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Research article

Cosine Exponential Distribution: Mathematical Properties and Applications to Real Data Sets

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Abstract: This study introduces a new probability distribution called the Cosine Exponential (CEX) Distribution, which combines the Cosine-G family of distributions with the Exponential distribution as the baseline model to create a more adaptable model. The aim is to improve modeling capabilities across various statistical applications. The paper presents expression of the density and distribution functions of the CEX model and investigates its key properties such as survival and hazard rate functions, reverse hazard function, cumulative hazard function, quantile function, moments, and moment generating function. It also outlines the methodology for estimating model parameters using maximum likelihood estimation. Through application to real datasets, the effectiveness of the proposed CEX distribution is demonstrated, showing significant enhancements over existing models. This paper highlights the potential of the CEX distribution as a robust tool for statistical modeling and analysis.

Mathematics Subject Classification: 00X00, 00G00, 00D00
Keywords: Cosine-G; Exponential Distribution; Trigonometric Distribution; Maximum Likelihood Estimate.

1 Introduction

Trigonometric probability distributions provide a unique framework for modeling random variables that exhibit periodic behavior (Isa et al., 2022). Unlike traditional probability distributions that are often based on continuous or discrete variables, trigonometric distributions are characterized by their periodicity, making them particularly useful in scenarios where cyclic patterns are prevalent (Mustapha et al., 2023). These distributions find applications in various fields such as signal processing, astronomy, engineering, and finance, where phenomena occur periodically or exhibit sinusoidal patterns. By leveraging the principles of trigonometry, these distributions offer a robust way to analyze and predict the behavior of cyclic phenomena in a probabilistic manner (Isa et al., 2023). Some of the recently developed trigonometric distribution include: Cosine Weibull by Wu et al., (2023), Sine Lomax distribution by Mustapha et al., (2023), Sine Modified Lindley distribution by Tomy and Chesneau (2021), Log-Cosine Power Unit distribution by Nasiru et al., (2024), Cosine Rayleigh distribution by Bashiru et al. (2025), Exponentiated Cosine Lomax distribution by Ali et al. (2025), Cosine Frechet distribution by Abonongo et al., (2024), Arctan Lomax distribution by Chaudhary and Kumar (2021), Sine Exponentiated Exponential by Ali et al. (2025), Extended Cosine Generalized Weibull distribution by Sayibu and Luguterah (2023), Sine Topp-Leone Exponentiated Exponential by Bashiru et al. (2025) and Sine Exponential distribution by Isa et al., (2022). These models were constructed in order to accurately model periodic phenomena. The trigonometric distributions are simple to present, efficient in computation, compatible with statistical methods, and the interpretability of their parameters. It allows researchers to create more flexible models without adding parameters. These benefits made trigonometric distributions a valuable tool for analyzing and understanding cyclic data in various fields of study (Mustapha et al., 2023).

2 Methodology

2.1 Cosine G Family of Distribution

The cdf of the Cosine G family of Probability Distribution as defined by Souza et al., (2015) is given by:

$$F(x) = 1 - \cos\left\{ \frac{\pi}{2} R(x) \right\} \tag{1}$$

with corresponding pdf given by

$$f(x) = \frac{\pi}{2} r(x) \sin\left\{ \frac{\pi}{2} R(x) \right\} \tag{2}$$

2.2 The Proposed Cosine-Exponential Distribution

The Cosine Exponential distribution is obtained by substituting the PDF and CDF of Exponential distribution into equation (2.1) and (2.2) respectively:

$$F(x) = 1 - \cos\left\{ \frac{\pi}{2} \left[1 - e^{-\lambda x} \right] \right\} \tag{3}$$

with corresponding pdf given by

$$f(x) = \frac{\pi}{2} \lambda e^{-\lambda x} \sin\left\{ \frac{\pi}{2} \left[1 - e^{-\lambda x} \right] \right\} \tag{4}$$

where $\lambda > 0$ is scale parameter.

