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EXPLORING IDEALS OF SEMIRING WITH INVOLUTION

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ABSTRACT. In this paper, we introduce the notion of involution in semirings. We define bi-ideal, quasi ideal, interior ideal, bi-quasi interior ideal, and bi-interior ideals of semirings with involution and study their properties.

1. INTRODUCTION

The notion of a semiring was introduced by Vandiver [35] in 1934, but semirings had appeared in earlier studies on the theory of ideals of rings. A universal algebra $(S, +, \cdot)$ is called a semiring if and only if $(S, +), (S, \cdot)$ are semigroups which are connected by distributive laws, $i.e., a(b + c) = ab + ac,\ (a + b)c = ac + bc,$ for all $a, b, c \in S$. The theory of rings and theory of semigroups have considerable impact on the development of theory of semirings. Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in the solution of graph theory, optimization theory, in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches of mathematics.

We know that the notion of a one sided ideal of any algebraic structure is a generalization of an ideal. The quasi ideals are generalizations of left ideals and right ideal whereas the bi-ideals are generalization of quasi ideals. In 1952, the concept of bi-ideals was introduced by Good and Hughes [3] for semigroups. The notion of bi-ideals in rings and semigroups were introduced by Lajos and Szasz [10,11]. Bi-ideal is a special case of (m-n) ideal. In 1976, the concept of interior ideals was introduced by Lajos [12] for semigroups. Steinfeld [34] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki and Izuka [5,6,7,8] introduced the concept of quasi ideal for a semiring. Quasi ideals bi-ideals in $\Gamma-$semirings studied by Jagtap and Pawar [9]. Henriksen [4] and Shabir et al. [33] studied ideals in semirings. Murali Krishna Rao et al. [27,28] studied ideals in $\Gamma-$semirings.

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2. PRELIMINARIES

In this section, we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. [15] A set S together with two associative binary operations called addition and multiplication (denoted by + and $\cdot$ respectively will be called semiring provided

(i) addition is a commutative operation.
(ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

Definition 2.2. [15] A semiring M is said to be commutative semiring if $xy = yx$, for all $x, y \in M$.

Definition 2.3. [15] Let M be a semiring. An element $1 \in M$ is said to be unity if for each $x \in M$, $x1 = 1x = x$.

Definition 2.4. [15] In a semiring M with unity 1, an element $a \in M$ is said to be left invertible (right invertible) if there exist $b \in M$ such that $ba = 1 (ab = 1)$.

Definition 2.5. [15] In a semiring M with unity 1, an element $a \in M$ is said to be invertible if there exist $b \in M, \in$ such that $ab = ba = 1$.

Definition 2.6. [15] A semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x$ and $0x = x0 = 0,$ for all $x \in M$.

Definition 2.7. [15] An element a in a semiring M is said to be idempotent if $a = aa$.

Definition 2.8. [15] Every element of M, is an idempotent of M then M is said to be idempotent semiring M.

Definition 2.9. [24] A semiring M is called a division semiring if for each non-zero element of M has multiplication inverse.

Definition 2.10. [24] A non-empty subset A of a semiring M is called

(i) a subsemiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $AA \subseteq A$.
(ii) a quasi ideal of M if A is a subsemiring of M and $AM \cap MA \subseteq A$.
(iii) a bi-ideal of M if A is a subsemiring of M and $AMA \subseteq A$.
(iv) an interior ideal of M if A is a subsemiring of M and $MAM \subseteq A$.
(v) a left (right) ideal of M if A is a subsemiring of M and $MA \subseteq A(AM \subseteq A)$.
(vi) an ideal if A is a subsemiring of M, $AM \subseteq A$ and $MA \subseteq A$.
(vii) a $k-$ideal if A is a subsemiring of $M, AM \subseteq A, MA \subseteq A$ and $x \in M,\ x + y \in A, y \in A$ then $x \in A$.
(viii) a bi-interior ideal of M if A is a subsemiring of M and $MBM \cap BMB \subseteq B$.
(ix) a left bi-quasi ideal (right bi-quasi ideal) of M if A is a subsemigroup of $(M, +)$ and $MA \cap AMA \subseteq A$ $(AM \cap AMA \subseteq A)$.
(x) a bi-quasi ideal of M if B is a subsemiring of M and B is a left bi-quasi ideal and a right bi-quasi ideal of M.
(xi) a left quasi-interior ideal (right quasi-interior ideal) of M if A is a subsemiring of M and $MAMA \subseteq A$ $(AMAM \subseteq A)$.
(xii) a quasi-interior of M if B is a subsemiring of M and B is a left quasi-interior ideal and a right quasi-interior ideal of M.
(xiii) a bi-quasi-interior ideal of M if A is a subsemiring of M and $BMBMB \subseteq B$.

