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ON PENDANT DOMINATION POLYNOMIAL IN THE CORONA OF SOME GRAPHS

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ABSTRACT. A dominating set S in G is called a *pendant dominating set* if $\langle S \rangle$ contains at least one pendant vertex. The minimum cardinality of a pendant dominating set is called the pendant domination number denoted by $\gamma_{pe}(G)$. The *pendant domination polynomial* of G is denoted by $D_{pe}(G,x)$ and is defined as $D_{pe}(G,x) = \sum_{i=\gamma_{pe}(G)}^{n} d_{pe}(G,i)x^i$, where $d_{pe}(G,i)x^i$ is the number of pendant dominating sets of size i. In this paper, we obtained the pendant domination number and pendant domination polynomial of the corona of some graphs, namely, $P_m \circ \overline{K_n}$, $C_m \circ \overline{K_n}$ and $K_m \circ \overline{K_n}$.

1. INTRODUCTION

Graph Theory is a branch of mathematics concerned with networks of points connected by lines. Generally, a graph comprises of vertices and edges which are studied in discrete mathematics. In the last three decades, hundreds of research article have been published in Graph Theory which have received good attention from mathematicians. Some of these are on graph Coloring and Labeling Graphs such as in [1, 2, 6, 9, 11, 12], Domination on Graphs such as in [7] and areas related to Algebraic Graph Theory in [10]. It is actively used in fields as varied as Biochemistry(Genomics), Electrical engineering, Computer science and Operations research, and etc. [8].

The mathematical study of Domination Theory in Graphs started around 1960. Its roots go back to 1862 when De Jaenisch studied the problems of determining the minimum number of queens necessary to cover an $n \times n$ chessboard in such way that every square is attacked by one of the queens where it can move any number of spaces vertically, horizontally, or diagonally. With this in mind, graph theoretical definitions will be related to the game of chess where it is applicable [10].

Nayaka S.R., Puttaswamy, and Purushothama S. introduced the concept of pendant domination polynomial and had obtained the results of the graphs like path graph P_n, cycle graph C_n, complete graph K_n, wheel graph W_n, star graph $K_{1,n-1}$, crown graph,

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helm graph H_n, cocktail party graph $K_{n\times 2}$, banana graph $B_{n,k}$, firecracker graph $F_{n,k}$, stacked book graph $B_{n,m}$, and jahangir graph $J_{n,m}$ [4, 5]. We extended their study and obtain new results of pendant domination polynomial on some corona graphs.

The investigation of the pendant domination polynomial in the corona of some graphs is significant as it enriches the theory of graph polynomials by linking domination concepts with structural graph operations. This study not only deepens the understanding of how pendant vertices affect domination properties but also provides a mathematical tool that may be applied in areas such as network design, optimization, and algorithmic analysis where domination parameters play a vital role.

2. PRELIMINARIES

Definition 2.1. [7] A subset S of $V(G)$ is a dominating set of G is for every $v \in V(G)\backslash S$, there exists $u \in S$ such that $uv \in E(G)$, that is, the closed neighbourhood of S is the vertex set of G. The domination number of G is denoted by $\gamma(G)$ which refers to the smallest cardinality of a dominating set of G. A dominating set of G with cardinality equal to $\gamma(G)$ is called a $\gamma-$set of G.

Definition 2.2. [5] A dominating set S in G is called a *pendant dominating set* if $\langle S\rangle$ contains at least one pendant vertex. The minimum cardinality of a pendant dominating set is called the pendant domination number denoted by $\gamma_{pe}(G)$. The *pendant domination polynomial* of G is denoted by $D_{pe}(G,x)$ and is defined as $D_{pe}(G,x) = \sum_{i=\gamma_{pe}(G)}^{n} d_{pe}(G,i)x^i$, where $d_{pe}(G,i)x^i$ is the number of pendant dominating sets of size i.

3. PENDANT DOMINATION POLYNOMIAL

This section presents the polynomial form of pendant dominating set of $P_m \circ \overline{K_n}$, $C_m \circ \overline{K_n}$, and $K_m \circ \overline{K_n}$.