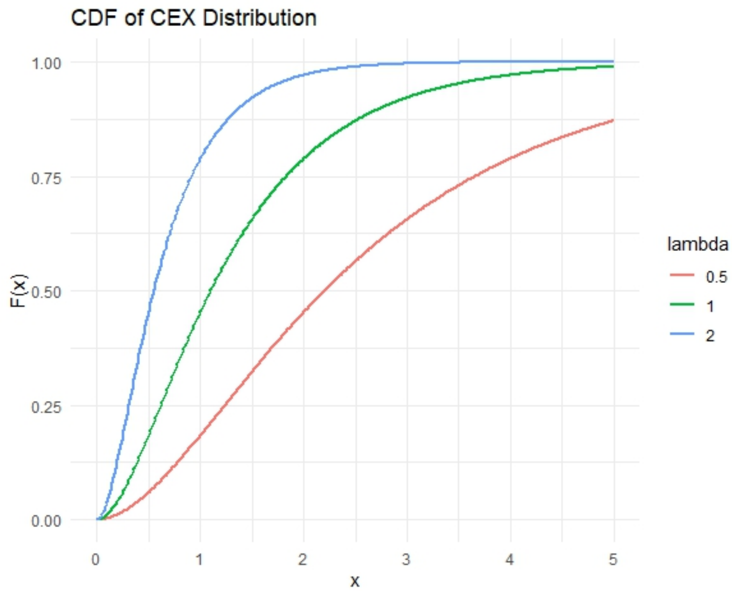

Figure 1: CDF PLOT

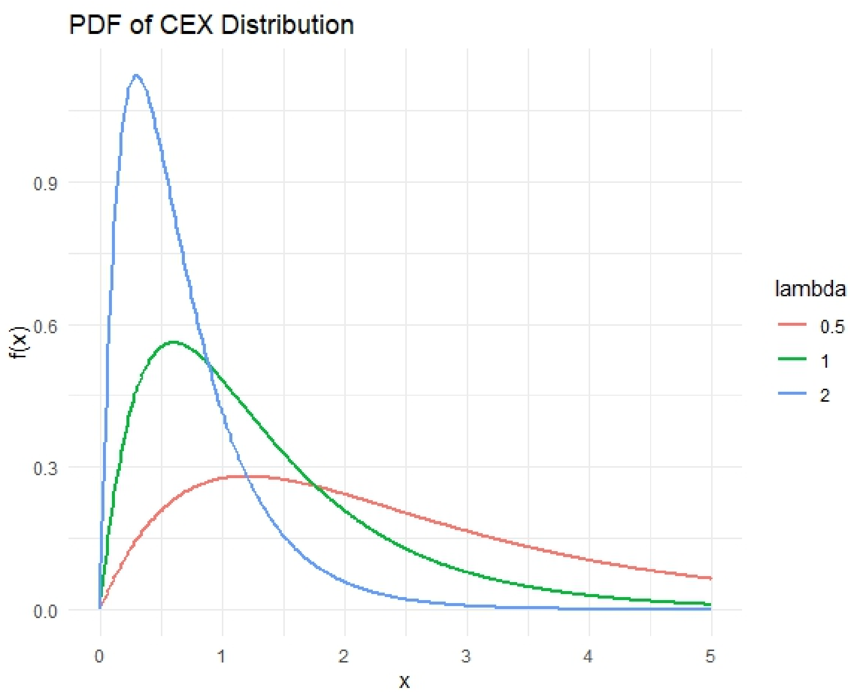

Figure 2: PDF plot of CEX Distribution

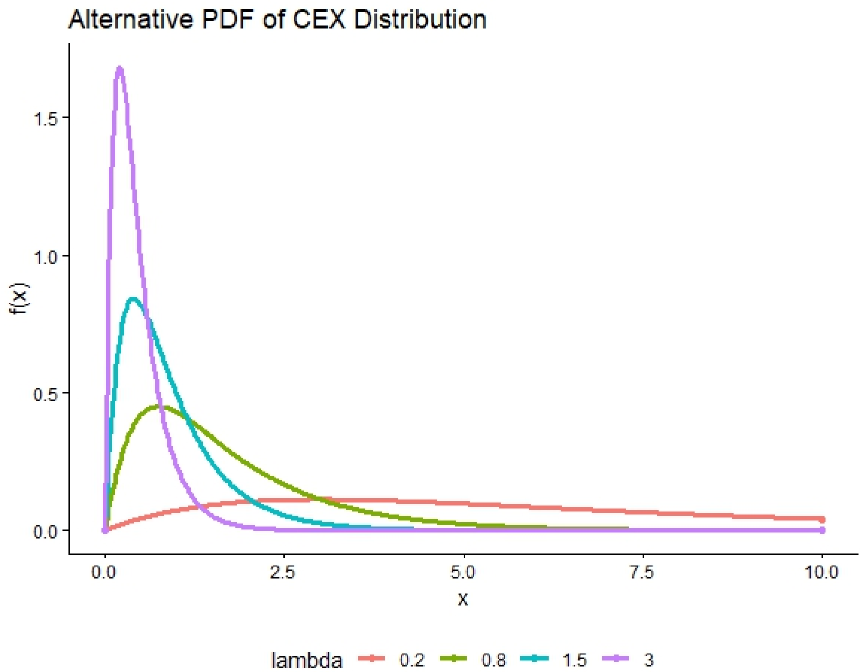

Figure 3: PDF Plot of CEX Distribution

2.3 The Survival Function

The survival function is the probability that a particular event has not occurred by a certain time point. It essentially describes the probability that an individual or object survives beyond a given time or until a specific event happens (Aalen et al., 2008). The survival function of the proposed Cosine Exponential distribution is given by:

$$S(x) = \cos\left\{ \frac{\pi}{2} \left[1 - e^{-\lambda x} \right] \right\} \tag{5}$$

2.4 Hazard Function

The hazard function represents the instantaneous rate at which events occur at a particular time, given that the individual or object has survived up to that time. It describes the likelihood of an event happening in