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Research article

On Lucas Product Cordial Labeling of Some Snake Graphs

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Abstract: An injective function $f : V(G) \to \{L_1, L_2, \ldots, L_n\}$, where L_j is the jth Lucas number, is called a Lucas product cordial labeling if the induced function satisfies $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$.

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1 Introduction

In this paper, the considered graphs are finite and undirected. The graph G consists of the set V of vertices and a collection E of unordered pairs. The concept of graph labeling is one of the growing fields in Graph theory. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to a few restrictions. The mid-1960s saw the first introduction of graph labelings [2]. Graph labeling are inspired by real-world issues, subject to certain restrictions. It is helpful as mathematical models for a variety of applications, including: coding theory, encompassing the creation of synch-set codes, missile guidance codes, excellent kinds codes , and optimum characteristics of auto correlation in convolutional codes [1]. In 1987, Cahit presented the idea of cordial labeling of graphs, while S.W. Golomb formalized the concept of numbering in graphs with each having a distinct function in graph theory and related fields. A graph is cordial if it is possible to label its vertices with 0's and 1's so that when the edges are labeled with the difference of the labels at their endpoints, the number of vertices labeled with ones and zeros differ at most by one. Researchers keep discovering additional variations of cordial labeling. See [3–5,7–9] for some of the variants of cordial labeling. Throughout this paper, we introduce a new concept of cordial labeling which is Lucas Product Cordial Labeling, inspired by the study of Salise and Pedrano [5] on Lucas Cordial Labeling.

2 Preliminaries

Definition 2.1 [6] The Lucas sequence of number can be defined as the linear recurrence relation satisfying the following condition:

$$L_1 = 1,\ L_2 = 3, \qquad \text{and} \qquad L_n = L_{(n-1)} + L_{(n-2)},\ n \geq 3.$$

This generates a sequence of integers in the following order: 1, 3, 4, 7, 11, 18, ….
Definition 2.2 [5] An injective function $f : V(G) \to \{L_1, L_2, \dots, L_n\}$, where L_j is the jth Lucas number ($j = 1, 2, \dots, n$) is said to be Lucas cordial labeling if the induced function $f^* : E(G) \to \{0, 1\}$ defined by $f^*(uv) = (f(u) + f(v)) \pmod 2$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. A graph which admits Lucas cordial labeling is called Lucas cordial graph.
Definition 2.3 An injective function $f : V(G) \to \{L_1, L_2, \dots, L_n\}$, where L_j is the jth Lucas number ($j = 1, 2, \dots, n$) is said to be Lucas product cordial labeling if the induced function $f^* : E(G) \to \{0, 1\}$ defined by $f^*(uv) = (f(u) \cdot f(v)) \pmod 2$ satisfies the condition $|e_f^*(0) - e_f^*(1)| \leq 1$, where $e_f^*(0)$ and $e_f^*(1)$ is the number of edges labeled with 0 and 1, respectively. A graph which admits Lucas product cordial labeling is called Lucas product cordial graph.

3 Lucas Product Cordial Graphs

This section presents the results of Lucas product cordial labeling of the quadrilateral snake graph Q_n, cycle quadrilateral snake graph CQ_n, alternate triangular snake graph $A(T_n)$ and double alternate quadrilateral snake graph $DA(QS_n)$.
Theorem 3.1 The Alternate Triangular Snake Graph $A(T_n)$ admits Lucas Product Cordial Labeling for all $n \geq 2$.

Proof: Let $A(T_n)$ be an Alternate Triangular Snake Graph where $n \geq 2$.

Case 1: Suppose that n is even.
Consider the Alternate Triangular Snake Graph $A(T_n)$. Observe that

$$|V(A(T_n))| = \frac{3n}{2} \quad \text{and} \quad |E(A(T_n))| = 2n - 1.$$

Let $f : V(A(T_n)) \to \left\{L_1, L_2, L_3, \dots, L_{\frac{3n}{2}}\right\}$ then the induced edge labels are

$$f^*(v_i v_{i+1}) = 1, \qquad 1 \leq i \leq n - 1$$
$$f^*(v_{2i-1} u_i) = 0, \qquad 1 \leq i \leq \frac{n}{2}$$
$$f^*(v_{2i} u_i) = 0, \qquad 1 \leq i \leq \frac{n}{2}$$

Notice that, $e_f^*(0) = n$ and $e_f^*(1) = n - 1$. Thus,

$$|e_f^*(0) - e_f^*(1)| = |n - (n-1)| = |1| = 1 \leq 1.$$

Therefore, if n is even, $n \geq 2$, the Alternate Triangular Snake Graph $A(T_n)$ is a Lucas Product Cordial Graph.

Case 2: Suppose that n is odd, then $n \geq 3$.
Consider the Alternate Triangular Snake Graph $A(T_n)$. Observe that

$$|V(A(T_n))| = \frac{3n-1}{2} \quad \text{and} \quad |E(A(T_n))| = 2n - 2.$$

Let $f : V(A(T_n)) \to \left\{L_1, L_2, L_3, \ldots, L_{\frac{3n-1}{2}}\right\}$ then the induced edge labels are

$$\begin{aligned} f^*(v_i v_{i+1}) &= 1, \quad 1 \leq i \leq n-1 \\ f^*(v_{2i-1} u_i) &= 0, \quad 1 \leq i \leq \frac{n-1}{2} \\ f^*(v_{2i} u_i) &= 0, \quad 1 \leq i \leq \frac{n-1}{2} \end{aligned}$$

Notice that, $e_f^*(0) = n - 1$ and $e_f^*(1) = n - 1$. Thus,

$$|e_f^*(0) - e_f^*(1)| = |(n-1) - (n-1)| = |0| = 0 \leq 1.$$

Therefore, if n is odd, $n \geq 3$, the Alternate Triangular Snake Graph $A(T_n)$ is a Lucas Product Cordial Graph.