Lemma 3.1. For all $m \geq 1$ and $n \geq 1$, the pendant domination number of $P_m \circ \overline{K_n}$ is $\gamma_{pe}(P_m \circ \overline{K_n}) = m$.

Proof. Let $S = \bigcup_{i=1}^{m}\{u_i | u_i \in V(P_m)\}$. To prove that S is a dominating set, we have to show that $N[S] = V(P_m \circ \overline{K_n})$ where $N[S] = \bigcup_{u_i\in S} N[u_i]$;

For $i = 1$,

$$N[u_1] = \{u_1\} \cup \{u_2\} \cup \textstyle\bigcup_{j=1}^{n}\{v_j^1 | v_j^1 \in V(\overline{K_n})\}$$

and for $i = m$

$$N[u_m] = \{u_m\} \cup \{u_{m-1}\} \cup \textstyle\bigcup_{j=1}^{n}\{v_j^m | v_j^m \in V(\overline{K_n})\}$$

and for $2 \leq i \leq m-1$,

$$N[u_i] = \{u_{i-1}, u_{i+1}\} \cup \{u_i\} \cup \textstyle\bigcup_{j=1}^{n}\{v_j^i | v_j^i \in V(\overline{K_n^i})\} \text{ for } 2 \leq i \leq m-1$$

Observe that,

$$N[S] = \bigcup_{u_i\in S} N[u_i] = \{\{u_2\} \cup \bigcup_{j=1}^{n}\{v_j^1\}\} \cup \{\{u_m\} \cup \bigcup_{j=1}^{n}\{v_j^m\}\} \cup \{u_{m-1}, u_m\}$$
$$\cup \bigcup_{j=1}^{n}\{v_j^i|, 2 \leq i \leq m-1\} \cup \{u_{i-1}, u_i, u_{i+1}\}$$

Hence,

$$N[S] = \bigcup_{u_i \in S} N[u_i] = \bigcup_{i=1}^{m} \{u_i\} \cup \bigcup_{j=1}^{n} \{v_j^i | v_j^i \in V(\overline{K_n})\}$$
$$= V(P_m) \cup V(\overline{K_n}) = V(P_m \circ \overline{K_n})$$

Thus, S is a dominating set in $P_m \circ \overline{K_n}$.

To show that S is a pendant dominating set in $P_m \circ \overline{K_n}$; $\langle S \rangle = P_m$. By definition, a path graph with atleast two vertices has at least two pendant vertices. So in a path graph, P_m, there exist pendant vertices, namely, u_1 and u_m. Hence S contains at least one pendant vertex. Thus, S is a pendant dominating set in $P_m \circ \overline{K_n}$. Further, $|S| = m$. We want to show that S is a minimum, that is for every set $T \subset V(P_m \circ \overline{K_n})$ with $|T| = |S| - 1$ is not a pendant dominating set. To prove the theorem, we will consider the following cases:

Case 1: $T \cap S \neq \emptyset$ and $T \subset S$.

Let T be a pendant dominating set. Further suppose, $T \cap S \neq \emptyset$ and $T \subset S$. Then $N[T] = V(P_m \circ \overline{K_n})$ and $\langle T \rangle$ contains atleast one pendant vertex. Observe that, there exist $u_i \in S$ such that $u_i \notin T$ for some $i = 1, 2, 3, \ldots, m$. This implies that there are some vertices of $P_m \circ \overline{K_n}$ of the form v_i^j that is/are not adjacent to some vertices in T and so $\bigcup_{u_i \in S} N[u_i] \neq V(P_m \circ \overline{K_n})$. Hence, T is not a dominating set in $P_m \circ \overline{K_n}$. Since, T fails to be a dominating set on $P_m \circ \overline{K_n}$, it will follow that T is not a pendant dominating set.

Case 2: $T \cap S \neq \emptyset$ and $T \not\subset S$ with $|T| = |S| - 1$.