the next instant, given that it has not occurred up to the current time (Rossello et al., 2022). The hazard function of the proposed model is given by:

$$h(x) = \frac{\pi}{2}\, \lambda e^{-\lambda x} \tan\left\{ \frac{\pi}{2} \left[1 - e^{-\lambda x} \right] \right\} \tag{6}$$

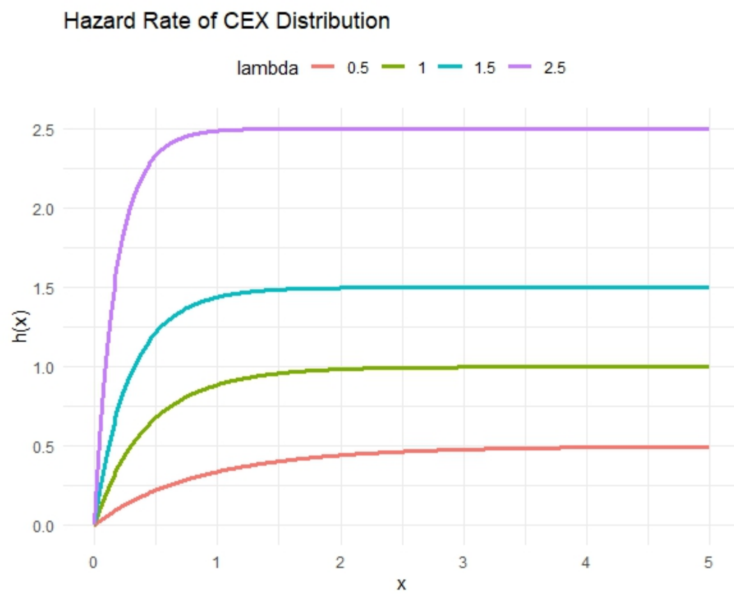

Figure 4: Hazard Function

2.5 Reverse Hazard Function

The reverse hazard function, also known as the survivor function, is a statistical concept that describes the probability of an individual or object surviving beyond a certain time, given that it has not experienced the event of interest up to that time (Hosmer *et al.*, 2008). The reverse hazard function of the Cosine Exponential distribution is given by:

$$r(x) = \frac{\frac{\pi}{2}\,\lambda e^{-\lambda x}\sin\left\{\frac{\pi}{2}\left[1-e^{-\lambda x}\right]\right\}}{1-\cos\left\{\frac{\pi}{2}\left[1-e^{-\lambda x}\right]\right\}} \tag{7}$$

