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Research article

On Logarithmic–Power Integrals and Their Extensions

K. Jyothi[1], **B. Ravi**[2]*, **A. Venkata Lakshmi**[3]

[1]Department of Mathematics, Osmania University Telangana State, 500007, India.
[2]Department of Mathematics, Government College(A), Anantapur, Andhra Pradesh, 515001, India.
[3] Department of Mathematics, Osmania University Telangana State, 500007, India.

*Corresponding Author: B. Ravi. Email: ravidevi19@gmail.com

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Abstract: This paper presents several integral formulas inspired by classical results and problems in the existing literature. We study a family of integrals and obtain closed-form expressions involving hypergeometric series and harmonic numbers.

Mathematics Subject Classification: 33B15, 33B20
Keywords: logarithmic integrals; logarithmic power integrals; definite integrals; open problems; series.

1 Introduction

The evaluation of definite integrals involving logarithmic and rational functions has long been a subject of great interest in mathematical analysis, owing to their frequent appearance in number theory, physics, and engineering. Classical texts [3–5] provides a collection of such evaluations, many of which are expressed in terms of special functions like the Gamma function, the Polygamma function, and the Riemann zeta function.

In recent years, there has been a focus on finding closed-form expressions for families of integrals that involve powers of logarithmic terms and rational denominators. For instance, Furdui [1] and Sondow [**?**] explored various integral representations related to Euler's constant and harmonic numbers. We refer to the recent studies in [7–11] More recently, Chesneau [6] provided new proofs and variants for a specific class of logarithmic-power integrals, specifically investigating forms such as:

$$\int_0^1 \frac{\ln(1+\alpha x)}{(1+\alpha x^2)^2}\, dx.$$

The primary objective of this paper is to extend these results by introducing a more general framework. We consider a broader class of integrals of the form:

$$\mathcal{I}(\alpha,\beta,\gamma,\delta,\nu)=\int_0^1 \frac{P(x)\,\ln^m(1+\alpha x)}{(1+\beta x^\gamma)^\delta}\,dx$$

where $P(x)$ is a polynomial and $m \in \{1,2\}$. By utilizing power series expansions and the dominated convergence theorem, we establish general theorems that express these integrals as infinite series involving the Gauss hypergeometric function ${}_2F_1$ and harmonic numbers H_n.

This work is organized as follows: In Section 2, we present our main theorems regarding the general framework of these integrals. We also provide several remarks illustrating how our general formulas reduce to the specific cases recently discussed in the literature [6], thereby providing a unified perspective on these recent results. Finally, we provide series sums that arise as a natural consequence of these integral evaluations.

2 Second section

Theorem 1: *Let $\beta > 0, \gamma > 0, \delta > 0$ and $\alpha \in (0,1)$. Then, we have*

$$\int_0^1 \frac{\ln(1+\alpha x)}{(1+\beta x^\gamma)^\delta}dx=\sum_{i=1}^\infty \frac{(-1)^{i+1}\alpha^i}{i(i+1)}\,{}_2F_1\left(\delta,\frac{i+1}{\gamma};\frac{i+\gamma+1}{\gamma};-\beta\right).$$

Proof: For $|\alpha x|<1$, it is well known that the power series expansion of $\ln(1+\alpha x)$ is given by

$$\ln(1+\alpha x)=\sum_{i=1}^\infty \frac{(-1)^{i+1}\alpha^i x^i}{i}.$$

Hence, exchanging the integral and sum signs by the dominated convergence theorem, we obtain

$$\int_0^1 \frac{\ln(1+\alpha x)}{(1+\beta x^\gamma)^\delta}dx=\sum_{i=1}^\infty \frac{(-1)^{i+1}\alpha^i}{i}\int_0^1 \frac{x^i}{(1+\beta x^\gamma)^\delta}dx. \tag{2.1}$$

Changing the variable $x=t^{1/\gamma}$, we obtain

$$\int_0^1 \frac{\ln(1+\alpha x)}{(1+\beta x^\gamma)^\delta}dx=\sum_{i=1}^\infty \frac{(-1)^{i+1}\alpha^i}{i\,\gamma}\int_0^1 \frac{t^{\frac{i+1}{\gamma}-1}}{(1+\beta t)^\delta}dt. \tag{2.2}$$

From [3, Entry 3.194], the integral in (2.2) can be evaluated to yield

$$= \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i\,\gamma} \cdot \frac{\gamma}{i+1} {}_2F_1\left(\delta, \frac{i+1}{\gamma}; \frac{i+\gamma+1}{\gamma}; -\beta\right)$$
$$= \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i(i+1)} {}_2F_1\left(\delta, \frac{i+1}{\gamma}; \frac{i+\gamma+1}{\gamma}; -\beta\right).$$