Suppose $T \cap S \neq \emptyset$ and $T \not\subset S$ then T contains some vertices of S and some vertices not in S. If $T \cap S \neq \emptyset$ and $|T| = |S| - 1$, we only have two options for set T , either

$$T_1 = \{u_i | u_i \in V(P_m)\} \backslash \{u_k | u_k \in S\}$$

or

$$T_2 = \{u_i | u_i \in V(P_m)\} \backslash \{u_a, u_b | u_a, u_b \in S\} \cup \{v_k^i \in V(\overline{K_n})\}.$$

But observe that, in either option, the set $T_1 \subset S$, a contradiction to our assumption that $T \not\subset S$. Also, the set T_2 is not a dominating set.

Case 3: $T \cap S = \emptyset$ with $|T| = |S| - 1$

Suppose $T \cap S = \emptyset$. Let $T = \bigcup_{j=1}^{n} \{v_j^i\}$. Observe that, $N[v_j^i] = \{u_i\} \cup \{v_j^i\}$ for $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$ which is equal to $V(P_m \circ \overline{K_n})$. Hence, T must contain all the vertices of the copies of $\overline{K_n}$. Since in $P_m \circ \overline{K_n}$ there are m copies of $\overline{K_n}$ it follows that $|T| = mn$ but $|T| = mn > m = |S|$ which is a contradiction. Hence, S is a minimum pendant dominating set.

Therefore, considering the cases above, we can say that, for all m and n, the pendant domination number of the corona graph $P_m \circ \overline{K_n}$ is $\gamma_{pe}(P_m \circ \overline{K_n}) = m$. □

Theorem 3.2. Let $P_m \circ \overline{K_n}$ be a corona graph, then for all $m \geq 1$ and $n \geq 1$ $D(P_m \circ \overline{K_n}, x) = x^m(1 + x)^{mn}$.

Proof. Suppose $P_m \circ \overline{K_n}$ be a corona graph with $mn+m$ vertices. By Lemma 2.1, the pendant domination number of $P_m \circ \overline{K_n}$, $\gamma_{pe}(P_m \circ \overline{K_n}) = m$. Observe that $\left|V(P_m \circ \overline{K_n})\right| = mn + m$. Thus, we have $(mn+m)-m=mn$ vertices to be chosen and to be added to pendant dominating sets having cardinality mn. This implies that, we must get a vertex from the set $\{v_1^i, v_2^i, v_3^i, ..., v_j^i\}$ to obtain the pendant dominating sets of $m+1$ and there are $\binom{mn}{1}$ ways to do this. Similarly, to get a pendant dominating sets of size $m+i$ where $1 \leq i \leq mn$, we have $\binom{mn}{i}$ ways. Thus,

$$D_{pe}(P_m \circ \overline{K_n}) = \binom{mn}{0}x^m + \binom{mn}{1}x^{m+1} + \binom{mn}{2}x^{m+2} + \binom{mn}{3}x^{m+3} + \ldots + \binom{mn}{mn}x^{mn+m}$$

$$= x^m \left[\sum_{i=0}^{mn} \binom{mn}{i} x^i\right] = x^m(1+x)^{mn}$$

Therefore, the $D_{pe}(P_m \circ \overline{K_n}, x) = x^m(1+x)^{mn}$.

□

Lemma 3.3. For all $m, n \geq 3$, the pendant domination number of $C_m \circ \overline{K_n}$ is $\gamma_{pe}(C_m \circ \overline{K_n}) = m+1$.