2.6 Cumulative Hazard Function

The cumulative hazard function represents the accumulated hazard up to time t, where the hazard function measures the instantaneous rate of failure at time t given survival up to that time. It is given by:

$$K(x) = \int_0^x \frac{f(t)}{1-F(t)}\,dt$$

The cumulative hazard function of the proposed model is derived as follows:

$$K(x) = \int_0^x \frac{\frac{\pi}{2}\lambda e^{-\lambda t}\sin\left\{\frac{\pi}{2}\left[1-e^{-\lambda t}\right]\right\}}{1-\cos\left\{\frac{\pi}{2}\left[1-e^{-\lambda t}\right]\right\}}\,dt$$

Using integration by substitution, the cumulative hazard function of the proposed Cosine Exponential distribution reduces to:

$$K(x) = -\ln\left\{1 - \cos\left[\frac{\pi}{2}\left(1 - e^{-\lambda x}\right)\right]\right\} \tag{8}$$

2.7 Quantile Function

The quantile function is used to determine the value at which a given proportion of a probability distribution lies below. It is also known as the inverse cumulative distribution function (CDF). To derive the quantile function of the CEX distribution, we begin by setting the CDF equal to u:

$$u = 1 - \cos\left[\frac{\pi}{2}\left(1 - e^{-\lambda x}\right)\right]$$

Next, we isolate the exponential term by rearranging the expression:

$$1 - e^{-\lambda x} = \frac{\cos^{-1}(1-u)}{\pi/2}$$

We then solve for $e^{-\lambda x}$:

$$e^{-\lambda x} = 1 - \frac{\cos^{-1}(1-u)}{\pi/2}$$

To remove the exponential, we take the natural logarithm of both sides:

$$-\lambda x = \log\left[1 - \frac{\cos^{-1}(1-u)}{\pi/2}\right]$$

Finally, dividing both sides by $-\lambda$ gives the quantile function of the proposed model:

$$Q(u) = -\frac{1}{\lambda}\log\left[1 - \frac{\cos^{-1}(1-u)}{\pi/2}\right] \tag{9}$$

Mixture Representation

The pdf of the proposed Cosine Exponential distribution in equation (2.4) can be expanded using the Maclaurin series for the sine and cosine functions:

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \qquad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

Using the sine expansion, we obtain:

$$\sin\left\{ \frac{\pi}{2}\left[1 - e^{-\lambda x}\right] \right\} = \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i+1)!}\left(\frac{\pi}{2}\right)^{2i+1}\left[1 - e^{-\lambda x}\right]^{2i+1}$$

Substituting this into equation (2.4), the pdf becomes:

$$f(x) = \frac{\pi}{2}\lambda e^{-\lambda x} \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i+1)!}\left(\frac{\pi}{2}\right)^{2i+1}\left[1 - e^{-\lambda x}\right]^{2i+1}$$

Next, we expand the term $\left[1 - e^{-\lambda x}\right]^{2i+1}$ using the binomial expansion:

$$\left[1 - e^{-\lambda x}\right]^{2i+1} = \sum_{j=0}^{\infty}(-1)^j \binom{2i+1}{j} e^{-\lambda x j}$$

Substituting this back into the pdf gives:

$$f(x) = \sum_{i,j=0}^{\infty} \frac{\lambda(-1)^{i+j}}{(2i+1)!}\left(\frac{\pi}{2}\right)^{2i+2}\binom{2i+1}{j} e^{-\lambda x(j+1)}$$

Thus, the pdf can be expressed compactly as:

$$f(x) = \sum_{i,j=0}^{\infty} \Theta_{i,j}\, e^{-\lambda x(j+1)} \tag{10}$$

where

$$\Theta_{i,j} = \frac{\lambda(-1)^{i+j}}{(2i+1)!}\left(\frac{\pi}{2}\right)^{2i+2}\binom{2i+1}{j}$$

CDF Expansion

The cdf can also be expanded using the cosine Maclaurin series:

$$F(x) = 1 - \cos\left\{\frac{\pi}{2}\left[1 - e^{-\lambda x}\right]\right\} = 1 - \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\left(\frac{\pi}{2}\right)^{2k}\left[1 - e^{-\lambda x}\right]^{2k}$$

Applying the binomial expansion:

$$\left[1 - e^{-\lambda x}\right]^{2k} = \sum_{l=0}^{\infty}(-1)^l \binom{2k}{l} e^{-\lambda x l}$$