□

Remark: *When $\beta = \alpha$, $\gamma = 2$ and $\delta = 2$, it follows from [6, Proposition 3.4] that*

$$\int_0^1 \frac{\ln(1+\alpha x)}{(1+\alpha x^2)^2} dx$$
$$= \frac{1}{4\sqrt{\alpha}} \arctan(\sqrt{\alpha}) \ln(1+\alpha) - \frac{1}{1+\alpha}\left\{\frac{1}{2}\sqrt{\alpha}\arctan(\sqrt{\alpha}) - \frac{3}{4}\ln(1+\alpha)\right\}.$$

Consequently, the corresponding series sum is given by

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i(i+1)} {}_2F_1\left(2, \frac{i+1}{2}; \frac{i+3}{2}; -\alpha\right)$$
$$= \frac{1}{4\sqrt{\alpha}} \arctan(\sqrt{\alpha}) \ln(1+\alpha) - \frac{1}{1+\alpha}\left\{\frac{1}{2}\sqrt{\alpha}\arctan(\sqrt{\alpha}) - \frac{3}{4}\ln(1+\alpha)\right\} + \frac{1}{2(1+\alpha)}.$$

Remark: *When $\beta = \alpha$, $\gamma = 2$ and $\delta = 1$, it follows from [6, Proposition 2.1] that*

$$\int_0^1 \frac{\ln(1+\alpha x)}{1+\alpha x^2}\, dx = \frac{1}{2\sqrt{\alpha}} \arctan(\sqrt{\alpha})\, \ln(1+\alpha).$$

Consequently, the corresponding series sum is given by

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i(i+1)} {}_2F_1\left(1, \frac{i+1}{2}; \frac{i+3}{2}; -\alpha\right) = \frac{1}{2\sqrt{\alpha}} \arctan(\sqrt{\alpha}) \ln(1+\alpha).$$

Theorem 2: *Let $\beta > 0, \gamma > 0, \delta > 0, \eta > 0, \nu > 0$ and $\alpha \in (0,1)$. Then, we have*

$$\int_0^1 \frac{(1-\eta x^\nu)\log(1+\alpha x)}{(1+\beta x^\gamma)^\delta}\, dx = \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i}\left[\frac{1}{i+1}\, {}_2F_1\left(\delta, \frac{i+1}{\gamma}; \frac{i+\gamma+1}{\gamma}; -\beta\right) - \frac{1}{i+\nu+1}\, {}_2F_1\left(\delta, \frac{i+\nu+1}{\gamma}; \frac{i+\nu+\gamma+1}{\gamma}; -\beta\right)\right].$$

Proof: Proof follows directly from Theorem 1. □

Remark: *When $\eta = \beta = \alpha$ and $\nu = \gamma = \delta = 2$, it follows from [6, Proposition 3.10] that*

$$\int_0^1 \frac{(1-\alpha x^2)\ \ln(1+\alpha x)}{(1+\alpha x^2)^2}\, dx = \frac{1}{1+\alpha}\left\{\frac{3}{2}\ln(1+\alpha) - \sqrt{\alpha}\ \arctan(\sqrt{\alpha})\right\}.$$

Consequently, the corresponding series sum is given by

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i}\left[\frac{1}{i+1}\, {}_2F_1\!\left(2,\, \frac{i+1}{2};\, \frac{i+3}{2};\, -\alpha\right) - \frac{1}{i+3}\, {}_2F_1\!\left(2,\, \frac{i+3}{2};\, \frac{i+5}{2};\, -\alpha\right)\right]$$
$$= \frac{1}{1+\alpha}\left\{\frac{3}{2}\ln(1+\alpha) - \sqrt{\alpha}\ \arctan(\sqrt{\alpha})\right\}.$$

Remark: *When $\beta = \alpha, \eta = \nu = 1$ and $\gamma = \delta = 2$, it follows from [6, Proposition 3.2] that*

$$\int_0^1 \frac{(1-x)\ \ln(1+\alpha x)}{(1+\alpha x^2)^2}\, dx = \frac{1}{2\sqrt{\alpha}}\left\{\frac{1}{2}\arctan(\sqrt{\alpha})\ln(1+\alpha) - \arctan(\sqrt{\alpha}) + \frac{1}{\sqrt{\alpha}}\ln(1+\alpha)\right\}.$$