Proof. Let $S = \bigcup_{i=i}^{m}\{u_i | u_i \in V(C_m)\} \cup \{v_j^i\}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$. To prove that S is a dominating set, we have to show that $N[S] = V(C_m \circ \overline{K_n})$ where $N[S] = \bigcup_{u_i \in S} N[u_i]$;

For $i = 1$,

$$N[u_1] = \{u_2, u_m\} \cup \{u_1\} \cup \textstyle\bigcup_{j=1}^{n}\{v_j^1 | v_j^1 \in V(\overline{K_n})\}$$

and for $2 \leq i \leq m-1$

$$N[u_i] = \{u_{i+1}, u_{i-1}\} \cup \{u_i\} \cup \textstyle\bigcup_{j=1}^{n}\{v_j^i | v_j^i \in V(\overline{K_n})\}$$

and for $i = m$

$$N[u_m] = \{u_{m-1}, u_1\} \cup \{u_m\} \cup \textstyle\bigcup_{j=1}^{n}\{v_j^m | v_j^m \in V(\overline{K_n})\}$$

Observe that,

$$N[S] = \bigcup_{u_i \in S} N[u_i] = \{\{u_1, u_2, u_m\} \cup \bigcup_{j=1}^{n}\{v_j^1\}\} \cup \bigcup_{j=1}^{n}\{v_j^i|, 2 \leq i \leq m-1\}$$
$$\cup \{u_{i-1}, u_i, u_{i+1}\} \cup \bigcup_{j=1}^{n}\{v_j^m\} \cup \{u_{m-1}, u_m, u_i\}$$

Hence,

$$N[S] = \bigcup_{u_i \in S} N[u_i] = \bigcup_{i=1}^{m}\{u_i\} \cup \bigcup_{j=1}^{n}\{v_j^i | v_j^i \in V(\overline{K_n})\}$$
$$= V(C_m) \cup V(\overline{K_n}) = V(C_m \circ \overline{K_n})$$

This implies that every vertex in $C_m \circ \overline{K_n}$ is adjacent to at least one vertex in S. Thus, S is a dominating set in $C_m \circ \overline{K_n}$.

To show that S is a pendant dominating set in $C_m \circ \overline{K_n}$ observe that $S = \bigcup_{i=1}^{m} \{u_i\} \cup \{v_j^i\}$. Hence, $u_i v_j^i \in E(C_m \circ \overline{K_n})$ for some $1 \leq i \leq m$ and $1 \leq j \leq n$. Thus, $\langle S \rangle$ is a cycle graph with one pendant vertex attached.

We want to show that S is minimum, that is, for every set $T \subset V(C_m \circ \overline{K_n^i})$ with $|T| = |S| - 1$ is not a pendant dominating set. To prove the theorem, we will consider the following cases:

Case 1: $T \cap S \neq \emptyset$ and $T \subset S$.

Suppose T is a dominating set, $T \cap S \neq \emptyset$, and $T \subset S$. Now, suppose $T = \bigcup_{i=1}^{m} \{u_i | u_i \in V(C_m)\}$. Observe that, the vertices $\bigcup_{j=1}^{n} \{v_j^i | v_j^i \in V(\overline{K_n^i})\}$ are adjacent to the vertices in T. Hence, T is a dominating set in $C_m \circ \overline{K_n}$. Notice that, $|T| = |S| - 1 = m + 1 - 1 = m$ and $\langle T \rangle = C_m$. Hence, we cannot find any pendant vertex in C_m. It follows that T is not a pendant dominating set.

Case 2: $T \cap S \neq \emptyset$ and $T \not\subset S$ with $|T| = |S| - 1$.

Suppose $T \cap S \neq \emptyset$ and $T \not\subset S$ then T contains some vertices of S and some vertices not in S. If $T \cap S \neq \emptyset$ and $|T| = |S| - 1$, we will consider two subcases for set T:

Subcase 2.1: $T = \bigcup_{i=1}^{m} \{u_i | u_i \in V(C_m)\}$

Suppose T is a pendant dominating set and in effect a dominating set with $|T| = |S| - 1$. Now, $|S| = m + 1$ then $|T| = m + 1 - 1 = m = V(C_m)$. Notice that, $\langle T \rangle = C_m$. Recall that any cycle graph C_m does not contain any pendant vertex. This means T is not a pendant dominating set.