Substituting back gives:

$$F(x) = 1 - \sum_{k,l=0}^{\infty} \frac{(-1)^{k+l}}{(2k)!}\left(\frac{\pi}{2}\right)^{2k}\binom{2k}{l} e^{-\lambda x l}$$

Thus, the cdf can be written as:

$$F(x) = 1 - \sum_{k,l=0}^{\infty} \psi_{k,l}\, e^{-\lambda x l} \tag{11}$$

where

$$\psi_{k,l} = \frac{(-1)^{k+l}}{(2k)!}\left(\frac{\pi}{2}\right)^{2k}\binom{2k}{l}$$

Mathematical Properties

2.8 Moment

Moments summarize key characteristics of a distribution, such as skewness, kurtosis, and variability (Ho and Yu, 2015). The r-th moment is defined as:

$$\mu_r = \int_{-\infty}^{\infty} x^r f(x)\, dx$$

Using the mixture form of the pdf in equation (2.10), we obtain:

$$\mu_r = \sum_{i,j=0}^{\infty} \Theta_{i,j} \int_0^{\infty} x^r e^{-\lambda x(j+1)}\, dx$$

Evaluating the integral gives:

$$\mu_r = \sum_{i,j=0}^{\infty} \Theta_{i,j} \frac{\Gamma(r+1)}{[\lambda(j+1)]^{r+1}} \tag{12}$$

The first four moments follow by substituting $r = 1, 2, 3, 4$:

$$\mu_1 = \sum_{i,j=0}^{\infty} \Theta_{i,j} \frac{\Gamma(2)}{[\lambda(j+1)]^{2}} \tag{13}$$

$$\mu_2 = \sum_{i,j=0}^{\infty} \Theta_{i,j} \frac{\Gamma(3)}{[\lambda(j+1)]^{3}} \tag{14}$$

$$\mu_3 = \sum_{i,j=0}^{\infty} \Theta_{i,j} \frac{\Gamma(4)}{[\lambda(j+1)]^{4}} \tag{15}$$

$$\mu_4 = \sum_{i,j=0}^{\infty} \Theta_{i,j} \frac{\Gamma(5)}{[\lambda(j+1)]^{5}} \tag{16}$$

2.9 Measures of Skewness and Kurtosis

Skewness measures the extent to which a distribution leans to one side of the mean, while kurtosis measures the flatness or peakedness of the probability curve. The skewness and kurtosis are denoted by β_1 and β_2 respectively, and are expressed as:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \tag{17}$$

and

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \tag{18}$$

2.10 Moment Generating Function

The moment generating function (MGF) provides a convenient way to derive moments of a probability distribution. It is defined as the expected value of e^{tX}, where t is a real-valued parameter:

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

Using the mixture representation of the pdf in equation (2.10), the MGF of the Cosine Exponential distribution becomes:

$$M_X(t) = \sum_{i,j=0}^{\infty} \Theta_{i,j} \int_0^{\infty} e^{tx} e^{-\lambda x(j+1)}\, dx$$

Evaluating the integral yields:

$$M_X(t) = \sum_{i,j=0}^{\infty} \Theta_{i,j} \frac{1}{\lambda(j+1) - t} \tag{19}$$

2.11 Shannon Entropy

The Shannon entropy of a continuous random variable X with pdf $f(x)$ is defined as:

$$H(X) = -\int_0^{\infty} f(x) \ln f(x)\, dx \tag{20}$$

Substituting the pdf in equation (2.4) into the entropy formula and breaking the expression into components gives:

$$H(X) = -\ln\left(\frac{\pi}{2}\right) + \lambda\mu_1 - \int_0^{\infty} f(x) \ln\left[\sin\left\{\frac{\pi}{2}(1 - e^{-\lambda x})\right\}\right] dx \tag{21}$$

where μ_1 is the mean of the CEX distribution.