Consequently, the corresponding series sum is given by

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i}\left[\frac{1}{i+1}\, {}_2F_1\!\left(2,\, \frac{i+1}{2};\, \frac{i+3}{2};\, -\alpha\right) - \frac{1}{i+2}\, {}_2F_1\!\left(2,\, \frac{i+2}{2};\, \frac{i+4}{2};\, -\alpha\right)\right]$$
$$= \frac{1}{2\sqrt{\alpha}}\left\{\frac{1}{2}\arctan(\sqrt{\alpha})\ln(1+\alpha) - \arctan(\sqrt{\alpha}) + \frac{1}{\sqrt{\alpha}}\ln(1+\alpha)\right\}.$$

Remark: *When $\beta = \eta = \alpha$, $\gamma = \delta = 2$ and $\nu = 1$, it follows from [6, Proposition 3.8] that*

$$\int_0^1 \frac{(1-\alpha x)\ \ln(1+\alpha x)}{(1+\alpha x^2)^2}\, dx = \frac{1}{4\sqrt{\alpha}}\ \arctan(\sqrt{\alpha})\ \ln(1+\alpha)$$
$$- \frac{1}{1+\alpha}\left\{\sqrt{\alpha}\ \arctan(\sqrt{\alpha}) + \alpha - \frac{5}{4}\ \ln(1+\alpha)\right\}.$$

Consequently, the corresponding series sum is given by

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i}\left[\frac{1}{i+1}\, {}_2F_1\!\left(2,\, \frac{i+1}{2};\, \frac{i+3}{2};\, -\alpha\right) - \frac{1}{i+2}\, {}_2F_1\!\left(2,\, \frac{i+2}{2};\, \frac{i+4}{2};\, -\alpha\right)\right]$$
$$= \frac{1}{4\sqrt{\alpha}}\ \arctan(\sqrt{\alpha})\ \ln(1+\alpha) - \frac{1}{1+\alpha}\left\{\sqrt{\alpha}\ \arctan(\sqrt{\alpha}) + \alpha - \frac{5}{4}\ \ln(1+\alpha)\right\}.$$

Theorem 3: *Let $n \in \mathbb{N}, \beta > 0, \gamma > 0, \delta > 0$ and $\alpha \in (0,1)$. Then, we have*

$$\int_0^1 \frac{\{\ln(1+\alpha x)\}^2}{(1+\beta x^{\gamma})^{\delta}}\, dx = \sum_{i=2}^{\infty} \frac{(-1)^{i+1}\alpha^i H_{i-1}}{i(i+1)}\, {}_2F_1\!\left(\delta,\, \frac{i+1}{\gamma};\, \frac{i+\gamma+1}{\gamma};\, -\beta\right).$$

Proof: Utilizing the series representation of $\{\ln(1+u)\}^2$ (see [3, Entry 1.516(1), p. 54]), we have

$$\{\ln(1+\alpha x)\}^2 = 2\sum_{i=2}^{\infty}(-1)^{i+1}\alpha^i \frac{H_{i-1}}{i}x^i.$$

Hence, exchanging the integral and sum signs by the dominated convergence theorem, we obtain

$$\int_0^1 \frac{\{\ln(1+\alpha x)\}^2}{(1+\beta x^{\gamma})^{\delta}}\,dx = 2\sum_{i=2}^{\infty}\frac{(-1)^{i+1}\alpha^i}{i}H_{i-1}\int_0^1 \frac{x^i}{(1+\beta x^{\gamma})^{\delta}}\,dx. \tag{2.3}$$

Changing the variable $x = t^{\frac{1}{\gamma}}$, we obtain

$$\int_0^1 \frac{\{\ln(1+\alpha x)\}^2}{(1+\beta x^{\gamma})^{\delta}}\,dx = 2\sum_{i=2}^{\infty}\frac{(-1)^{i+1}\alpha^i}{i\,\gamma}H_{i-1}\int_0^1 \frac{t^{\frac{i+1}{\gamma}-1}}{(1+\beta t)^{\delta}}\,dt. \tag{2.4}$$

From [3, Entry 3.194], the integral in (2.4) can be evaluated to yield

$$\begin{aligned}
&= 2\sum_{i=2}^{\infty}\frac{(-1)^{i+1}\alpha^i}{i\,\gamma}H_{i-1}\cdot\frac{\gamma}{i+1}\,{}_2F_1\!\left(\delta,\frac{i+1}{\gamma};\frac{i+\gamma+1}{\gamma};-\beta\right)\\
&= 2\sum_{i=2}^{\infty}\frac{(-1)^{i+1}\alpha^i H_{i-1}}{i(i+1)}\,{}_2F_1\!\left(\delta,\frac{i+1}{\gamma};\frac{i+\gamma+1}{\gamma};-\beta\right).
\end{aligned}$$

