Annals of Communications in Mathematics
Volume 9, Number 1 (2026), 3
DOI: 10.62072/acm.2026.09003
ISSN: 2582-0818
https://www.technoskypub.com/journal/acm/

Research article

On Edouard Product Cordial Labeling of Some Graphs

Jan Carl M. Vertudes

Department of Mathematics and Statistics, College of Arts and Sciences, University of Southeastern Philippines, Davao City, 8000, Philippines.

*Corresponding Author: Jan Carl M. Vertudes. Email: jancarlvertudes2002@gmail.com

Received: 01 January 2025; Accepted: 14 February 2026; Published: 31 March 2026

Citation: J. C. M. Vertudes, On Edouard Product Cordial Labeling of Some Graphs. Ann. Commun. Math. 9 **(2026)**, 3. https://doi.org/10.62072/acm.2026.09003

Abstract: Let G be a graph. An Edouard Product Cordial Labeling (EPCL) of a graph G with $|V(G)| = n$ is an injective function $f : V(G) \to \{E_0, E_1, E_2, \dots, E_{n-1}\}$ where E_i is the ith Edouard number $(i = 0, 1, 2, 3, \dots, n)$ that induces a function f^* defined by

$$f^*(uv) = (f(u)f(v)) \pmod 2$$

for all edge $e = uv$ such that $|e_f^*(0) - e_f^*(1)| \leq 1$ where $e_f^*(0)$ is the number of vertices labeled with 0 and $e_f^*(1)$ is the number of vertices labeled with 1. The graph that satisfies the condition of an edouard product cordial labeling is called an edouard product cordial graph (EPCG).

Mathematics Subject Classification: 05C75,05C78.
Keywords: Edouard Number,; Edouard Product Cordial Labeling; Cordial Labeling.

1 Introduction

Euler's Konigsberg Bridge Problem is considered as the birth of graph theory. Graph theory is a branch of discrete mathematics with deals with the study of graphs and its characteristics. Graph Theory is considered as the intensively researched area of mathematics. Graph Theory has different area of topic such as graph domination, labeling, mathing, coloring, etc. Graph labeling refers to an assignment of integers (or sometimes other symbols) to the vertices, edges, or both, of a graph under specific conditions. The concept was first introduced by Rosa in 1967 through the study of graceful labeling, which initiated a wide range of labeling schemes such as magic[11], harmonious [2], cordial [4], prime [3], and divisor labelings [6]. Numerous types of labeling being discovered, codial labeling is one of them. Cordial Labeling is a graph labeling in which the possible choices to be labeled is either 0 or 1 of the edges and vertices of a certain graph. Cordial Labeling comes in many ways depending to the condition that must be considered. This concept was introduced by Cahit in 1987 as a weaker version of harmonious and graceful graphs [2]. Several cordial labeling exist, see [1,5,7,9,10].

As a reference of existing variants of cordial labeling, this study explore its new concept using the other types of number sequence which is Edouard number.

2 Basic Concepts

Definition 1: *A star graph $S_n = K_1 + \overline{K_n}$ is a tree on n nodes with one node having vertex degree $n-1$ and the other $n-1$ having vertex degree* 1.

Definition 2: *[8]*A generalized Edouard sequence $\{E_n\}_{n\geq 0} = \{E_n(E_0, E_1, E_2)\}_{n\geq 0}$ is defined by the third-order recurrence relation

$$W_n = 7E_{n-1} - 7E_{n-2} + E_{n-3}$$

with the initial values E_0, E_1, E_2 not all being zero. If we take $E_0 = 0, E_1 = 1, E_2 = 7$, then $\{E_n\}$ is the Edouard sequence.

Definition 3: Let G be a graph. An Edouard Product Cordial Labeling (EPCL) of a graph G with $|V(G)| = n$ is an injective function $f : V(G) \rightarrow \{E_0, E_1, E_2, \dots, E_{n-1}\}$ where E_i is the ith Edouard number ($i = 0, 1, 2, 3, \dots, n$) that induces a function f^* defined by

$$f^*(uv) = (f(u)f(v)) \pmod 2$$

for all edge $e = uv$ such that $|e_f^*(0) - e_f^*(1)| \leq 1$ where $e_f^*(0)$ is the number of vertices labeled with 0 and $e_f^*(1)$ is the number of vertices labeled with 1. The graph that satisfies the condition of an Edouard product cordial labeling is called an Edouard product cordial graph (EPCG).