2.12 Order Statistics

The probability density function of the rth order statistic $X_{(r)}$ from a sample of size n is:

$$f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!} [F(x)]^{r-1} [1 - F(x)]^{n-r} f(x) \tag{22}$$

Substituting the pdf and cdf from equations (2.3) and (2.4), we obtain:

$$f_{X_{(r)}}(x) = \frac{n!}{(r-1)!(n-r)!}\left[1-\cos\left\{\frac{\pi}{2}(1-e^{-\lambda x})\right\}\right]^{r-1}\left[\cos\left\{\frac{\pi}{2}(1-e^{-\lambda x})\right\}\right]^{n-r} \times \frac{\pi}{2}\lambda e^{-\lambda x}\sin\left\{\frac{\pi}{2}(1-e^{-\lambda x})\right\} \quad (23)$$

2.13 Probability Weighted Moments (PWMs)

The rth probability weighted moment is defined as:

$$\beta_r = \int_0^{\infty} x\,[F(x)]^r f(x)\,dx$$

Parameter Estimation

2.14 Method of Maximum Likelihood

The parameter of the model was estimated using the method of maximum likelihood. Using the pdf in equation (2.4), the log-likelihood function for a sample $x_1, x_2, \ldots, x_n$ is given by:

$$\ell(\lambda) = n\ln\left(\frac{\pi}{2}\right) + n\ln(\lambda) + \sum_{i=1}^{n}\lambda x_i + \sum_{i=1}^{n}\ln\left[\sin\left\{\frac{\pi}{2}\left(1-e^{-\lambda x_i}\right)\right\}\right] \quad (24)$$

To obtain the maximum likelihood estimate of λ, we differentiate $\ell(\lambda)$ with respect to λ:

$$\frac{\partial \ell}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\left[\frac{\pi}{2}\lambda e^{-\lambda x_i}\cot\left\{\frac{\pi}{2}\left(1-e^{-\lambda x_i}\right)\right\}\right] \quad (25)$$

Equation (2.15) provides the likelihood equation whose solution yields the maximum likelihood estimate of the parameter λ.

Assessing the Consistency of the Parameter Estimates

To evaluate the performance of the proposed Cosine Exponential Distribution, a simulation study was conducted to compute the mean, bias, and root mean square error (RMSE) of the maximum likelihood estimates. The simulated data were generated using the quantile function in equation (2.9) for various sample sizes: $n = 20,\ 50,\ 70,\ 100,\ 150,\ 200,\ 350,\ 500,\ 1000$ each was replicated 1000 times. The parameter values considered were: $\lambda = 0.1,\ 0.5,\ 1.5,\ 2.5$ This simulation framework allows for assessing how well the estimator performs as the sample size increases, thereby examining the consistency and stability of the maximum likelihood estimates.

The table shows the estimates of the parameter λ for the new CEX Distribution, along with their Bias and Root Mean Square Error (RMSE) for different sample sizes and varying values of λ (0.1, 0.5, 1.5, and 2.5).

Table 1: Estimate, Bias and RMSE of the new Cosine Exponential Distribution

n	Properties	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 1.5$	$\lambda = 2.5$
20	Mean	0.1037	0.5186	1.5559	2.5931
	Bias	0.0037	0.0186	0.0559	0.0931
	RMSE	0.0186	0.0928	0.2785	0.4641
50	Mean	0.1018	0.5087	1.5262	2.5437
	Bias	0.0018	0.0087	0.0262	0.0437
	RMSE	0.0112	0.0562	0.1685	0.2808
70	Mean	0.1012	0.5058	1.5173	2.5288
	Bias	0.0012	0.0058	0.0173	0.0288
	RMSE	0.0092	0.0461	0.1383	0.2305
100	Mean	0.1007	0.5033	1.5099	2.5166
	Bias	0.0007	0.0033	0.0099	0.0166
	RMSE	0.0075	0.0377	0.1130	0.1883
150	Mean	0.1003	0.5017	1.5052	2.5087
	Bias	0.0003	0.0017	0.0052	0.0087
	RMSE	0.0061	0.0303	0.0908	0.1514
200	Mean	0.1001	0.5017	1.5052	2.5087
	Bias	0.0001	0.0017	0.0052	0.0087
	RMSE	0.0039	0.2690	0.0807	0.1346
350	Mean	0.1001	0.5004	1.5013	2.5022
	Bias	0.0001	0.0004	0.0013	0.0022
	RMSE	0.0033	0.0197	0.0590	0.0984

The results indicate that the mean estimates are close to the true values, and both bias and RMSE decrease as the sample size increases, demonstrating that the estimator is nearly unbiased and consistent. This trend suggests that the efficiency of the estimator is consistent for the CEX distribution.