□

Theorem 4: *Let $n\in\mathbb{N}$, $\beta>0$, $\gamma>0$, $\delta>0$, $\nu>0$ and $\alpha\in(0,1)$. Then, we have*

$$\int_0^1 x^{\nu}\frac{\ln(1+\alpha x)}{(1+\beta x^{\gamma})^{\delta}}\,dx = \sum_{i=1}^{\infty}\frac{(-1)^{i+1}\alpha^i}{i(i+\nu+1)}\,{}_2F_1\!\left(\delta,\frac{i+\nu+1}{\gamma};\frac{i+\gamma+\nu+1}{\gamma};-\beta\right).$$

Proof: For $|\alpha x|<1$, it is well known that the power series expansion of $\ln(1+\alpha x)$ is given by

$$\ln(1+\alpha x) = \sum_{i=1}^{\infty}\frac{(-1)^{i+1}\alpha^i x^i}{i}.$$

Hence, exchanging the integral and sum signs by the dominated convergence theorem, we obtain

$$\int_0^1 x^{\nu}\frac{\ln(1+\alpha x)}{(1+\beta x^{\gamma})^{\delta}}\,dx = \sum_{i=1}^{\infty}\frac{(-1)^{i+1}\alpha^i}{i}\int_0^1 \frac{x^{i+\nu}}{(1+\beta x^{\gamma})^{\delta}}\,dx. \tag{2.5}$$

Changing the variable $x = t^{\frac{1}{\gamma}}$, we obtain

$$= \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i\,\gamma} \int_0^1 \frac{t^{\frac{i+\nu+1}{\gamma}-1}}{(1+\beta t)^{\delta}}\,dt. \tag{2.6}$$

From [3, Entry 3.194], the integral in (2.6) can be evaluated to yield

$$= \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i\,\gamma} \cdot \frac{\gamma}{i+\nu+1}\,{}_2F_1\left(\delta, \frac{i+\nu+1}{\gamma}; \frac{i+\gamma+\nu+1}{\gamma}; -\beta\right)$$

$$= \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i(i+\nu+1)}\,{}_2F_1\left(\delta, \frac{i+\nu+1}{\gamma}; \frac{i+\gamma+\nu+1}{\gamma}; -\beta\right).$$

□

Remark: *When $\beta = \alpha$ and $\gamma = \delta = \nu = 2$, it follows from [6, Proposition 3.3] that*

$$\int_0^1 x^2 \frac{\ln(1+\alpha x)}{(1+\alpha x^2)^2}\,dx$$
$$= \frac{1}{(1+\alpha)\sqrt{\alpha}}\left\{\frac{1}{2}\arctan(\sqrt{\alpha}) - \frac{3}{4\sqrt{\alpha}}\log(1+\alpha)\right\} + \frac{1}{4\alpha\sqrt{\alpha}}\arctan(\alpha)\log(1+\alpha).$$

Consequently, the corresponding series sum is given by

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i(i+3)}\,{}_2F_1\left(2, \frac{i+3}{2}; \frac{i+5}{2}; -\alpha\right)$$
$$= \frac{1}{(1+\alpha)\sqrt{\alpha}}\left\{\frac{1}{2}\arctan(\sqrt{\alpha}) - \frac{3}{4\sqrt{\alpha}}\log(1+\alpha)\right\} + \frac{1}{4\alpha\sqrt{\alpha}}\arctan(\alpha)\log(1+\alpha).$$

Remark: *When $\beta = \alpha$, $\nu = 1$ and $\gamma = \delta = 2$, it follows from [6, Proposition 3.5] that*

$$\int_0^1 x \frac{\ln(1+\alpha x)}{(1+\alpha x^2)^2}\,dx = \frac{1}{4\alpha(1+\alpha)}\left\{2\sqrt{\alpha}\arctan(\sqrt{\alpha}) + (\alpha-2)\log(1+\alpha)\right\}$$

Consequently, the corresponding series sum is given by

$$\sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i(i+2)}\,{}_2F_1\left(2, \frac{i+2}{2}; \frac{i+4}{2}; -\alpha\right) = \frac{1}{4\alpha(1+\alpha)}\left\{2\sqrt{\alpha}\arctan(\sqrt{\alpha}) + (\alpha-2)\log(1+\alpha)\right\}.$$