3 Results and Discussion

Theorem 1: *Let E_n be the nth term of the Edouard sequence. Then for every $n \in \mathbb{N}$,*

$$E_n = \begin{cases} odd, & if\ n \equiv 1 \pmod 4 \ \ or \ \ n \equiv 2 \pmod 4 \\ even, & if\ n \equiv 0 \pmod 4 \ \ or \ \ n \equiv 3 \pmod 4 \end{cases}.$$

Proof: Let E_n be the nth term of the Edouard sequence. To prove this, let us separate into cases.
Case 1: $n \equiv 1 \pmod 4$

Assume that E_k is odd whenever $k \equiv 1 \pmod 4$ for some $k \in \mathbb{W}$. We need to show that the assumption holds up to E_{k+4} where $k + 4 \equiv 1 \pmod 4$. Now,

$$\begin{aligned} E_{k+4} &= E_{k+3} + E_{k+2} + E_{k+1} \\ &= (7E_{k+2} - 7E_{k+1} + E_k) + E_{k+2} + E_{k+1} \\ &= 8E_{k+2} + 6E_{k+1} + E_k \end{aligned}$$

By inductive hypothesis, E_k is odd. Also, $8E_{k+2}$ and $6E_{k+1}$ are both even. Implying that E_{k+4} is odd whenever $n \equiv 1 \pmod 4$.

Case 2: $n \equiv 2 \pmod 4$

Assume that E_k is odd whenever $k \equiv 2 \pmod 4$ for some $k \in \mathbb{W}$. We need to show that the assumption holds up to E_{k+4} where $k + 4 \equiv 2 \pmod 4$. Now,

$$\begin{aligned} E_{k+4} &= E_{k+3} + E_{k+2} + E_{k+1} \\ &= (7E_{k+2} - 7E_{k+1} + E_k) + E_{k+2} + E_{k+1} \\ &= 8E_{k+2} + 6E_{k+1} + E_k \end{aligned}$$

By inductive hypothesis, E_k is odd. Also, $8E_{k+2}$ and $6E_{k+1}$ are both even. Implying that E_{k+4} is odd whenever $n \equiv 0 \pmod 4$.

Case 3: $n \equiv 0 \pmod 4$.

Assume that E_k is even whenever $k \equiv 0 \pmod 4$ for some $k \in \mathbb{W}$. We need to show that the assumption holds up to E_{k+4} where $k + 4 \equiv 0 \pmod 4$. Now,

$$\begin{aligned} E_{k+4} &= E_{k+3} + E_{k+2} + E_{k+1} \\ &= (7E_{k+2} - 7E_{k+1} + E_k) + E_{k+2} + E_{k+1} \\ &= 8E_{k+2} + 6E_{k+1} + E_k \end{aligned}$$

By inductive hypothesis, E_k is even. Also, $8E_{k+2}$ and $6E_{k+1}$ are both even. Implying that E_{k+4} is even whenever $n \equiv 0 \pmod 4$.

Case 4: $n \equiv 3 \pmod 4$.

Assume that E_k is even whenever $k \equiv 3 \pmod 4$ for some $k \in \mathbb{W}$. We need to show that the assumption holds up to E_{k+4} where $k + 4 \equiv 3 \pmod 4$. Now,

$$\begin{aligned} E_{k+4} &= E_{k+3} + E_{k+2} + E_{k+1} \\ &= (7E_{k+2} - 7E_{k+1} + E_k) + E_{k+2} + E_{k+1} \\ &= 8E_{k+2} + 6E_{k+1} + E_k \end{aligned}$$

By inductive hypothesis, E_k is even. Also, $8E_{k+2}$ and $6E_{k+1}$ are both even. Implying that E_{k+4} is even whenever $n \equiv 3 \pmod 4$.

