



Research article

Pendant Domination Polynomial of the Corona of a Wheel and an Independent Graph

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Abstract: Let W_M be the wheel graph on $M \geq 4$ vertices and let \overline{K}_n be the independent graph on $n \geq 1$ vertices. We study the corona product $W_M \circ \overline{K}_n$ and obtain an explicit formula for its pendant domination polynomial. The computation starts from the domination polynomial and subtracts a correction term that counts dominating sets whose induced subgraph contains no vertex of degree 1. For the wheel, the correction term reduces to counting subsets of the rim cycle for which the selected rim vertices are not isolated on the rim. We also determine the pendant domination number for this family.

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1 Introduction

Domination in graphs concerns vertex sets whose closed neighborhoods cover all vertices and has many variants and applications; see [1]. The domination polynomial encodes the number of dominating sets by size and has been studied for many graph operations, including corona products [2,3]; see also [4,5] for further results on domination polynomials. A refinement called the pendant domination polynomial counts dominating sets whose induced subgraph contains at least one pendant (degree-1) vertex, introduced and studied in [6].

The present work investigates the family arising from the classical corona operation [2]:

$$W_M \circ \overline{K}_n \quad (M \geq 4, n \geq 1),$$

obtained by attaching n leaves to each vertex of the wheel W_M .

Background and contribution. The domination polynomial of a corona graph is available in closed form via a standard identity [3]. The pendant domination polynomial, however, requires controlling when the



induced subgraph on a dominating set contains a degree-1 vertex [6]. For $W_M \circ \overline{K_n}$ we (i) isolate a general correction term for $G_0 \circ \overline{K_n}$ in terms of base-graph subsets whose induced subgraph has no degree-1 vertex, (ii) evaluate the resulting subset-count $a_k(W_M)$ explicitly by counting cyclic runs on the rim cycle, and (iii) determine $\gamma_{pe}(W_M \circ \overline{K_n})$.

2 Preliminaries and notation

All graphs are finite, simple, and undirected. For a graph G , let $V(G)$ and $E(G)$ denote its vertex and edge sets. For $S \subseteq V(G)$, the induced subgraph on S is $G[S]$. For $v \in V(G)$, write $N(v)$ for the open neighborhood and $N[v] = N(v) \cup \{v\}$ for the closed neighborhood.

Definition 1 (Dominating set [1]): *A set $S \subseteq V(G)$ is a dominating set if $N[S] = V(G)$. The domination number is $\gamma(G) = \min\{|S| : S \text{ is dominating}\}$.*

Definition 2 (Domination polynomial [3]): *Let $d(G, k)$ be the number of dominating sets of G of size k . The domination polynomial is*

$$D(G, x) = \sum_{k=\gamma(G)}^{|V(G)|} d(G, k) x^k.$$

Definition 3 (Pendant domination polynomial [6]): *A dominating set S is a pendant dominating set if $G[S]$ contains at least one vertex of degree 1. Let $d_{pe}(G, k)$ be the number of pendant dominating sets of size k . The pendant domination polynomial is*

$$D_{pe}(G, x) = \sum_{k=\gamma_{pe}(G)}^{|V(G)|} d_{pe}(G, k) x^k,$$

where $\gamma_{pe}(G)$ is the minimum size of a pendant dominating set.

Remark 1 (Pendant vertices in induced subgraphs). A pendant vertex in $G[S]$ means a vertex of degree 1 in the induced subgraph $G[S]$. Such a vertex need not be a leaf in the ambient graph G . This distinction is particularly relevant for corona constructions, where base vertices may become pendant in $G[S]$ depending on the choice of S .

2.1 Wheel graphs and the corona product

Let W_M denote the wheel on $M \geq 4$ vertices: a cycle C_{M-1} (the rim) plus a hub adjacent to every rim vertex (see, e.g., [7]). Let $\overline{K_n}$ be the independent graph on n vertices. Throughout, we write

$$\ell := M - 1$$

for the rim length of W_M .

Definition 4 (Corona product [2]): Let G and H be graphs. The corona $G \circ H$ is obtained by taking one copy of G and $|V(G)|$ disjoint copies of H , and joining the i -th vertex of G to every vertex in the i -th copy of H .