(xiv) a left tri-ideal (right tri-ideal) of M if A is a subsemiring of M and $AMAA \subseteq A$ ($AAMA \subseteq A$).

(xv) a tri- ideal of M if A is a subsemiring of M and $AMAA \subseteq A$ and $AAMA \subseteq A$.

(xvi) a left(right) weak-interior ideal of M if B is a subsemiring of M and $MBB \subseteq B$($BBM \subseteq B$).

(xvii) a weak-interior ideal of M if B is a subsemiring of M and B is a left weak-interior ideal and a right weak-interior ideal)of M.

3. IDEALS OF SEMIRINGS WITH INVOLUTION

In this section, we introduce the notion of semiring with involution and ideals of semiring with involution.

Definition 3.1. Let $(M, *)$ be a semiring with involution.Then A^* is said to be a quasi ideal of $(M, *)$, if $(MA^*) \cap (A^*M) \subseteq A^*$.

Definition 3.2. Let $(M, *)$ be a semiring with involution. Then A^* is said to be a left(right) ideal of $(M, *)$, if $MA^* \subseteq A^*(A^*M) \subseteq A^*)$.

Definition 3.3. Let $(M, *)$ be a semiring with involution. Then B^* is said to be an interior ideal of $(M, *)$, if $MB^*M \subseteq B^*$.

Definition 3.4. Let $(M, *)$ be a semiring with involution. Then B^* is said to be a bi-ideal of $(M, *)$, if $B^*MB^* \subseteq B^*$.

Definition 3.5. Let $(M, *)$ be a semiring with involution. Then B^* is said to be a bi-interior ideal of $(M, *)$, if $B^*MB^* \cap MB^*M \subseteq B^*$.

Definition 3.6. Let $(M, *)$ be a semiring with involution. Then B^* is said to be a left(right) tri-ideal of $(M, *)$, if $B^*MB^*B^* \subseteq B^*(B^*B^*MB^* \subseteq B^*)$.

Definition 3.7. Let $(M, *)$ be a semiring with involution. Then B^* is said to be a bi-quasi interior ideal of $(M, *)$, if $B^*MB^*MB^* \subseteq B^*$.

Theorem 3.1. *Let M be a semiring with involution. Then A^* is a right ideal of the semiring with involution $(M, *)$, if A is a left ideal of M.*

Proof. Let A be a left ideal of M. Then $MA \subseteq A \Rightarrow (MA)^* \subseteq A^* \Rightarrow A^*M^* \subseteq A^* \Rightarrow A^*M \subseteq A^*$. Hence A^* is a right ideal of M. □

Theorem 3.2. *Let M be a semiring with involution $*$. Then*

(i) $(xMy)^* = y^*Mx^*$

(ii) $(MxM)^* = Mx^*M$, *for all* $x, y \in M$.

Theorem 3.3. *Let M be a semiring with involution if A is a quasi ideal of M. Then A^* is a quasi ideal of $(M, *)$.*

Proof. Suppose A is a quasi ideal of M. Then $(AM) \cap (MA) \subseteq A \Rightarrow [(AM) \cap (MA)]^* \subseteq A^* \Rightarrow (AM)^* \cap (MA)^* \subseteq A^* \Rightarrow (M^*A^*) \cap (A^*M^*) \subseteq A^* \Rightarrow (MA^*) \cap (A^*M) \subseteq A^*$. Hence A^* is a quasi ideal of the $(M, *)$. □