By Principle of Mathematical Induction, we show that E_n is odd whenever $n \equiv 1 \pmod 4$ or $n \equiv 2 \pmod 4$ and E_n is even whenever $n \equiv 0 \pmod 4$ or $n \equiv 3 \pmod 4$. $\square$

Theorem 2: *The Path Graph P_n is an Edouard Cordial Graph except when $n = 4k + 1$ for $n \in \mathbb{N}$.*

Proof: Suppose $V(P_n) = \{v_i : 1 \le i \le n\}$ and $E(P_n) = \{v_i v_{i+1} : 1 \le i \le n-1\}$. The order and size of the Path Graph are $|V(P_n)| = n$ and $|E(P_n)| = n - 1$, respectively. To prove this theorem, consider the following cases.

Case 1: $n = 4k$ for some $k \in \mathbb{N}$.

Define the function $f : V(P_n) \to \{E_0, E_1, \dots, E_{n-1}\}$ by

$$\begin{aligned} f(v_{2i-1}) &= E_{4i-3}, & 1 \le i \le \frac{n}{4} \\ f(v_{2i}) &= E_{4i-2}, & 1 \le i \le \frac{n}{4} \\ f(v_{(2i-1+\frac{n}{2})}) &= E_{4i-4}, & 1 \le i \le \frac{n}{4} \\ f(v_{(2i+\frac{n}{2})}) &= E_{4i-1}, & 1 \le i \le \frac{n}{4} \end{aligned}$$

Then the induced edge labels are

$$\begin{aligned} f^*(v_{2i-1}v_{2i}) &= 1, & 1 \le i \le \frac{n}{4} \\ f^*(v_{2i}v_{2i+1}) &= 1, & 1 \le i \le \frac{n-4}{4} \\ f^*(v_{(\frac{n}{4}+2i+1)}v_{(\frac{n}{4}+2i+2)}) &= 0, & 1 \le i \le \frac{n}{4} \\ f^*(v_{(\frac{n}{4}+2i+2)}v_{(\frac{n}{4}+2i+3)}) &= 0, & 1 \le i \le \frac{n}{4} \end{aligned}$$

Observe that $|e_f(0) - e_f(1)| = |(\frac{n}{4} + \frac{n}{4}) - (\frac{n}{4} + \frac{n-4}{4})| = |\frac{n}{2} - (\frac{n-2}{2})| = 1 \le 1$. Hence, the Path Graph P_n is an Edouard Product Cordial Graph when $n = 4k$ for some $k \in \mathbb{N}$.

Case 2: $n = 4k + 2$ for some $k \in \mathbb{N}$.

Define the function $f : V(P_n) \to \{E_0, E_1, \dots, E_{n-1}\}$ by

$$\begin{aligned} f(v_{2i-1}) &= E_{4i-3}, & 1 \le i \le \frac{n+2}{4} \\ f(v_{2i}) &= E_{4i-2}, & 1 \le i \le \frac{n-2}{4} \\ f(v_{(2i-1+\frac{n}{2})}) &= E_{4i-4}, & 1 \le i \le \frac{n+2}{4} \\ f(v_{(2i+\frac{n}{2})}) &= E_{4i-1}, & 1 \le i \le \frac{n-2}{4} \end{aligned}$$

Then the induced edge labels are

$$f^*(v_{2i-1}v_{2i}) = 1, \qquad 1 \le i \le \frac{n-2}{4}$$
$$f^*(v_{2i}v_{2i+1}) = 1, \qquad 1 \le i \le \frac{n-2}{4}$$
$$f^*(v_{(\frac{n+2}{4}+2i)}v_{(\frac{n+2}{4}+2i+1)}) = 0, \qquad 1 \le i \le \frac{n+2}{4}$$
$$f^*(v_{(\frac{n+2}{4}+2i+1)}v_{(\frac{n+2}{4}+2i+2)}) = 0, \qquad 1 \le i \le \frac{n-2}{4}$$

Observe that $|e_f(0) - e_f(1)| = |(\frac{n+2}{4} + \frac{n-2}{4}) - (\frac{n-2}{4} + \frac{n-2}{4})| = |\frac{n}{2} - (\frac{n-2}{2})| = 1 \le 1$. Hence, the Path Graph P_n is an Edouard Product Cordial Graph when $n = 4k + 2$ for some $k \in \mathbb{N}$.

