L}$ is a left hyperideal of $\mathcal{H}$, then $\mathfrak{L} \circ \mathfrak{R}$ is a left(right) quasi-interior hyperideal of $\mathcal{H}$.

Proof: Let $\mathfrak{L}$ be a left hyperideal of $\mathcal{H}$ and $\mathfrak{R}$ be a right hyperideal of $\mathcal{H}$.
First we prove that $\mathfrak{L} \circ \mathfrak{R}$ is a hypersubsemigroup of $\mathcal{H}$.

$$(\mathfrak{L} \circ \mathfrak{R}) \circ (\mathfrak{L} \circ \mathfrak{R}) \subseteq (\mathfrak{L} \circ \mathfrak{R}) \circ \mathcal{H} \subseteq \mathfrak{L} \circ (\mathfrak{R} \circ \mathcal{H}) \subseteq \mathfrak{L} \circ \mathfrak{R}$$

Thus, $(\mathfrak{L} \circ \mathfrak{R}) \circ (\mathfrak{L} \circ \mathfrak{R}) \subseteq (\mathfrak{L} \circ \mathfrak{R})$. Now consider,

$$\begin{aligned}
\mathcal{H} \circ (\mathfrak{L} \circ \mathfrak{R}) \circ \mathcal{H} \circ (\mathfrak{L} \circ \mathfrak{R}) &= (\mathfrak{L} \circ \mathfrak{R}) \circ (\mathcal{H} \circ \mathfrak{L}) \circ \mathfrak{R} \\
&\subseteq (\mathfrak{L} \circ \mathfrak{R}) \circ (\mathfrak{L} \circ \mathfrak{R}) \\
&\subseteq (\mathfrak{L} \circ \mathfrak{R}).
\end{aligned}$$

Hence, $\mathfrak{L} \circ \mathfrak{R}$ is a left quasi-interior hyperideal of $\mathcal{H}$.
Similarly, if $\mathfrak{L}$ is a left hyperideal and $\mathfrak{R}$ is a right hyperideal of $\mathcal{H}$, then $\mathfrak{R} \circ \mathfrak{L}$ is a right quasi-interior hyperideal of $\mathcal{H}$. □

3.19 Theorem

Let $\mathcal{H}$ be a intra-regular hypersemigroup. If $\mathfrak{B}$ is a left(right) quasi-interior hyperideal of $\mathcal{H}$, then $\mathfrak{B}$ is a bi-hyperideal of $\mathcal{H}$.

Proof: Let $\mathfrak{B}$ be a left quasi-interior hyperideal of $\mathcal{H}$. Then $\mathfrak{B} \subseteq \mathcal{H} \circ \mathfrak{B}^2 \circ \mathcal{H}$. Now,

$$\begin{aligned}\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} &\subseteq \mathcal{H} \circ \mathfrak{B}^2 \circ \mathcal{H} \circ \mathcal{H} \circ \mathfrak{B} \\ &\subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\ &\subseteq \mathfrak{B}.\end{aligned}$$

Thus, $\mathfrak{B}$ is a bi-hyperideal of $\mathcal{H}$.
Similarly, If $\mathfrak{B}$ is a right quasi-interior hyperideal of $\mathcal{H}$, then $\mathfrak{B}$ is a bi-hyperideal of $\mathcal{H}$. □

3.20 Theorem

Let $\mathcal{H}$ be a hypersemigroup. If $\mathcal{H}$ is simple, then $\mathcal{H}$ is quasi-interior simple.

Proof: Suppose, $\mathcal{H}$ is simple. Let $\mathfrak{B}$ be a left quasi-interior hyperideal of $\mathcal{H}$. Then $H \circ \mathfrak{B}$ is a left hyperideal of $\mathcal{H}$. Since, $\mathcal{H}$ is simple, $\mathcal{H} \circ \mathfrak{B} = \mathcal{H}$. Now

$$\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} = \mathcal{H} \circ \mathcal{H} = H.$$

Let $\mathfrak{B}$ be a right quasi-interior hyperideal of $\mathcal{H}$. Then $\mathfrak{B} \circ H$ is a right hyperideal of $\mathcal{H}$. Since, $\mathcal{H}$ is simple, $\mathfrak{B} \circ \mathcal{H} = \mathcal{H}$. Now

$$\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} = \mathcal{H} \circ \mathcal{H} = H.$$