Subcase 2.2: $T = \{v_j^i\}$ or $T = \{u_k\} \cup \{v_j^i\}$

Suppose T is a dominating set and $T = \{v_j^i\}$ or $T = \{u_k\} \cup \{v_j^i\}$ for some $u_k \in S$ and $1 \leq i \leq m$ and $1 \leq j \leq n$. Now, since $|T| = |S| - 1 = m$ then we are force to say that T must contain atleast $m - 1$ $u_k{}'s$ such that T is a dominating set. However, this means that there exist vertices of the form v_r^i adjacent to u_r where $u_r \notin T$. It follows that T is not a pendant dominating set.

Case 3: $T \cap S = \emptyset$ with $|T| = |S| - 1$.

Suppose $T \cap S = \emptyset$. Let $T = \{v_j^i\}$ where $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$ such that $v_k^i \notin T$ where $v_k^i \in S$. It follows that $|T| = mn - 1 > m + 1 = |S|$, which is a contradiction. This implies that T is not a pendant dominating set. Thus, S is a minimum pendant dominating set.

Therefore, considering the cases above, we can say that, for all $m, n \geq 3$, the pendant domination number of the corona graph $C_m \circ \overline{K_n}$ is $\gamma_{pe}(C_m \circ \overline{K_n}) = m + 1$.

□

Theorem 3.4. Let $C_m \circ \overline{K_n}$ be a corona graph, then for all $m, n \geq 3$, $D(C_m \circ \overline{K_n}, x) = x^m(1 + x)^{mn} - x^m$.

Proof. Suppose $C_m \circ \overline{K_n}$ be a corona graph with $mn + m$ vertices. By Lemma 2.2, the pendant domination number of $C_m \circ \overline{K_n}$, $\gamma_{pe}(C_m \circ \overline{K_n}) = m + 1$. Note that the pendant dominating set is obtained from all vertices of C_m and one vertex from any of the copies of $\overline{K_n}$. Thus we have mn vertices to be chosen such that the pendant dominating set have the cardinality $m+1$, that is $\binom{mn}{1}$. Also, to get the pendant dominating set of size $m+2$, we have $\binom{mn}{2}$ ways to do this. Similarly, to get a pendant dominating sets of size $m+i$ where $1 \leq i \leq mn$, we have $\binom{mn}{i}$ ways. Thus,

$$\begin{aligned} D_{pe}(C_m \circ \overline{K_n}) &= \left[\binom{mn}{1}x^{m+1}\right] + \left[\binom{mn}{2}x^{m+2} + \binom{mn}{3}x^{m+3} + \ldots + \right. \\ &\left. \binom{mn}{mn}x^{m+mn}\right] \\ &= x^m \left[\binom{mn}{1}x + \binom{mn}{2}x^2 + \binom{mn}{3}x^3 + \ldots + \right. \\ &\left. \binom{mn}{mn}x^{mn} + 1 - 1\right] \\ &= x^m \left[\sum_{i=0}^{mn} \binom{mn}{i} x^i\right] - x^m = x^m(1+x) - x^m \end{aligned}$$

Therefore, the $D_{pe}(C_m \circ \overline{K_n}, x) = x^m(1+x)^{mn} - x^m$.

$\square$

Lemma 3.5. For all $m, n \geq 3$, the pendant domination number of $K_m \circ \overline{K_n}$ is $\gamma_{pe}(K_m \circ \overline{K_n}) = m+1$.

Proof. Let $S = \bigcup_{i=1}^{m}\{u_i | u_i \in V(K_m)\} \cup \bigcup_{j=1}^{n}\{v_j^i | v_j^i \in V(\overline{K_n})\}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$. To prove that S is a dominating set, we have to show that $N[S] = V(K_m \circ \overline{K_n})$ where $N[S] = \bigcup_{u_i \in S} N[u_i]$;