Case 3: $n = 4k + 3$ for some $k \in \mathbb{N}$.

Define the function $f : V(P_n) \to \{E_0, E_1, \dots, E_{n-1}\}$ by

$$f(v_{2i-1}) = E_{4i-3}, \qquad 1 \le i \le \frac{n+1}{4}$$
$$f(v_{2i}) = E_{4i-2}, \qquad 1 \le i \le \frac{n+1}{4}$$
$$f(v_{(2i-1+\frac{n+1}{2})}) = E_{4i-4}, \qquad 1 \le i \le \frac{n+1}{4}$$
$$f(v_{(2i+\frac{n+1}{2})}) = E_{4i-1}, \qquad 1 \le i \le \frac{n-3}{4}$$

Then the induced edge labels are

$$f^*(v_{2i-1}v_{2i}) = 1, \qquad 1 \le i \le \frac{n+1}{4}$$
$$f^*(v_{2i}v_{2i+1}) = 1, \qquad 1 \le i \le \frac{n-3}{4}$$
$$f^*(v_{(\frac{n+1}{4}+2i+1)}v_{(\frac{n+1}{4}+2i+2)}) = 0, \qquad 1 \le i \le \frac{n+1}{4}$$
$$f^*(v_{(\frac{n+1}{4}+2i+2)}v_{(\frac{n+1}{4}+2i+3)}) = 0, \qquad 1 \le i \le \frac{n-3}{4}$$

Observe that $|e_f(0) - e_f(1)| = |(\frac{n+1}{4} + \frac{n-3}{4}) - (\frac{n+1}{4} + \frac{n-3}{4})| = |\frac{n}{2} - (\frac{n}{2})| = 0 \le 1$. Hence, the Path Graph P_n is an Edouard Product Cordial Graph when $n = 4k + 3$ for some $k \in \mathbb{N}$.

Considering the cases above, this proves that the Path Graph P_n is an Edouard Product Cordial Graph except when $n = 4k + 1$ for $k \in \mathbb{N}$. □

Theorem 3: *The Cycle Graph $C_n, n \ge 3$, is an Edouard Cordial Graph when $n = 4k + 3$ for $n \in \mathbb{N}$.*

Proof: Suppose the vertex set $V(C_n) = \{v_i : 1 \leq i \leq n\}$ and the edge set $E(C_n) = \{v_i v_{i+1}, v_1 v_{n+1} : 1 \leq i \leq n-1\}$. The order and size of are $|V(C_n)| = n$ and $|E(C_n)| = n$, respectively. Define the function $f : V(C_n) \to \{E_0, E_1, \dots, E_{n-1}\}$ by

$$\begin{aligned} f(v_{2i-1}) &= E_{4i-3}, && 1 \leq i \leq \frac{n+1}{4} \\ f(v_{2i}) &= E_{4i-2}, && 1 \leq i \leq \frac{n+1}{4} \\ f(v_{(2i-1+\frac{n+1}{2})}) &= E_{4i-4}, && 1 \leq i \leq \frac{n+1}{4} \\ f(v_{(2i+\frac{n+1}{2})}) &= E_{4i-1}, && 1 \leq i \leq \frac{n-3}{4} \end{aligned}$$

Then the induced edge labels are

$$\begin{aligned} f^*(v_1, v_n) &= 0, \\ f^*(v_{2i-1} v_{2i}) &= 1, && 1 \leq i \leq \frac{n+1}{4} \\ f^*(v_{2i} v_{2i+1}) &= 1, && 1 \leq i \leq \frac{n-3}{4} \\ f^*(v_{(\frac{n+1}{4}+2i+1)} v_{(\frac{n+1}{4}+2i+2)}) &= 0, && 1 \leq i \leq \frac{n+1}{4} \\ f^*(v_{(\frac{n+1}{4}+2i+2)} v_{(\frac{n+1}{4}+2i+3)}) &= 0, && 1 \leq i \leq \frac{n-3}{4} \end{aligned}$$