Thus, $\mathcal{H}$ is quasi-interior simple. □

3.21 Theorem

Let $\mathcal{H}$ be an interior simple hypersemigroup. Every left(right) quasi-interior hyperideal of $\mathcal{H}$ is a left(right) hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be an interior simple hypersemigroup. Then $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} = \mathcal{H}$, where $\mathfrak{B}$ is a left quasi-interior hyperideal, $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq B$.
Since, $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} = \mathcal{H}$. Therefore,

$$\begin{aligned}\mathcal{H} \circ \mathfrak{B} &\subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\ &\subseteq \mathfrak{B}\end{aligned}$$

Similarly, we can prove that every right quasi-interior hyperideal of $\mathcal{H}$ is a right hyperideal of $\mathcal{H}$.
Hence, theorem. □

3.22 Theorem

Let $\mathcal{H}$ be a hypersemigroup. If $\mathcal{H}$ is left simple, then every left quasi-interior hyperideal of H is right hyperideal of $\mathcal{H}$.

Proof: Suppose, $\mathcal{H}$ is left simple. Let $\mathfrak{B}$ be a left quasi-interior hyperideal of $\mathcal{H}$. Then $H \circ \mathfrak{B}$ is a left hyperideal of $\mathcal{H}$. Since, $\mathcal{H}$ is simple, $\mathcal{H} \circ \mathfrak{B} = \mathcal{H}$. Now

$$\begin{aligned}\mathfrak{B} \circ \mathcal{H} &= \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\ &\subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\ &\subseteq \mathfrak{B}\end{aligned}$$

Hence, $\mathfrak{B}$ is a right hyperideal of $\mathcal{H}$. □

3.23 Theorem

Let $\mathcal{H}$ be a regular hypersemigroup. If $\mathfrak{B}$ is a left(right) quasi-interior ideal of $\mathcal{H}$, then $\mathfrak{B}$ is a left(right) hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be a regular hypersemigroup, and $\mathfrak{B}$ be a left quasi-interior hyperideal of $\mathcal{H}$. Since $\mathcal{H}$ is regular, $\mathfrak{B} \subseteq \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B}$. Now

$$\mathcal{H} \circ \mathfrak{B} \subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}.$$

Thus, $\mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Hence, $\mathfrak{B}$ is a left hyperideal of $\mathcal{H}$.
Similarly, If $\mathfrak{B}$ is a right quasi-interior ideal of $\mathcal{H}$, then $\mathfrak{B}$ is a right hyperideal of $\mathcal{H}$. □

3.24 Theorem

Let $\mathcal{H}$ be a hypersemigroup, $\mathfrak{B}$ is a left quasi-interior
hyperideal, $\mathfrak{I}$ is an hyper ideal, $\mathfrak{L}$ is a left hyperideal of $\mathcal{H}$. $\mathcal{H}$ is a regular
hypersemigroup if and only if $\mathfrak{B} \cap \mathfrak{I} \cap \mathfrak{L} \subseteq \mathfrak{B} \circ \mathfrak{I} \circ \mathfrak{L}$.

Proof: Let $\mathcal{H}$ be a hypersemigroup, $\mathfrak{B}$ is a left quasi-interior hyperideal, $\mathfrak{I}$ is an hyper ideal, $\mathfrak{L}$ is a left hyperideal of H.
Let $a \in \mathfrak{B} \cap \mathfrak{I} \cap \mathfrak{L}$, then $a \in a \circ \mathcal{H} \circ a$.(Since $\mathcal{H}$ is a regular hypersemigroup).

$$\begin{aligned}a \in a \circ \mathcal{H} \circ a &\subseteq a \circ \mathcal{H} \circ a \circ \mathcal{H} \circ a \\ &\subseteq \mathfrak{B} \circ \mathfrak{I} \circ \mathfrak{L}.\end{aligned}$$