In this work, $H = \overline{K_n}$, so each base vertex receives n leaves.

3 Domination polynomial for the corona family

The following corona domination-polynomial identity is known.

Theorem 2 (Corona domination polynomial [3, Theorem 4]): Let G and H be nonempty graphs with $|V(G)| = N$ and $|V(H)| = m$. Then

$$D(G \circ H, x) = (x(1+x)^m + D(H, x))^N.$$

Corollary 3: For $M \geq 4$ and $n \geq 1$,

$$D(W_M \circ \overline{K_n}, x) = (x(1+x)^n + x^n)^M.$$

Proof: In $\overline{K_n}$ every vertex is isolated, so the only dominating set is $V(\overline{K_n})$; hence $D(\overline{K_n}, x) = x^n$. Apply Theorem 2 with $G = W_M$ and $H = \overline{K_n}$. \square

Remark 4. Corollary 3 is an immediate specialization of the corona identity. Accordingly, the main work is to isolate and evaluate the correction term needed to pass from $D(\cdot, x)$ to $D_{pe}(\cdot, x)$.

4 A correction-term framework for $D_{pe}(G_0 \circ \overline{K_n}, x)$

We first record a reduction that applies to any base graph G_0 . Let $G := G_0 \circ \overline{K_n}$ and write $B := V(G_0)$ for the base vertices. For each $u \in B$, let L_u be the set of its attached n leaves; thus $|L_u| = n$ and every $\ell \in L_u$ is adjacent only to u .

Lemma 5 (Leaf-selection condition): Let S be a dominating set of G . For each base vertex $u \in B$, either $u \in S$, or $u \notin S$ and then necessarily $L_u \subseteq S$.

Proof: If $u \notin S$, then any leaf $\ell \in L_u$ has only neighbor u , so ℓ can be dominated only by belonging to S . Thus all leaves in L_u must be selected. If $u \in S$, it dominates all vertices in L_u . \square

Lemma 6 (Size decomposition): Let S be a dominating set of G and let $T := S \cap B$ with $|T| = k$. For each $u \in T$, set $p_u := |S \cap L_u|$ and let $p := \sum_{u \in T} p_u$. Then

$$|S| = k + n(|B| - k) + p = n|B| - (n-1)k + p,$$

where $0 \leq p \leq nk$. Moreover, $p = 0$ if and only if no leaf adjacent to a selected base vertex is selected.

Proof: There are $|B| - k$ base vertices outside T , and by Lemma 5 each contributes all n of its leaves to S , contributing $n(|B| - k)$ vertices. The remaining contribution consists of the k selected base vertices plus the p selected leaves in $\bigcup_{u \in T} L_u$. \square

It is convenient to isolate those dominating sets whose induced subgraph contains no pendant vertex.

Definition 5: For a graph G_0 and $0 \leq k \leq |V(G_0)|$, let $a_k(G_0)$ be the number of k -subsets $T \subseteq V(G_0)$ such that the induced subgraph $G_0[T]$ has no vertex of degree 1.

Lemma 7 (No-pendant dominating sets in $G_0 \circ \overline{K_n}$): Let $G := G_0 \circ \overline{K_n}$ and let $|V(G_0)| = N$. The generating function of dominating sets S for which the induced subgraph $G[S]$ contains no vertex of degree 1 is

$$\sum_{k=0}^N a_k(G_0) x^{nN - (n-1)k}.$$

Proof: Fix a k -subset $T \subseteq V(G_0)$ such that $G_0[T]$ has no vertex of degree 1. Define S by selecting exactly T on the base and, for every $u \in V(G_0) \setminus T$, selecting all leaves L_u . Also select no leaves adjacent to T . Then S is dominating by Lemma 5.

In the induced subgraph $G[S]$, every selected leaf in S is adjacent only to an *unselected* base vertex, hence is isolated in $G[S]$ and has degree 0. The induced subgraph on selected base vertices is exactly $G_0[T]$, which has no degree-1 vertex by assumption. Thus $G[S]$ contains no vertex of degree 1. Moreover, by Lemma 6 we have $p = 0$ in this construction, hence $|S| = nN - (n-1)k$.