Observe that $|e_f(0) - e_f(1)| = |(\frac{n+1}{4} + \frac{n-3}{4} + 1) - (\frac{n+1}{4} + \frac{n-3}{4})| = |\frac{n+2}{2} - (\frac{n}{2})| = 1 \leq 1$. Hence, the Cycle Graph C_n is an Edouard Product Cordial Graph when $n = 4k+3$ for some $k \in \mathbb{N}$. □

Theorem 4: *The Star Graph* $S_n \cong K_1 + \overline{K}_n$ *is an Edouard Product Cordial Graph except when* $n = 4k+1$ *for* $k \in \mathbb{W}$.

Proof: Suppose $V(S_n) = \{u_1, v_i : 1 \leq i \leq n\}$ and $E(S_n) = \{u_1 v_i, : 1 \leq i \leq n\}$ be the vertex set and edge set of Star Graph. The order and size are $|V(S_n)| = n+1$ and $|E(S_n)| = n$, respectively. To prove this theorem, consider the following cases.

Case 1: $n = 4k$ for some $k \in \mathbb{W}$.

Define the function $f : V(S_n) \to \{E_0, E_1, \dots, E_n\}$ by

$$f(u_1) = E_1, \qquad f(v_1) = E_0, \qquad f(v_i) = E_i, \quad 2 \leq i \leq n$$

Then the induced edge labels are

$$f^*(u_1v_1) = 1,$$
$$f^*(u_1v_2) = 0,$$
$$f^*(u_1v_{4j-1}) = 0, \qquad 1 \le j \le \frac{n}{4}$$
$$f^*(u_1v_{4j}) = 0, \qquad 1 \le j \le \frac{n-4}{4}$$
$$f^*(u_1v_{4j+1}) = 1, \qquad 1 \le j \le \frac{n-4}{4}$$
$$f^*(u_1v_{4j+2}) = 1, \qquad 1 \le j \le \frac{n-4}{4}$$

Observe that $|e_f(0) - e_f(1)| = |(\frac{n}{4} + \frac{n-4}{4} + 1) - (\frac{n-4}{4} + \frac{n-4}{4} + 1)| = |(\frac{n-2}{2} + 1) - (\frac{n-4}{2} + 1)| = 1 \le 1$. Hence, the Star Graph S_n is an Edouard Product Cordial Graph when $n = 4k$ for some $k \in \mathbb{N}$.
Case 2: $n = 4k + 2$ for some $k \in \mathbb{W}$.
Define the function $f : V(S_n) \to \{E_0, E_1, \dots, E_n\}$ by

$$f(u_1) = E_1, \qquad f(v_1) = E_0, \qquad f(v_i) = E_i, \qquad 2 \le i \le n$$

Then the induced edge labels are

$$f^*(u_1v_1) = 1,$$
$$f^*(u_1v_2) = 0,$$
$$f^*(u_1v_{4j-1}) = 0, \qquad 1 \le j \le \frac{n-2}{4}$$
$$f^*(u_1v_{4j}) = 0, \qquad 1 \le j \le \frac{n-2}{4}$$
$$f^*(u_1v_{4j+1}) = 1, \qquad 1 \le j \le \frac{n-2}{4}$$
$$f^*(u_1v_{4j+2}) = 1, \qquad 1 \le j \le \frac{n-6}{4}$$

Observe that $|e_f(0) - e_f(1)| = |(\frac{n-2}{4} + \frac{n-2}{4} + 1) - (\frac{n-2}{4} + \frac{n-6}{4} + 1)| = |(\frac{n-2}{2} + 1) - (\frac{n-4}{2} + 1)| = 1 \le 1$. Hence, the Star Graph S_n is an Edouard Product Cordial Graph when $n = 4k + 2$ for some $k \in \mathbb{N}$.
Case 3: $n = 4k + 3$ for some $k \in \mathbb{W}$.
Define the function $f : V(S_n) \to \{E_0, E_1, \dots, E_n\}$ by

