



FUZZY WEAK-INTERIOR IDEALS OF SEMIRINGS

M. MURALI KRISHNA RAO

ABSTRACT. In this paper, we introduce the notion of a fuzzy weak interior ideal as a generalization of a fuzzy ideal of a semiring. We characterize the regular semiring in terms of fuzzy weak interior ideals of a semiring.

1. INTRODUCTION

The notion of a semiring was introduced by Vandiver [21] in 1934, but semirings had appeared in earlier studies on the theory of ideals of rings. The theory of rings and theory of semigroups have considerable impact on the development of theory of semirings. Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in the solution of graph theory, optimization theory, in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches of mathematics. Ideals play an important role in advance studies and uses of algebraic structures. Generalization of ideals in algebraic structures is necessary for further study algebraic structures. Many mathematicians proved important results and characterization of algebraic structures by using the concept and the properties of generalization of ideals in algebraic structures. Henriksen [3] and Shabir et al. [19] studied ideals in semirings. We know that the notion of a one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi ideals are generalization of left ideal and right ideal whereas the bi-ideals are generalization of quasi ideals. In 1952, the concept of bi-ideals was introduced by Good and Hughes [2] for semigroups. The notion of bi-ideals in rings and semirings were introduced by Lajos [5] and Szasz [11, 12]. Bi-ideal is a special case of $(m-n)$ ideal. In 1956, Steinfeld [20] first introduced the notion of quasi ideals for semigroups and then for rings. The fuzzy set theory was developed by L. A. Zadeh [22] in 1965. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc. The fuzzification of algebraic structure was introduced by A. Rosenfeld [18] and he introduced the notion

2010 *Mathematics Subject Classification.* 16Y60, 16Y99, 03E72.

Key words and phrases. Interior ideal, Weak interior ideal, Fuzzy Weak interior ideal, Regular Semiring.

Received: April 12, 2025. Accepted: June 19, 2025. Published: June 30, 2025.

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of fuzzy subgroups in 1971. N. Kuroki studied fuzzy semigroups D. Mandal [6] studied fuzzy ideals and fuzzy interior ideals in an ordered semiring.

Marapureddy Murali Krishna Rao [7,8,9,10,11,12] introduced and studied the properties of bi quasi ideals, bi interior ideals, bi quasi interior ideals, quasi interior ideals, tri-quasi ideals, and weak interior ideals of semirings and semigroups. Marapureddy Marapureddy Murali Krishna Rao et.al [13,14,15,16,17] introduced the notions of fuzzy bi quasi ideals, fuzzy bi interior ideals, fuzzy tri-quasi ideals, Fuzzy tri ideals of semirings and semigroups and studied their properties. In this paper, we study the some of the properties of fuzzy weak interior ideals of semirings.

2. PRELIMINARIES

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1 (1). A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called semiring provided

- (i) addition is a commutative operation.
- (ii) multiplication distributes over addition both from the left and from the right.
- (iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

Definition 2.2 (10). Let M be the set of all natural numbers. Then $(M, \max, \min)$ is a semiring.

Definition 2.3 (10). Let M be a semiring. If there exists $1 \in M$ such that $a \cdot 1 = 1 \cdot a = a$, for all $a \in M$, is called an unity element of M then M is said to be semiring with unity.

Definition 2.4 (10). An element a of a semiring S is called a regular element if there exists an element b of S such that $a = aba$.

Definition 2.5 (10). A semiring S is called a regular semiring if every element of S is a regular element.

Definition 2.6 (10). An element a of a semiring S is called a multiplicatively idempotent (an additively idempotent) element if $aa = a(a + a = a)$.

Definition 2.7 (10). An element b of a semiring M is called an inverse element of a of M if $ab = ba = 1$.

Definition 2.8 (10). A semiring M is called a division semiring if for each non-zero element of M has multiplication inverse.

Definition 2.9 (10). A semiring M is called a division semiring if for each non-zero element of M has multiplication inverse.

Definition 2.10 (7,8,9,10,11,12). A non-empty subset A of a semiring M is called

- (i) a subsemiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $AA \subseteq A$.
- (ii) a quasi ideal of M if A is a subsemiring of M and $AM \cap MA \subseteq A$.
- (iii) a bi-ideal of M if A is a subsemiring of M and $AMA \subseteq A$.
- (iv) an interior ideal of M if A is a subsemiring of M and $MAM \subseteq A$.
- (v) a left (right) ideal of M if A is a subsemiring of M and $MA \subseteq A$ ($AM \subseteq A$).
- (vi) an ideal if A is a subsemiring of M , $AM \subseteq A$ and $MA \subseteq A$.
- (vii) a k -ideal if A is a subsemiring of M , $AM \subseteq A$, $MA \subseteq A$ and $x \in M, x + y \in A, y \in A$ then $x \in A$.
- (viii) a bi-interior ideal of M if A is a subsemiring of M and $MBM \cap BMB \subseteq B$.