Conversely, let S be a dominating set such that $G[S]$ has no vertex of degree 1. If $u \in T := S \cap V(G_0)$ and $\ell \in L_u \cap S$, then ℓ has degree 1 in $G[S]$ (its only neighbor is $u \in S$), a contradiction. Hence no leaf adjacent to T can be selected, so the parameter p in Lemma 6 must equal 0. Then Lemma 5 forces $L_u \subseteq S$ for every $u \notin T$, so S is uniquely reconstructed from T . Finally, $G[S]$ has no degree-1 vertex if and only if $G_0[T]$ has no degree-1 vertex. Summing over all such T yields the stated generating function. \square

Corollary 8 (Key identity): Let $G := G_0 \circ \overline{K_n}$ with $|V(G_0)| = N$. Then

$$D_{pe}(G, x) = D(G, x) - \sum_{k=0}^N a_k(G_0) x^{nN - (n-1)k}.$$

Remark 9. Corollary 8 reduces the computation of $D_{pe}(G_0 \circ \overline{K_n}, x)$ to the evaluation of the quantities $a_k(G_0)$ from Definition 5. Henceforth we take $G_0 = W_M$ and compute $a_k(W_M)$ explicitly via enumerations on the rim cycle.

5 Counting $a_k(W_M)$ via the rim cycle

Let the hub of W_M be c and let the rim be the cycle C_ℓ where $\ell = M - 1$. Any subset $T \subseteq V(W_M)$ either contains c or does not. Write

$$a_k(W_M) = a_k^{(0)} + a_k^{(1)},$$

where $a_k^{(0)}$ counts valid T of size k with $c \notin T$, and $a_k^{(1)}$ counts valid T of size k with $c \in T$.

5.1 Case $c \notin T$: independent sets on the rim or the whole rim

If $c \notin T$, then $T \subseteq V(C_\ell)$ and the induced subgraph is $C_\ell[T]$.

Lemma 10: *Let $T \subseteq V(C_\ell)$. The induced subgraph $C_\ell[T]$ has no vertex of degree 1 if and only if either (i) T is an independent set, or (ii) $T = V(C_\ell)$.*

Proof: Every component of $C_\ell[T]$ is a path (possibly a single vertex) or the whole cycle. Any path component on at least two vertices has endpoints of degree 1, which is forbidden. Hence all components are isolated vertices (so T is independent) or else T is the full cycle. \square

Let $I_k(C_\ell)$ be the number of independent sets of size k in C_ℓ .

Lemma 11 (Independent k -sets in a cycle): *For $\ell \geq 3$,*

$$I_0(C_\ell) = 1, \quad I_k(C_\ell) = \frac{\ell}{k} \binom{\ell - k - 1}{k - 1} \quad (1 \leq k \leq \lfloor \ell/2 \rfloor),$$

and $I_k(C_\ell) = 0$ otherwise.

Proof: Fix $k \geq 1$ and count independent k -subsets of a labeled cycle (cf. [8]). Choose a distinguished selected vertex v and break the cycle at v ; the remaining $\ell - 1$ vertices form a path. To preserve independence, the two neighbors of v cannot be chosen, leaving a path of $\ell - 3$ eligible vertices on which we must choose $k - 1$ vertices with no adjacency. The number of $(k - 1)$ -subsets with no adjacency on a path of length L equals $\binom{L - (k - 1) + 1}{k - 1}$, hence here it is $\binom{\ell - k - 1}{k - 1}$. Thus counting pairs (independent set T , distinguished vertex $v \in T$) gives

$$k I_k(C_\ell) = \ell \binom{\ell - k - 1}{k - 1},$$

and the stated formula follows. \square

Therefore,

$$a_k^{(0)} = \begin{cases} I_k(C_\ell), & 0 \leq k \leq \lfloor \ell/2 \rfloor, \\ 1, & k = \ell, \\ 0, & \text{otherwise.} \end{cases} \quad (5.1)$$

5.2 Case $c \in T$: rim subsets with no isolated selected rim vertex

Now suppose $c \in T$ and set $R := T \cap V(C_\ell)$, so $|R| = r = k - 1$. In $W_M[T]$, each rim vertex in R is adjacent to c , and also to its rim neighbors that lie in R . Hence a rim vertex in R has degree 1 in $W_M[T]$ exactly when it has *no* selected rim neighbor in R . Thus $W_M[T]$ has no degree-1 vertex if and only if every selected rim vertex has at least one selected rim neighbor.