$$f(u_1) = E_1, \qquad f(v_1) = E_0, \qquad f(v_i) = E_i, \qquad 2 \le i \le n$$

Then the induced edge labels are

$$f^*(u_1v_1) = 1,$$
$$f^*(u_1v_2) = 0,$$
$$f^*(u_1v_{4j-1}) = 0, \qquad 1 \leq j \leq \frac{n-3}{4}$$
$$f^*(u_1v_{4j}) = 0, \qquad 1 \leq j \leq \frac{n-3}{4}$$
$$f^*(u_1v_{4j+1}) = 1, \qquad 1 \leq j \leq \frac{n-3}{4}$$

$$f^*(u_1v_{4j+2}) = 1, \qquad 1 \leq j \leq \frac{n-3}{4}$$

Observe that $|e_f(0) - e_f(1)| = |(\frac{n-3}{4} + \frac{n-3}{4} + 1) - (\frac{n-3}{4} + \frac{n-3}{4} + 1)| = |\frac{n-1}{2} - \frac{n-1}{2}| = 0 \leq 1$. Hence, the Star Graph S_n is an Edouard Product Cordial Graph when $n = 4k + 3$ for some $k \in \mathbb{N}$.

Considering the cases above, this concludes that the Star Graph $S_n \cong K_1 + P_n$ is an Edouard Cordial Graph except when $n = 4k + 3$. for $k \in \mathbb{N}$. □

4 Conclusions

This paper determine its necessary or sufficient condition of the graphs considered to admit Edouard Product Cordial Labeling. This result would apply in real world particularly in network design, chemistry, electronics engineering, etc.

Acknowledgement: The author acknowledges the Department of Science and Technology–Science Education Institute (DOST-SEI) for its support under the STRAND Scholarship Program.

Funding Statement: The author(s) received no specific funding for this study.

Data Availability Statement: Not applicable.

Ethics Approval: Not applicable

Use of Generative-AI tools declaration: The author declares that no Artificial Intelligence (AI) tools were used in the creation of this article.

References

1. J. D. Andoyo and R. F. Rulete, On Legendre cordial labeling of some graphs, *Eur. J. Math. Stat.* 6(5) **(2025)**, 1–7. https://doi.org/10.24018/ejmath.2025.6.5.409
2. I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, *Ars Combin.* **(1987)**, 201–207.
3. H. L. Fu and K. C. Huang, On prime labellings, *Discrete Math.* **(1994)**. https://doi.org/10.1016/0012-365X(92)00477-9.
4. G. V. Ghodasara and S. G. Sonchhatra, Cordial labeling of fan related graphs, *Int. J. Sci. Eng. Res.* 4(8) **(2013)**, 470–476.

5. M. V. Gudin and A. C. Pedrano, Tribonacci cordial labeling of some snake graphs, *Ann. Commun. Math.* 8(3) **(2025)**, 393–405. https://doi.org/10.62072/acm.2025.080306
6. A. Parthiban, G. Samdanielthomson, and D. Sharma, Divisor labeling of some special graphs, *J. Phys. Conf. Ser.* 1531 **(2020)**. https://doi.org/10.1088/1742-6596/1531/1/012076.
7. J. C. C. Prudencio and R. F. Rulete, On Legendre product cordial labeling of some graphs, *Eur. J. Math. Stat.* 6(6) **(2025)**, 1–12. https://doi.org/10.24018/ejmath.2025.6.6.418
8. Y. Soykan, A study on dual hyperbolic generalized Edouard numbers, *Asian J. Adv. Res. Rep.* 19(7) **(2025)**, 301–324. https://doi.org/10.9734/jamcs/2023/v38i31748.
9. J. E. Sulayman and A. C. Pedrano, On Fibonacci cordial labeling of some snake graphs, *Eur. J. Math. Stat.* **(2023)**. https://eu-opensci.org/index.php/ejmath/article/view/14193.
10. J. C. Vertudes and A. C. Pedrano, On topological cordial labeling of some graphs, *Eur. J. Math. Stat.* **(2025)**. https://doi.org/10.24018/ejmath.2025.6.4.404.
11. W. D. Wallis, E. T. Baskoro, M. Miller, and Slamin, Edge-magic total labelings, *Australas. J. Combin.* 22 **(2000)**, 177–190.