Hence, $\mathfrak{B} \cap \mathfrak{I} \cap \mathfrak{L} \subseteq \mathfrak{B} \circ \mathfrak{I} \circ \mathfrak{L}$.
Conversely, suppose that $\mathfrak{B} \cap \mathfrak{I} \cap \mathfrak{L} \subseteq \mathfrak{B} \circ \mathfrak{I} \circ \mathfrak{L}$, for any left quasi-interior hyperideal, hyper ideal, left hyperideal of $\mathcal{H}$. Let $\mathfrak{R}$ be a right hyperideal and $\mathfrak{L}$ be a left hyperideal of $\mathcal{H}$. Then

$$\begin{aligned}\mathfrak{R} \cap \mathfrak{L} &= \mathfrak{R} \cap \mathcal{H} \cap \mathfrak{L} \\ &\subseteq \mathfrak{R} \circ \mathcal{H} \circ \mathfrak{L} \\ &\subseteq \mathfrak{R} \circ \mathfrak{L}.\end{aligned}$$

Since, $\mathfrak{R} \circ \mathfrak{L} \subseteq \mathfrak{R}$ and $\mathfrak{R} \circ \mathfrak{L} \subseteq \mathfrak{L}$, then $\mathfrak{R} \circ \mathfrak{L} \subseteq \mathfrak{R} \cap \mathfrak{L}$. Therefore, $\mathfrak{R} \circ \mathfrak{L} = \mathfrak{R} \cap \mathfrak{L}$. Hence, $\mathcal{H}$ is a regular hypersemigroup. □

3.25 Theorem

If the hypersemigroup $\mathcal{H}$ is regular, then $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} = \mathfrak{B}$, for every left(right) quasi-interior hyperideal $\mathfrak{B}$ of $\mathcal{H}$.

Proof: Let $\mathfrak{B}$ be a left quasi-interior hyperideal of $\mathcal{H}$.
Then $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Since, $\mathcal{H}$ is regular,

$$\begin{aligned}\mathfrak{B} &\subseteq \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \\ &\subseteq \mathfrak{B} \circ \mathrm{H} \circ \mathfrak{B} \circ \mathcal{H} \\ &\subseteq \mathfrak{B}.\end{aligned}$$

Hence, $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} = \mathfrak{B}$.
Similarly, If the hypersemigroup $\mathcal{H}$ is regular, then $\mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} = \mathfrak{B}$, for every right quasi-interior hyperideal $\mathfrak{B}$ of $\mathcal{H}$. □

3.26 Theorem

Let $\mathcal{H}$ be a regular hypersemigroup. Then every left quasi-interior hyperideal is a hyperideal.

Proof: Let $\mathfrak{B}$ be a left quasi interior ideal hyper ideal. Then $\mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Since, $\mathcal{H}$ is a regular hypersemigroup, then $\mathfrak{B} \subseteq \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Now, $\mathcal{H} \circ \mathfrak{B} \subseteq \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$. Hence, $\mathcal{H} \circ \mathfrak{B} \subseteq \mathfrak{B}$.
Therefore, $\mathfrak{B}$ is a left hyper ideal. And
$\mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B} \circ \mathcal{H} \circ \mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$.
Therefore, $\mathfrak{B} \circ \mathcal{H} \subseteq \mathfrak{B}$. Hence, $\mathfrak{B}$ is a right hyperideal.
Therefore, $\mathfrak{B}$ is a hyper ideal. □

3.27 Definition

A nonempty left hyperideal of a hypersemigroup $\mathcal{H}$ is said to be a minimal left hyperideal if it contains no other left hyperideal other than itself.

3.28 Theorem

Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{L}$ is a minimal left hyperideal of $\mathcal{H}$. If $\mathfrak{B} = \mathfrak{L} \circ \mathfrak{L}$, then $\mathfrak{B}$ is a minimal left quasi-interior hyperideal of $\mathcal{H}$.