Theorem 5: *Let $\beta > 0$, $\gamma > 0$, $\delta > 0$, $\nu > 0$ and $\alpha \in (0,1)$. Then, we have*

$$\int_0^1 \frac{(1+\nu x)^2 \ln(1+\alpha x)}{(1+\beta x^\gamma)^\delta}\,dx = \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i}\Bigg[\frac{1}{i+1}\,{}_2F_1\!\left(\delta, \frac{i+1}{\gamma}; \frac{i+\gamma+1}{\gamma}; -\beta\right) + \frac{2\nu}{i+2}\,{}_2F_1\!\left(\delta, \frac{i+2}{\gamma}; \frac{i+\gamma+2}{\gamma}; -\beta\right) + \frac{\nu^2}{i+3}\,{}_2F_1\!\left(\delta, \frac{i+3}{\gamma}; \frac{i+\gamma+3}{\gamma}; -\beta\right)\Bigg].$$

Proof: Using the series expansion of $\ln(1+\alpha x)$ for $|\alpha x| < 1$, we have

$$\ln(1+\alpha x) = \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i x^i}{i}.$$

Hence, by the dominated convergence theorem, we can exchange the sum and integral:

$$\int_0^1 \frac{(1+\nu x)^2 \ln(1+\alpha x)}{(1+\beta x^\gamma)^\delta}\,dx = \sum_{i=1}^{\infty} \frac{(-1)^{i+1}\alpha^i}{i} \int_0^1 \frac{(1+\nu x)^2 x^i}{(1+\beta x^\gamma)^\delta}\,dx. \tag{2.7}$$

Expanding $(1+\nu x)^2 = 1 + 2\nu x + \nu^2 x^2$, we get

$$\int_0^1 \frac{(1+\nu x)^2 x^i}{(1+\beta x^\gamma)^\delta}\,dx = \int_0^1 \frac{x^i}{(1+\beta x^\gamma)^\delta}\,dx + 2\nu \int_0^1 \frac{x^{i+1}}{(1+\beta x^\gamma)^\delta}\,dx + \nu^2 \int_0^1 \frac{x^{i+2}}{(1+\beta x^\gamma)^\delta}\,dx.$$

Now, using the same change of variable $x = t^{1/\gamma}$ as in Theorem 4, and from [3, Entry 3.194], each integral becomes

$$\int_0^1 \frac{x^{i+k}}{(1+\beta x^\gamma)^\delta}\,dx = \frac{1}{\gamma}\int_0^1 \frac{t^{\frac{i+k+1}{\gamma}-1}}{(1+\beta t)^\delta}\,dt = \frac{1}{i+k+1}\,{}_2F_1\!\left(\delta, \frac{i+k+1}{\gamma}; \frac{i+k+\gamma+1}{\gamma}; -\beta\right),$$

where $k = 0, 1, 2$.

Substituting back into (2.7) yields the stated result. □

Remark: *When $\beta = \alpha = \nu$, and $\gamma = \delta = 2$, it follows from [6, Proposition 3.6] that*

$$\int_0^1 (1+\alpha x)^2 \frac{\ln(1+\alpha x)}{(1+\alpha x^2)^2}\,dx = \frac{1}{4}\left\{\frac{\alpha+1}{\sqrt{\alpha}}\arctan(\sqrt{\alpha})\log(1+\alpha) - \log(1+\alpha) + 2\sqrt{\alpha}\arctan(\sqrt{\alpha})\right\}$$

Consequently, the corresponding series sum is given by

$$\sum_{i=1}^{\infty}\frac{(-1)^{i+1}\alpha^i}{i}\left[\frac{1}{i+1}\,{}_2F_1\left(2,\frac{i+1}{2};\frac{i+3}{2};-\alpha\right)+\frac{2\alpha}{i+2}\,{}_2F_1\left(2,\frac{i+2}{2};\frac{i+4}{2};-\alpha\right)\right.$$
$$\left.+\frac{\alpha^2}{i+3}\,{}_2F_1\left(2,\frac{i+3}{2};\frac{i+5}{2};-\alpha\right)\right]$$
$$=\frac{1}{4}\left\{\frac{\alpha+1}{\sqrt{\alpha}}\arctan(\sqrt{\alpha})\log(1+\alpha)-\log(1+\alpha)+2\sqrt{\alpha}\arctan(\sqrt{\alpha})\right\}.$$

3 Conclusions

In this paper, we developed a unified framework for evaluating a broad class of logarithmic–power integrals involving rational functions. By employing power series expansions and the dominated convergence theorem, we derived general representations in terms of hypergeometric functions and harmonic numbers. Several special cases were obtained as direct consequences of the main results, recovering and extending recently established formulas in the literature. The derived identities also led to new infinite series representations, which may be of independent interest.

The results presented here provide a systematic approach to handle logarithmic integrals of this type and open avenues for further research, particularly in the study of higher-order logarithmic powers, extensions to more general functional forms, and applications in analytic number theory and special functions.

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