Theorem 3.4. *Let M be a semiring . Then A^* is a left ideal of $(M, *)$, for any right ideal A of M.*

Proof. Let A be a right ideal of the semiring M. Then $AM \subseteq A$ and $M^* = M \Rightarrow MA^* = M^*A^* = (AM)^* \subseteq A^*$. Thus A^* is a left ideal of the semiring M with involution. □

Theorem 3.5. *Let M be a semiring with involution. If A is a bi-quasi ideal of M then A^* is a bi-quasi ideal of M.*

Proof. Let A be a bi-quasi ideal of M. Then $AM \cap AMA \subseteq A \Rightarrow (AM \cap AMA)^* \subseteq A^* \Rightarrow A^*M^*A^* \cap M^*A^* \subseteq A^* \Rightarrow A^*MA^* \cap MA^* \subseteq A^*$. Hence A^* is a bi-quasi ideal of M. □

Theorem 3.6. *Let M be a semiring with involution. Then $A = A^*A^*$, for all subsemirings of M if and only if M is a regular semiring.*

Proof. Let A be a left ideal of M with involution. Then $A = A^*A^* \subseteq A^* \subseteq A$ Therefore $A^* = A$. Let A be a right ideal and B be a left ideal of M. Then $A = A^*A$ and $B = B^*$. Now $A^*B^* = A \cap B \Rightarrow AB \subseteq AM \subseteq A \Rightarrow AB \subseteq MB \subseteq B$. Therefore $AB \subseteq A \cap B$. Now $A^*B^* = A \cap B \Rightarrow AB = (AB)(AB) \subseteq (A \cap B)(A \cap B) \Rightarrow AB \subseteq A \cap B \Rightarrow BA \subseteq A \cap B$. Now $A^*B^* = A \cap B \Rightarrow A \cap B = (A \cap B)(A \cap B) \subseteq AB$. Therefore $AB = A \cap B$. Hence M is a regular semiring. □

Theorem 3.7. *Let M be a semiring with involution $*$. If $\{A_i \mid i \in I\}$ is a family of left ideals of semiring then the $\cap A_i \neq \phi$ is a left ideals of semiring M.*

Proof. Let $\{A_i \mid i \in I\}$ is a family of left ideals of semiring M. Then

$$\begin{aligned} & M \cap A_i^* \subseteq MA_i^*, \quad \text{for all } i \\ & \qquad \subseteq A_i^*, \quad \text{for all } i \\ \Rightarrow \ & M \cap A_i^* \subseteq \cap A_i \end{aligned}$$

□

Theorem 3.8. *Let M be a semiring with involution $*$. Then A^* is a bi-interior ideal of M if A is a bi-interior ideal of M.*

Proof. Suppose A is a bi-interior ideal of M. Then $AMA \cap MAM \subseteq A \Rightarrow (AMA \cap MAM)^* \subseteq A^* \Rightarrow A^*MA^* \cap MA^*M \subseteq A^*$. Hence A^* is a bi-interior ideal of M. □

Corollary 3.9. *Let M be a semiring with involution $*$. If A is a left ideal and right ideal of M then A^* is an ideal of M with involution.*

Theorem 3.10. *Let M be a semiring with involution if A is a quasi ideal of M. Then A^* is a quasi ideal of M.*

Proof. Suppose A is a quasi ideal of M. Then $(AM) \cap (MA) \subseteq A \Rightarrow [(AM) \cap (MA)] \subseteq A^* \Rightarrow (AM)^* \cap (MA)^* \subseteq A^* \Rightarrow (M^*A^*) \cap (A^*M^*) \subseteq A^* \Rightarrow (MA^*) \cap (A^*M) \subseteq A^*$. Hence A^* is a quasi ideal of M. □

Theorem 3.11. *Let M be a semiring with involution $*$. Then A^* is a left (right) ideal for any right ideal A of M.*

Proof. Let A be a right ideal of the semiring M. Then $AM \subseteq A$ and $M^* = M$. $MA^* = M^*A^* = (AM)^* \subseteq A^*$. Thus A^* is a left ideal of the semiring M with involution. □