Based on the cases above, we could say that the Alternate Triangular Snake Graph $A(T_n)$ is a Lucas Product Cordial Graph for all $n \geq 2$. □

Theorem 3.2 The Quadrilateral Snake Graph Q_n admits Lucas Product Cordial Labeling for all $n \geq 2$.

Proof: Consider the Quadrilateral Snake Graph Q_n. Observe that

$$|V(Q_n)| = 3n - 2 \quad \text{and} \quad |E(Q_n)| = 4n - 4.$$

Let $f : V(Q_n) \to \{L_1, L_2, L_3, \ldots, L_{3n-2}\}$ then the induced edge labels, where $1 \leq i \leq n-1$, are

$$\begin{aligned} f^*(v_i v_{i+1}) &= 1, \qquad f^*(v_i u_i) = 1, \\ f^*(v_{i+1} w_i) &= 0, \qquad f^*(u_i w_i) = 0. \end{aligned}$$

Notice that, $e_f^*(0) = 2n - 2$ and $e_f^*(1) = 2n - 2$. Thus,

$$|e_f^*(0) - e_f^*(1)| = |(2n-2) - (2n-2)| = |0| = 0 \leq 1.$$

Therefore, the Quadrilateral Snake Graph Q_n is a Lucas Product Cordial Graph for all $n \geq 2$. □

Theorem 3.3 The Cycle Quadrilateral Snake Graph CQ_n admits Lucas Product Cordial Labeling for all $n \geq 3$.

Proof: Consider the Cycle Quadrilateral Snake Graph CQ_n. Observe that

$$|V(CQ_n)| = 3n \text{ and } |E(CQ_n)| = 4n.$$

Let $f : V(CQ_n) \to \{L_1, L_2, L_3, \dots, L_{3n}\}$ then the induced edge labels are

$$\begin{aligned} f^*(v_iv_{i+1}) &= 1, && 1 \leq i \leq n-1 \\ f^*(v_1v_n) &= 1 \\ f^*(u_iw_i) &= 0, && 1 \leq i \leq n \\ f^*(v_iu_i) &= 1, && 1 \leq i \leq n \\ f^*(v_{i+1}w_i) &= 0, && 1 \leq i \leq n-1 \\ f^*(v_1w_n) &= 0 \end{aligned}$$

Notice that, $e_f^*(0) = 2n$ and $e_f^*(1) = 2n$. Thus,

$$|e_f^*(0) - e_f^*(1)| = |(2n) - (2n)| = |0| = 0 \leq 1.$$

Therefore, the Cycle Quadrilateral Snake Graph CQ_n is a Lucas Product Cordial Graph for all $n \geq 3$. □

4 Conclusions

This study has established that Lucas product cordial labeling can be successfully applied to selected classes of snake graphs, namely the quadrilateral snake, cycle quadrilateral snake, and alternate triangular snake graphs. By constructing explicit labeling schemes based on Lucas numbers, the authors demonstrated that these graph families satisfy the required balance condition between vertex and edge labels induced by the product rule. The results confirm that the structural regularity and repetitive patterns inherent in snake graphs play a crucial role in enabling such labelings.

Moreover, the work enriches the existing body of graph labeling theory by extending Lucas product cordial labeling to new graph classes that had not been previously explored under this framework. The findings highlight the flexibility of Lucas-based labelings and suggest that similar techniques may be adapted to other graph constructions derived from paths and cycles. Overall, this study not only contributes concrete results for specific snake graphs but also opens avenues for further investigation into Lucas product cordial labeling on broader families of graphs and other graph operations.

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References

1. J. L. Bennet and S. C. Kumar, A study on graph labeling problems, *UGC Care J. Dep. Math.* 40(60) **(2020)**.
2. J. A. Gallian, A dynamic survey of graph labeling, *Electron. J. Combin.* 27 **(2024)**, 1–712.
3. M. V. S. Gudin and A. C. Pedrano, Tribonacci cordial labeling of some snake graphs, *Ann. Commun. Math.* 8(3) **(2025)**, 393–405. https://doi.org/10.62072/acm.2025.080306.
4. A. C. Pedrano and R. F. Rulete, On the total product cordial labeling on the cartesian product of $P_m \times C_n$, $C_m \times C_n$ and the generalized Petersen graph $P(m, n)$, *Malaya J. Math.* 5(2) **(2017)**, 194–201.
5. E. R. Salise Jr. and A. C. Pedrano, On Lucas cordial labeling of some snake graphs, *Ann. Commun. Math.* 8(4) **(2025)**, 451–480. https://doi.org/10.62072/acm.2025.080403.
6. A. Sugumaran and K. Rajesh, Lucas divisor cordial labeling, *Int. J. Pure Appl. Math.* 113(6) **(2017)**, 233–241.
7. J. E. Sulayman and A. C. Pedrano, On Fibonacci cordial labeling of some snake graphs, *Eur. J. Math. Stat.* 4(2) **(2023)**, 29–35. https://doi.org/10.24018/ejmath.2023.4.2.193.
8. J. C. G. Valdehueza and A. C. Pedrano, On the total edge product cordial labeling of some corona graphs, *Int. J. Math. Trends Technol.* 67(10) **(2021)**, 1–6.
9. J. C. M. Vertudes and A. C. Pedrano, On topological cordial labeling of some graphs, *Eur. J. Math. Stat.* 6(4) **(2025)**, 1–7. https://doi.org/10.24018/ejmath.2025.6.4.404.