Let $J_r(C_\ell)$ be the number of r -subsets $R \subseteq V(C_\ell)$ such that every vertex in R has a neighbor in R along the cycle (equivalently, in the cyclic 0–1 indicator string, every 1 belongs to a block of 1's of length at least 2).

Theorem 12 (Cycle subsets with no isolated selected vertices): *Let $\ell \geq 3$. For $0 \leq r \leq \ell$,*

$$J_0(C_\ell) = J_\ell(C_\ell) = 1, \quad J_1(C_\ell) = 0,$$

and for $2 \leq r \leq \ell - 1$ with $z := \ell - r$,

$$J_r(C_\ell) = \sum_{t=1}^{\lfloor r/2 \rfloor} \frac{\ell}{t} \binom{r-t-1}{t-1} \binom{z-1}{t-1},$$

where t is the number of cyclic blocks of consecutive selected vertices.

Proof: Represent R as a cyclic binary word of length ℓ with r ones. The constraint says every run of ones has length at least 2. Fix $t \geq 1$ as the number of runs (blocks) of ones. Necessarily $2t \leq r$ and also $t \leq z$ (there must be at least one 0 between consecutive blocks).

Let the block lengths be (a_1, \dots, a_t) with $a_i \geq 2$ and $\sum a_i = r$. The number of ordered solutions is $\binom{r-t-1}{t-1}$ (set $a'_i = a_i - 2 \geq 0$ and use stars-and-bars). Let the zero-gap lengths between successive blocks be (b_1, \dots, b_t) with $b_i \geq 1$ and $\sum b_i = z$. The number of ordered solutions is $\binom{z-1}{t-1}$.

Given ordered (a_i) and (b_i) , choosing a start vertex on the labeled cycle yields a unique labeled subset, giving a factor ℓ . Each labeled subset is counted exactly t times, corresponding to which of its t blocks is declared the first block in the cyclic order; hence we divide by t . Summing over feasible t yields the formula. \square

Thus,

$$a_k^{(1)} = J_{k-1}(C_\ell) \quad (0 \leq k \leq M), \quad (5.2)$$

with the convention $J_{-1}(C_\ell) = 0$.

Proposition 13 (Explicit formula for $a_k(W_M)$): Let $M \geq 4$ and $\ell = M - 1$. For $0 \leq k \leq M$,

$$a_k(W_M) = a_k^{(0)} + a_k^{(1)},$$

where $a_k^{(0)}$ is given by (5.1) and $a_k^{(1)}$ is given by (5.2) together with Theorem 12.

Corollary 14: Let $M \geq 4$ and $\ell = M - 1$. For $0 \leq k \leq M$,

$$a_k(W_M) = I_k(C_\ell) + \mathbf{1}_{\{k=\ell\}} + J_{k-1}(C_\ell),$$

where $I_k(C_\ell)$ denotes the number of independent k -subsets of C_ℓ (in particular, $I_\ell(C_\ell) = 0$), and the indicator term $\mathbf{1}_{\{k=\ell\}}$ accounts for the full-rim subset $T = V(C_\ell)$ in (5.1); moreover, $J_r(C_\ell)$ is given by Theorem 12 and $J_{-1}(C_\ell) = 0$.

6 Main result: pendant domination polynomial

Combine Corollary 3, Corollary 8 (with $G_0 = W_M$ and $N = M$), and Proposition 13.

Theorem 15 (Pendant domination polynomial of $W_M \circ \overline{K_n}$): Let $M \geq 4$ and $n \geq 1$, and set $\ell = M - 1$. Then

$$D_{pe}(W_M \circ \overline{K_n}, x) = (x(1+x)^n + x^n)^M - \sum_{k=0}^M a_k(W_M) x^{nM-(n-1)k},$$

where $a_k(W_M)$ is as in Definition 5 and is evaluated in Proposition 13.