For $i = 1$

$$N[u_1] = \{u_2, u_3, \ldots, u_m\} \cup \{u_1\} \cup \textstyle\bigcup_{j=1}^{n}\{v_j^1 | v_j^1 \in V(\overline{K_n})\}$$

and for $2 \leq i \leq m-1, 1 \leq j \leq n$

$$N[u_i] = \{u_{i-1}\} \cup \{u_i\} \cup \textstyle\bigcup_{k=i+1}^{m}\{u_k\} \cup \bigcup_{j=1}^{n}\{v_j^i | v_j^i \in V(\overline{K_n})\}$$

and for $i = m$

$$N[u_m] = \{u_{m-1}, u_{m-2}, \ldots, u_1\} \cup \{u_m\} \cup \textstyle\bigcup_{j=1}^{n}\{v_j^m | v_j^m \in V(\overline{K_n})\}$$

Observe that,

$$\begin{aligned} N[S] = \bigcup_{u_i \in S} N[u_i] = \{\{u_2, u_3, \ldots, u_m\} \cup \bigcup_{j=1}^{n}\{v_j^1\}\} \cup \{\{u_{i-1}\} \cup \bigcup_{k=i+1}^{m}\{u_k\} \\ \cup \bigcup_{j=1}^{n}\{v_j^i|, 2 \leq i \leq m-1\}\} \cup \{\{u_{m-1}, u_{m-2}, \ldots, u_1\} \cup \bigcup_{j=1}^{n}\{v_j^m\}\} \\ \cup \{u_{m-1}, u_m, u_{m+1}\} \end{aligned}$$

Hence,

$$N[S] = \bigcup_{u_i \in S} N[u_i] = \bigcup_{i=1}^{m} \{u_i\} \cup \bigcup_{j=1}^{n} \{v_j^i | v_j^i \in V(\overline{K_n})\}$$
$$= V(K_m) \cup V(\overline{K_n}) = V(K_m \circ \overline{K_n})$$

This implies that every vertex in $K_m \circ \overline{K_n}$ is adjacent to at least one vertex in S. Thus, S is a dominating set in $K_m \circ \overline{K_n}$.

To show that S is a pendant dominating set in $K_m \circ \overline{K_n}$ observe that $S = \bigcup_{i=1}^{m} \{u_i\} \cup \{v_j^i\}$. Hence, $u_i v_j^i \in E(K_m \circ \overline{K_n})$ for some $1 \leq i \leq m$ and $1 \leq j \leq n$. Thus, $\langle S \rangle$ is a complete graph with one pendant vertex attached.

We want to show that S is a minimum, that is, for every set $T \subset V(K_m \circ \overline{K_n^i})$ with $|T| = |S| - 1$ is not a pendant dominating set. To prove the theorem, we will consider the following cases:

Case 1: $T \cap S \neq \emptyset$ and $T \subset S$.

Suppose T is a dominating set. Suppose further that $T \cap S \neq \emptyset$ and $T \subset S$. Now, let $T = \bigcup_{i=1}^{m} \{u_i | u_i \in V(K_m)\}$. Observe that, the vertices $\bigcup_{j=1}^{n} \{v_j^i | v_j^i \in V(\overline{K_n^i})\}$ are adjacent to the vertices in T. Hence, T is a dominating set in $K_m \circ \overline{K_n}$. Notice that, $|T| = |S| - 1 = m + 1 - 1 = m$ and $\langle T \rangle = K_m$. Hence, we cannot find any pendant vertex in K_m. It follows that T is not a pendant dominating set. Thus, S is a minimum pendant dominating set.

Case 2: $T \cap S \neq \emptyset$ and $T \not\subset S$ with $|T| = |S| - 1$.

Suppose $T \cap S \neq \emptyset$ and $T \not\subset S$ then T contains some vertices of S and some vertices not in S. If $T \cap S \neq \emptyset$ and $|T| = |S| - 1$, we will consider two casesfor set T:

Subcase 2.1: $T = \{v_j^i\}$ or $T = \bigcup_{i=1}^{m} \{u_i | u_i \in V(K_m)\}$

Suppose T is a dominating set and $|T| = |S| - 1$. Now, $|S| = m + 1$ then $|T| = m + 1 - 1 = m = V(K_m)$. Notice that, $\langle T \rangle = K_m$. By definition, K_m does not contain any pendant vertex. Hence, S is a pendant dominating set.