Theorem 3.12. *Let M be a semiring with involution $*$. If B is an interior ideal of M, then B^* is an interior ideal of M.*

Proof. Let B be a interior ideal of the semiring M. Then $MBM \subseteq B$ and $M^* = M \Rightarrow M^*B^*M^* = (MBM)^* \subseteq B^*$. Hence B^* is an interior ideal of M □

Theorem 3.13. *Let M be a semiring with involution $*$. If B is a bi-ideal of M, then B^* is a bi-ideal of M.*

Proof. Let B be a bi-ideal of M. Then $BMB \subseteq B$ and $M^* = M$. $\Rightarrow$ $B^*M^*B^* = (BMB)^* \subseteq B^*$. Hence B^* is a bi-ideal of M. □

Theorem 3.14. *Let M be a semiring with involution. If A is a bi quasi interior ideal of M then A^* is a bi quasi interior ideal of M.*

Proof. Let A be a bi quasi interior ideal of the semiring with involution$M$. Then $AMAMA \subseteq A \Rightarrow (AMAMA)^* \subseteq A^* \Rightarrow A^*M^*A^*M^*A^* \subseteq A^* \Rightarrow A^*MA^*MA^* \subseteq A^*$. Hence A^* is a bi-quasi interior ideal of M. □

Theorem 3.15. *Let M be a semiring with involution $*$. If $\{A_i/i \in I\}$ is a family of bi-interior ideals of semiring M then $\cap A_i^* \neq \phi$ is a bi-interior ideal of a semiring M.*

Proof. Let $\{A_i/i \in I\}$ be a family of bi-interior ideals of semiring M with involution. Now $A_iMA_i \cap MA_iM \subseteq A_i \Rightarrow A_i^*MA_i^* \cap MA_i^*M \subseteq A_i^* \Rightarrow (\cap A_i^*M \cap A_i^*) \bigcap (M \cap A_i^*M) \subseteq A_i^*M \cap A_i^* \bigcap (MA_i^*M) \subseteq A_i \Rightarrow (\cap A_i^*M \cap A_i^*) \bigcap (M \cap A_i^*M) \subseteq \cap A_i^*$. Hence $\cap A_i^*$ is a bi-interior ideal of a semiring M with involution. □

Theorem 3.16. *Let M be a semiring with involution. Then $A = AA$, for all left ideals and right deals of M, if and only if M is a regular semiring.*

Proof. Let A be a left ideal of M with involution. Then $A = AA \Rightarrow (AA)^* = A^* \subseteq A \Rightarrow A^*A^* = AA \Rightarrow A^* \subseteq AA) \subseteq A$. Therefore $A^* = A$. Let A be a right ideal and B be a left ideal of M. Then $A = A^*$ and $B = B^*$. Now $AB \subseteq AM \subseteq A$ and $AB \subseteq MB \subseteq B$. Therefore $AB \subseteq A \cap B$. Now

$$\begin{aligned} A^*B^* &\subseteq A^* \cap B^* \\ AB &= (AB)(AB) \\ &\subseteq (A \cap B)(A \cap B) \\ AB &\subseteq AB \\ BA &\subseteq AB. \end{aligned}$$

Now $A \cap B = (A \cap B) \cap (A \cap B) \subseteq AB$. Therefore $AB = A \cap B$. Hence M is a regular semiring. □

Theorem 3.17. *Let M be a semiring with involution $*$. If $\{A_i \mid i \in I\}$ is a family of right ideals of semiring, then $\cap A_i^* \neq \phi$ is a left ideals of semiring M.*

Proof. Let $\{A_i \mid i \in I\}$ is a family of right ideals of semiring M. Then

$$\begin{aligned} M \cap A_i^* &\subseteq MA_i^*, \text{ for all } i \\ &\subseteq A_i^*, \text{ for all } i \\ \Rightarrow M \cap A_i^* &\subseteq \cap A_i^*. \end{aligned}$$

Hence $\cap A_i^*$ is a left ideal of M. □

Corollary 3.18. *Let M be a semiring with involution $*$. If $\{A_i \mid i \in I\}$ is a family of left ideals of semiring then $\cap A_i^* \neq \phi$ is a right ideal of semiring M.*