Corollary 16 (Expanded form): Let $M \geq 4$ and $n \geq 1$, and set $\ell := M - 1$. Then

$$D_{pe}(W_M \circ \overline{K_n}, x) = (x(1+x)^n + x^n)^M - \sum_{k=0}^M \left(I_k(C_\ell) + \mathbf{1}_{\{k=\ell\}} + J_{k-1}(C_\ell) \right) x^{nM-(n-1)k},$$

where $\mathbf{1}_{\{k=\ell\}}$ equals 1 if $k = \ell$ and 0 otherwise, and the quantities $I_k(C_\ell)$ and $J_r(C_\ell)$ are given explicitly as follows.

(i) Independent k -subsets of the cycle. For $\ell \geq 3$,

$$I_0(C_\ell) = 1, \quad I_k(C_\ell) = \frac{\ell}{k} \binom{\ell-k-1}{k-1} \quad (1 \leq k \leq \lfloor \ell/2 \rfloor), \quad I_k(C_\ell) = 0 \text{ otherwise.}$$

(ii) Cycle r -subsets with no isolated selected vertex. Set $J_{-1}(C_\ell) := 0$. For $0 \leq r \leq \ell$,

$$J_0(C_\ell) = J_\ell(C_\ell) = 1, \quad J_1(C_\ell) = 0,$$

and for $2 \leq r \leq \ell - 1$ (writing $z := \ell - r$),

$$J_r(C_\ell) = \sum_{t=1}^{\lfloor r/2 \rfloor} \ell \binom{r-t-1}{t-1} \binom{z-1}{t-1}.$$

7 Pendant domination number

The pendant domination number is determined by a case distinction at $n = 1$.

Corollary 17: *Let $M \geq 4$ and $n \geq 1$. Then*

$$\gamma_{pe}(W_M \circ \overline{K_n}) = \begin{cases} M, & n = 1, \\ M + 1, & n \geq 2. \end{cases}$$

Proof: Let $G := W_M \circ \overline{K_n}$, and for each base vertex $u \in V(W_M)$ let L_u be its attached set of n leaves.

Lower bound. To dominate the vertices of L_u , a dominating set S must satisfy either $u \in S$ or $L_u \subseteq S$ (since each leaf has only neighbor u). Thus each base vertex forces at least one selected vertex, so $|S| \geq M$ for any dominating set S .

Case $n = 1$. We construct a pendant dominating set of size M . Let c be the hub, pick exactly one rim vertex v , and for every other base vertex $u \neq c, v$ include its unique leaf. Then $|S| = M$, and S dominates G by Lemma 5. In the induced subgraph $G[S]$, the hub c is adjacent to v and to no other selected base vertex, hence $\deg_{G[S]}(c) = 1$. Therefore S is pendant dominating and $\gamma_{pe}(G) = M$.

Case $n \geq 2$. Assume S is dominating with $|S| = M$. If some base vertex $u \notin S$, then domination of L_u forces $L_u \subseteq S$, contributing $n \geq 2$ vertices. Even if all other base vertices contribute the minimum 1, we get $|S| \geq (M - 1) + n \geq M + 1$, contradicting $|S| = M$. Hence every dominating set of size M must contain all base vertices and no leaves, so $G[S] \cong W_M$. Since W_M has minimum degree 3 for $M \geq 4$, $G[S]$ contains no pendant vertex, so no pendant dominating set has size M . Thus $\gamma_{pe}(G) \geq M + 1$.

Finally, $V(W_M) \cup \{\lambda\}$ (where λ is any leaf of G) is a pendant dominating set of size $M + 1$. Hence $\gamma_{pe}(G) = M + 1$. \square

8 Conclusions

We obtained an explicit formula for the pendant domination polynomial of $W_M \circ \overline{K_n}$. The domination polynomial component follows from the corona identity in Theorem 2, while the pendant restriction is captured by a correction term governed by the quantities $a_k(W_M)$. These quantities were evaluated via enumerations on the rim cycle, yielding Theorem 15 and the expanded form in Corollary 16. We also determined the pendant domination number for this family. Future work includes extending the correction-term method to $G_0 \circ \overline{K_n}$ for additional base graphs G_0 and studying algebraic properties (e.g., roots or coefficient patterns) of the resulting pendant domination polynomials.

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