Subcase 2.2: $T = \{u_k\} \cup \{v_j^i\}$

Suppose T is a dominating set and $T = \{u_k\} \cup \{v_j^i\}$ for some $u_k \in S$ and $1 \leq i \leq m$ and $1 \leq j \leq n$. Now, since $|T| = |S| - 1 = m$ then we are force to say that T must contain atleast $m - 1$ $u_k's$ such that T is a dominating set. However, this means that there exist vertices of the form v_r^i adjacent to u_r where $u_r \notin T$. It follows that T is not a dominating set. Hence, S is a minimum pendant dominating set.

Case 3: $T \cap S = \emptyset$ with $|T| = |S| - 1$.

Suppose $T \cap S = \emptyset$. Let $T = \{v_j^i\}$ where $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$ such that $v_k^i \notin T$ where $v_k^i \in S$. It follows that $|T| = mn - 1 > m + 1 = |S|$, which is a contradiction. Thus, S is a minimum pendant dominating set.

Hence, considering the cases above, we can say that, for all $m, n \geq 3$, the pendant domination number of the corona graph $K_m \circ \overline{K_n}$ is $\gamma_{pe}(K_m \circ \overline{K_n}) = m + 1$. □

Theorem 3.6. Let $K_m \circ \overline{K_n}$ be a corona graph, then for all $m, n \geq 3$, $D(K_m \circ \overline{K_n}, x) = x^m(1 + x)^{mn} - x^m$.

Proof. Suppose $K_m \circ \overline{K_n}$ be a corona graph with $mn + m$ vertices. By Lemma 2.3, the pendant domination number of $K_m \circ \overline{K_n}$, $\gamma_{pe}(K_m \circ \overline{K_n}) = m + 1$. Note that the pendant dominating set is obtained from all vertices of K_m and one vertex from any of the copies of $\overline{K_n}$. Thus we have mn vertices to be chosen such that the pendant dominating set have the cardinality $m + 1$, that is $\binom{mn}{1}$. Also, to get the pendant dominating set of size $m + 2$, we have $\binom{mn}{2}$ ways to do this. Similarly, to get a pendant dominating sets of size $m + i$ where $1 \leq i \leq mn$, we have $\binom{mn}{i}$ ways. Hence,

$$\begin{aligned} D_{pe}(K_m \circ \overline{K_n}) &= \left[\binom{mn}{1}x^{m+1}\right] + \left[\binom{mn}{2}x^{m+2} + \binom{mn}{3}x^{m+3} + \ldots + \right. \\ &\left.\binom{mn}{mn}x^{m+mn}\right] \\ &= x^m\left[\binom{mn}{1}x + \binom{mn}{2}x^2 + \binom{mn}{3}x^3 + \ldots + \right. \\ &\left.\binom{mn}{mn}x^{mn} + 1 - 1\right] \\ &= x^m\left[\sum_{i=0}^{mn}\binom{mn}{i}x^i\right] - x^m = x^m(1+x) - x^m \end{aligned}$$

Therefore, the $D_{pe}(K_m \circ \overline{K_n}, x) = x^m(1 + x)^{mn} - x^m$.

□

4. Conclusion

The study on the pendant domination polynomial in the corona of some graphs reveals significant relationships between the structural properties of the base graphs and the resulting polynomial expressions. By examining the corona operation, it is shown that the pendant domination polynomial effectively captures domination characteristics influenced by pendant vertices. These findings provide deeper insights into domination-related parameters and open new avenues for analyzing polynomial invariants in more complex graph operations. Moreover, this work opens several promising directions for future research, including the extension of pendant domination polynomials to other graph operations such as joins, Cartesian and lexicographic products, vertex and edge subdivisions, and graph complements. Further investigations may also explore comparative studies with other domination polynomials, algorithmic aspects of computing these polynomials for large graph classes, and potential applications in network modeling where pendant structures naturally arise.

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