Corollary 3.19. *Let M be a semiring with involution $*$. If $\{A_i \mid i \in I\}$ is a family of ideals of semiring then the $\cap A_i^* \neq \phi$ is an ideal of semiring M*

Theorem 3.20. *Let M be a semiring with involution $*$. Then A^* is a bi-interior ideal of M, if A is a bi-interior ideal of M.*

Proof. Suppose A is a bi-interior ideal of M. Then $AMA \cap MAM \subseteq A(AMA \cap MAM)^* \subseteq A^* \Rightarrow A^*MA^* \cap MA^*M \subseteq A^*$. Hence A^* is a bi-interior ideal of M. □

Theorem 3.21. *Let M be a semiring with involution $*$. Then A^* is a right tri-ideal of M, if A is a left tri-ideal of M.*

Proof. Let A be a left tri-ideal of M. Then $AMAA \subseteq A \Rightarrow (AMAA)^* \subseteq A^* \Rightarrow A^*A^*M^*A^* \subseteq A^* \Rightarrow A^*A^*MA^* \subseteq A^*$. Hence A^* is a right tri-ideal of M. □

Corollary 3.22. *Let M be a semiring with involution $*$. Then A^* is a left tri-ideal of M, if A is a right tri-ideal of M.*

4. Conclusion

In this work, we have introduced the notion of involution in semirings and extended the existing theory by defining various types of ideals including bi-ideal, quasi ideal, interior ideal, bi-quasi interior ideal, and bi-interior ideal within the framework of semirings with involution. We have systematically studied their properties and explored how these structures interact under the defined operations. The results presented contribute to a deeper understanding of the algebraic structure of semirings with involution and provide a foundation for further research in this area.

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References

[1] P. J. Allen, *A fundamental theorem of homomorphism for semirings,* Proc. Amer. Math. Sco., **21** (1969), 412–416.
[2] T. K. Dutta and S.K. Sardar, *On the operator semirings of a $\Gamma-$semiring,* Southeast Asian Bull. Math., **26** (2002), 203-213.
[3] R. A. Good and D. R. Hughes, *Associated groups for a semigroup,* Bull. Amer. Math. Soc., **58** (1952), 624-625.
[4] M. Henriksen, *Ideals in semirings with commutative addition,* Amer. Math. Soc. Noti,, **5** (1958), 321.
[5] K. Iseki, *Quasi-ideals in semirings without zero,* Proc. Japan Acad., **34** (1958), 79-84.
[6] K. Iseki, *Ideal theory of semiring,* Proc. Japan Acad., **32** (1956), 554-559.
[7] K. Iseki, *Ideal in semirings,* Proc. Japan Acad., **34** (1958), 29-31.
[8] K. Izuka, *On the Jacobson radical of a semiring,* Tohoku, Math. J., **11(2)** (1959), 409-421.
[9] R. D. Jagatap, Y.S. Pawar, *Quasi-ideals and minimal quasi-ideals in $\Gamma-$semirings,* Novi Sad J. Math., **39(2)** (2009), 79-87.
[10] S. Lajos and F. A. Szasz, *On the bi-ideals in associative ring,* Proc. Japan Acad., **46** (1970), 505-507.
[11] S. Lajos, *On the bi-ideals in semigroups,* Proc. Japan Acad., **45** (1969), 710-712.
[12] S. Lajos, (m; k; n)-ideals in semigroups. In: Notes on Semigroups II, Karl Marx Univ. Econ., Dept. Math. Budapest (1976), No. 1, 12-19.
[13] H. Lehmer , *A ternary analogue of abelian groups,* Amer. J. of Math., **59** (1932), 329-338.
[14] G. Lister , *Ternary rings,* Trans. of Amer. Math. Soc., **154** (1971), 37-55.
[15] M. Murali Krishna Rao, *$\Gamma-$semirings-I,* Southeast Asian Bull. Math., **19 (1)** (1995), 49–54.
[16] M. Murali Krishna Rao, *Γ-semirings-II,* Southeast Asian Bulletin of Mathematics, **21(3)**(1997,) 281-287
[17] M. Murali Krishna Rao, *The Jacobson radical of Γ-semiring,* Southeast Asian Bull. of Math., **23** (1999), 127-134
[18] M. Murali Krishna Rao and B. Venkateswarlu, *Regular $\Gamma-$incline and field $\Gamma-$semiring,* Novi Sad J. of Math., **45 (2),** (2015), 155-171.