Proof: Let $\mathcal{H}$ be a hypersemigroup and $\mathfrak{L}$ is a minimal left hyperideal of $\mathcal{H}$. Then $\mathfrak{B} = \mathfrak{L} \circ \mathfrak{L}$. Let $\mathcal{Q}$ be a left quasi-interior hyperideal of $\mathcal{H}$,
such that $\mathcal{Q} \subseteq \mathfrak{B}$.
We have $\mathcal{H} \circ \mathcal{Q}$ is a minimal left hyperideal of $\mathcal{H}$. Then
$\mathcal{H} \circ \mathcal{Q} \subseteq \mathcal{H} \circ \mathfrak{B} = \mathcal{H} \circ \mathfrak{L} \circ \mathfrak{L} \subseteq \mathfrak{L}$ (Since $\mathfrak{L}$ is a left hyperideal of $\mathcal{H}$.)
Hence, $\mathcal{H} \circ \mathcal{Q} = \mathfrak{L}$.
And $\mathfrak{B} = \mathcal{H} \circ \mathcal{Q} \circ \mathcal{H} \circ \mathcal{Q} \subseteq \mathcal{Q}$.
Therefore, $\mathcal{Q} = \mathfrak{B}$.
Hence, $\mathfrak{B}$ is a minimal left quasi-interior hyperideal of $\mathcal{H}$. □

3.29 Theorem

If $\mathcal{H}$ be a regular hypersemigroup, then the following hyperideals are equivalent
(i) $\mathfrak{B}$ is a left(right) hyperideal of $\mathcal{H}$.
(ii) $\mathfrak{B}$ is a Interior hyperideal of $\mathcal{H}$.
(iii) $\mathfrak{B}$ is a left(right) quasi-interior hyperideal of $\mathcal{H}$.

Proof: The equivalent proofs of the theorems are as follows:

$(i) \Rightarrow (ii)$ *by Theorem* 3.15.

$(ii) \Rightarrow (iii)$ *by Theorem* 3.14.

$(iii) \Rightarrow (i)$ *by Theorem* 3.23.

$(i) \Rightarrow (iii)$ *by Theorem* 3.11.

□

4 Conclusions

In this study, we introduced and examined the notion of quasi-interior hyperideals in hypersemigroups. By analyzing their structural properties, we established several equivalent characterizations and highlighted their relationships with other types of hyperideals. In particular, it was shown that quasi-interior hyper-ideals

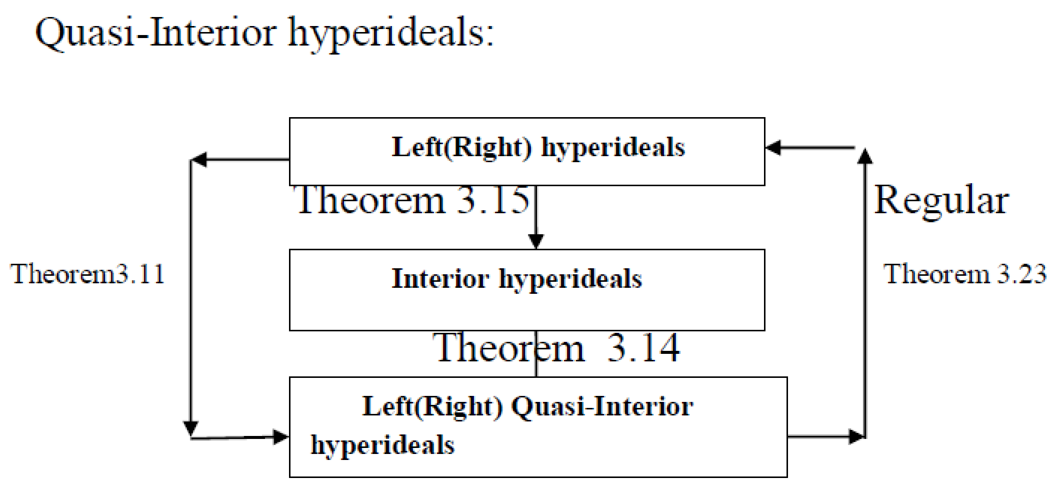

Figure 1: Quasi -Interior hyperideals

can be represented through intersections and products of left and right hyper ideals, thereby generalizing well-known results from semigroup theory to the framework of hypersemigroups. We studied the properties of quasi interior hyperideals of hypersemigroups and characterized the quasi-interior simple hypersemigroups and quasi-interior hyperideals in regular and intra-regular hypersemigroups. Hypersemigroups have wide applications in automata, probability, geometry, topology, cryptography and coding theory, lattices, binary relations, graphs, hypergraphs . We further wish to exetend and study tri-hyperideals and fuzzy tri-hyperideals in hypersemigroups.

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