Simulation and Evaluation of Model Performance

A random sample of size $n = 200$ was generated from the CEX distribution using the quantile function in equation (2.9) by setting the parameter $\lambda = 0.5$.

Table 2: MLE and AIC of the Simulated Data from Cosine Exponential Distribution

Models	λ	θ	LL	AIC
CEX	1.4762	–	-57.8183	117.637
EX	1.0977	–	-72.5485	147.097
IEX	0.6247	–	-72.3135	146.627
TIEX	2.6583	4.2983	-71.2365	144.475

Table 2 presents the MLE and AIC values for different models based on simulated data from the Cosine Exponential Distribution. The CEX model has an estimated parameter $\lambda = 1.4762$ with a log-likelihood (LL) of −57.8183 and the lowest AIC of 117.637, suggesting it is the best fit among the models presented. Figure 4 also validates the superiority of the CEX distribution.

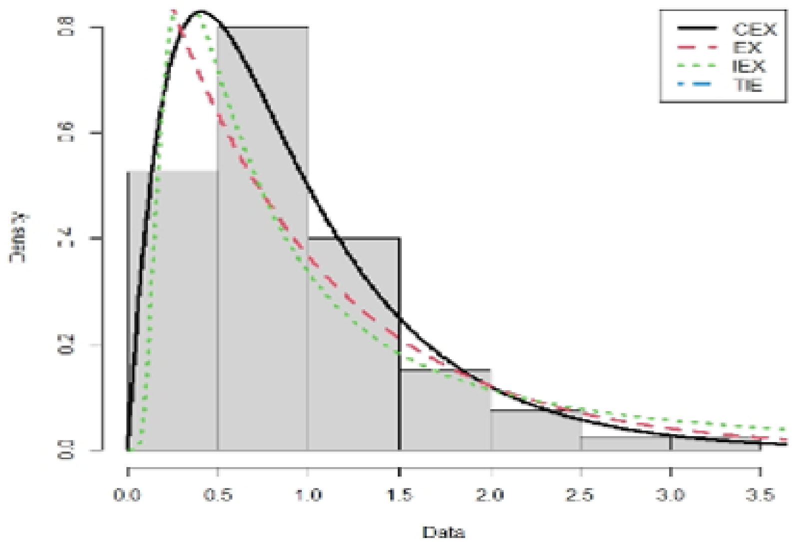

Figure 5: Density Plot

Application

In this section, we consider a real dataset on the Breaking Stress of Carbon Fibres of 50 mm length (GPa) to assess the flexibility of the CEX distribution compared with several well-known generalizations of the Exponential distribution. The results were compared with the baseline Exponential (EX) distribution, the Inverse Exponential (IEX) distribution, and the Transmuted Inverse Exponential (TIEX) distribution. For each dataset, the maximum likelihood method was used to estimate the unknown parameters of each distribution. Additionally, the Akaike Information Criterion (AIC) was computed for both the proposed model and its competitors.

The dataset is as follows:

0.39, 0.85, 1.08, 1.25, 1.47, 1.57, 1.61, 1.61, 1.69, 1.80, 1.84, 1.87, 1.89, 2.03, 2.03, 2.05, 2.12, 2.35, 2.41, 2.43, 2.48, 2.50, 2.53, 2.55, 2.55, 2.56, 2.59, 2.67, 2.73, 2.74, 2.79, 2.81, 2.82, 2.85, 2.87, 2.88, 2.93, 2.95, 2.96, 2.97, 3.09, 3.11, 3.11, 3.15, 3.15, 3.19, 3.22, 3.22, 3.27, 3.28 3.31, 3.31, 3.33, 3.39, 3.39, 3.56, 3.60, 3.65,3.68,3.70, 3.75, 4.20, 4.38, 4.42, 4.70, 4.90

Table 3: Descriptive Statistics of Breaking Strength of Carbon Fibre Data Set

Dataset	N	Min	Max	Mean	SD	Skewness	Kurtosis
Breaking Strength of Carbon Fibre	66	0.390	4.900	2.759	4.030	0.410	-0.380

The descriptive statistics reveal that the distribution is slightly positively skewed, with a skewness value of 0.410, indicating a minor asymmetry and a longer right tail. This suggests the presence of a few values higher than the mean. The kurtosis value of −0.380 indicates that the distribution is slightly platykurtic, meaning it has lighter tails and is flatter than a normal distribution, implying fewer extreme values than expected under normality.