[19] M. Murali Krishna Rao, $\Gamma-$ *semiring with identity,* Disc. Math. Gen. Alg. and Appl., **37** (2017,) 189-207, doi:10.7151/dmgaa.1276.
[20] M. Murali Krishna Rao, *Ideals in ordered* $\Gamma-$*semirings,* Disc. Math. Gen. Alg. and Appl., **38** (2018), 47-68 doi:10.7151/dmgaa.1284
[21] M. Murali Krishna Rao, *Bi-interior Ideals in semigroups,* Disc. Math. Gen. Alg. and Appl., **38** (2018),69-78 doi:10.7151/dmgaa.1284
[22] M. Murali Krishna Rao, *Left bi-quasi ideals of semirings,* Bull. Int. Math. Virtual Inst , **8** (2018), 45-53 . 10.7251/BIMVI1801045R
[23] M.Murali Krishna Rao, *Bi-quasi-ideals and fuzzy bi-quasi ideals of* $\Gamma-$*semigroups*, Bull. Int. Math. Virtual Inst., **7(2)** (2017), 231-242.
[24] Marapureddy Murali Krishna Rao, *A study of bi-quasi interior ideal as a new generalization of ideal of generalization of semiring,* Bull. Int. Math. Virtual Inst , **8** (2018), 519-535.
[25] M.Murali Krishna Rao, *Quasi- interior ideals of semirings*, Bull. Int. Math. Virtual Inst., **9(1)** (2018), 231-242.
[26] M. Murali Krishna Rao, *A study of generalization of bi- ideal, quasi- ideal and interior ideal of semigroup*, Mathematica Morovica, Vol.22,2(2018),103-115.
[27] M. Murali Krishna Rao,Ideal Theory of Gamma-semirings, LAMBERT ACADEMIC PUBLISHING,ISBN: 978-620-0-22278-7 June 2019
[28] M. Murali Krishna Rao, B. Venkateswarlu and N.Rafi, *Left bi-quasi-ideals of* $\Gamma-$*semirings,* Asia Paci. J. of Math., **4(2)** (2017), 144-153 ISSN 2357-2205.
[29] M. Murali Krishna Rao and B. Venkateswarlu, *Bi-interior Ideals in* $\Gamma-$ *semirings* , Disc. Math. Gen. Alg. and Appl., **38(2)** (2018) 209-221 doi:10.7151/dmgaa.1258
[30] M. Murali Krishna Rao and B. Venkateswarlu, *Right derivation of ordered* $\Gamma-$*semirings* , Disc. Math. Gen. Alg. and Appl., **36** (2016) 209-221 doi:10.7151/dmgaa.1258
[31] M. Murali Krishna Rao and B. Venkateswarlu, *On generalized right derivations of incline,* J. Of The Int. Math. Virtual Inst., **6** (2016), 31-47.
[32] N. Nobusawa, *On a generalization of the ring theory,* Osaka. J.Math., **1** (1964), 81 – 89.
[33] M. K. Sen, *On* $\Gamma-$*semigroup,* Proc. of International Conference of algebra and its application, (1981), Decker Publicaiton, New York, 301–308
[34] A. M. Shabir , A Batod, *A note on quasi ideal in semirings,* Southeast Asian Bull. Math., **7** (2004), 923-928.
[35] O. Steinfeld, *Uher die quasi ideals, Von halbgruppend* , Publ. Math., Debrecen, **4** (1956), 262 - 275.
[36] H. S. Vandiver, *Note on a simple type of algebra in which cancellation law of addition does not hold,* Bull. Amer. Math. Soc.(N.S.), **40** (1934), 914–920.

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