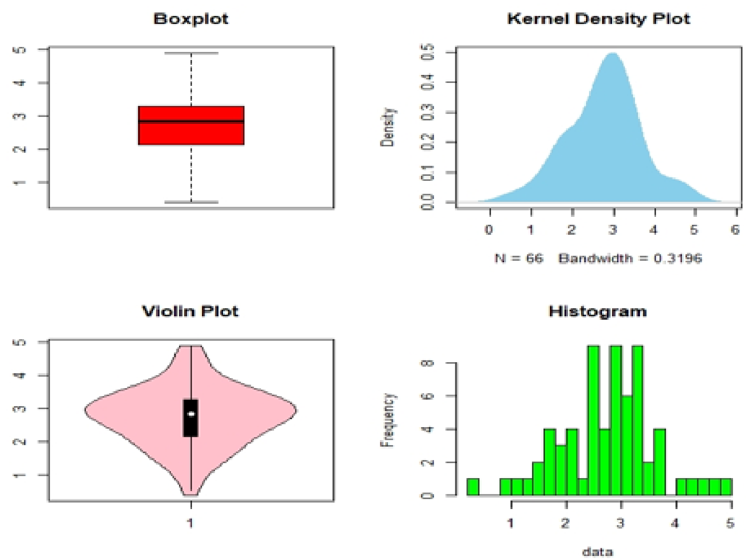

Figure 6: Boxplot, Kernel Plot, Violin Plot and Histogram showing the features of the dataset

Table 4: MLE and AIC of the Breaking Strength of Carbon Fibre Data Set

Models	λ	θ	LL	AIC
CEX	0.4674	–	-115.5132	233.0263
EX	0.3624	–	-132.9944	267.9887
IEX	2.2990	–	-136.0285	274.0570
TIEX	1.0493	1.0861	-135.8570	275.7140

Table 4 presents the Maximum Likelihood Estimates (MLE) and Akaike Information Criterion (AIC) values for the different models fitted to the Breaking Strength of Carbon Fibre dataset. The proposed Cosine Exponential (CEX) distribution achieves the lowest AIC value of 233.0263, indicating that it provides the best fit among the models considered. This result demonstrates the superior performance and suitability of the CEX distribution for modeling the dataset.

3 Conclusion

This study has introduced the Cosine Exponential (CEX) distribution, a new member of the trigonometric family of probability models. The model was constructed by integrating the Cosine G framework with the exponential baseline, the CEX distribution provides a flexible structure capable of capturing complex data behaviors, particularly those with periodic or skewed patterns. Theoretical exploration established its fundamental properties, including the probability density and cumulative distribution functions, survival and hazard functions, quantile function, mixture representation and higher order moments. These derivations confirm that the CEX distribution is mathematically sound and offers a rich set of tools for statistical analysis. Parameter estimation was carried out using maximum likelihood methods and the performance of the estimator was evaluated through simulation studies. The results demonstrated that the estimator is consistent and nearly unbiased, with both bias and root mean square error decreasing as sample size increases. This finding highlights the reliability of the estimation procedure and its suitability for practical applications. The usefulness of the CEX distribution was further demonstrated using both simulated and real data sets. When applied to the Breaking Strength of Carbon Fibre dataset, the CEX distribution provided a superior fit compared to several established generalizations of the exponential distribution, as evidenced by lower Akaike Information Criterion (AIC) values. This empirical validation highlights the distribution's ability to model real world phenomena with greater accuracy and efficiency. The Cosine Exponential distribution combines theoretical robustness with practical applicability. It enriches the growing class of trigonometric probability models and offers researchers a versatile option for analyzing complex datasets. Future work may extend this contribution by exploring Bayesian estimation approaches, multivariate extensions, and applications in fields such as reliability analysis, survival studies and environmental modeling. The evidence presented here suggests that the CEX distribution has strong potential to become a valuable tool in both theoretical research and applied statistics.

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