(ix) a left bi-quasi ideal (right bi-quasi ideal) of M if A is a subsemigroup of $(M, +)$ and $MA \cap AMA \subseteq A(AM \cap AMA \subseteq A)$.

(x) a left quasi-interior ideal (right quasi-interior ideal) of M if A is a subsemiring of M and $MAMA \subseteq A(AMAM \subseteq A)$.

(xi) a bi-quasi-interior ideal of M if A is a subsemiring of M and $BMBMB \subseteq B$.

(xii) a left tri-ideal (right tri-ideal) of M if A is a subsemiring of M and $AMAA \subseteq A(AAMA \subseteq A)$.

(xiii) a tri-ideal of M if A is a subsemiring of M and $AMAA \subseteq A$ and $AAMA \subseteq A$.

(xiv) a left(right) weak-interior ideal of M if B is a subsemiring of M and $MBB \subseteq B(BBM \subseteq B)$.

(xv) a weak-interior ideal of M if B is a subsemiring of M and B is a left weakinterior ideal and a right weak-interior ideal of M .

Definition 2.11 (15). Let f be a fuzzy subset of a non-empty set M , for $t \in [0, 1]$ the set $f_t = \{x \in M | f(x) \geq t\}$ is called a level subset of M with respect to f .

Definition 2.12 (15). Let M be a semiring. A fuzzy subset μ of M is said to be fuzzy subsemiring of M if it satisfies the following conditions

- (i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$
- (ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in M$.

Definition 2.13 (15). A fuzzy subset μ of a semiring M is called a fuzzy left (right) ideal of M if for all $x, y \in M$ it satisfies the following conditions

- (i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$
- (ii) $\mu(xy) \geq \mu(y)(\mu(x))$, for all $x, y \in M$.

Definition 2.14 (15). A fuzzy subset μ of a semiring M is called a fuzzy ideal of M if it satisfies the following conditions

- (i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$
- (ii) $\mu(xy) \geq \max\{\mu(x), \mu(y)\}$, for all $x, y \in M$.

Definition 2.15. [15] For any two fuzzy subsets λ and μ of M , $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in M$.

Definition 2.16. [15] Let f and g be fuzzy subsets of a semiring M . Then $f \circ g, f + g, f \cup g, f \cap g$, are defined by

$$f \circ g(z) = \begin{cases} \sup_{z=xy} \{\min\{f(x), g(y)\}\}, \\ 0 & \text{otherwise} \end{cases}, \quad f + g(z) = \begin{cases} \sup_{z=x+y} \{\min\{f(x), g(y)\}\}, \\ 0 & \text{otherwise} \end{cases}$$

$$f \cup g(z) = \max\{f(z), g(z)\}; f \cap g(z) = \min\{f(z), g(z)\}$$

$x, y, z \in M$.

Definition 2.17 (15). Let A be a non-empty subset of M . The characteristic function of A is a fuzzy subset of M , defined by

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A \end{cases}$$

3. FUZZY WEAK- INTERIOR IDEALS OF SEMIRINGS

In this section, we introduce the notion of a fuzzy right(left) weak- interior ideal as a generalization of a fuzzy interior-ideal of a semiring and study the properties of fuzzy right(left) weak- interior ideals.

Definition 3.1. A fuzzy subset μ of a semiring M is called a fuzzy right(left)weakinterior ideal if,

- (i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in M$.
- (ii) $\mu \circ \mu \circ \chi_M \subseteq \mu(\chi_M \circ \mu \circ \mu \subseteq \mu)$.

Definition 3.2. A fuzzy subset μ of a semiring M is called a fuzzy weak- interior ideal if it is both left and right weak- interior ideal of M .

Example 3.3. Let Q be the set of all rational numbers, $M = \left\{ \begin{pmatrix} 0 & a \\ b & c \end{pmatrix} \mid a, b, c \in Q \right\}$. Then M is a semiring with respect to usual addition of matrices and ternary operation is defined as the usual matrix multiplication. If $A = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a, 0 \neq b \in Q \right\}$ then A is a right weak interior ideal but not a bi-ideal of a semiring M . Define $\mu : M \rightarrow [0, 1]$ such that

$$\mu(x) = \begin{cases} 1, & \text{if } x \in A, \\ 0 & \text{otherwise} \end{cases}$$

Then μ is a fuzzy right weak- interior ideal of M .

Theorem 3.1. Every fuzzy right ideal of a semiring M is a fuzzy right weak- interior ideal of M .

Proof. Let μ be a fuzzy right ideal of the semiring M and $x \in M$.

$$\begin{aligned} \mu \circ \chi_M(x) &= \sup_{x=ab} \min\{\mu(a), \chi_M(b)\} a, b \in M, \\ &= \sup_{x=ab} \mu(a) \\ &\leq \sup_{x=ab} \mu(ab) \\ &= \mu(x) \end{aligned}$$

Therefore $\mu \circ \chi_M(x) \leq \mu(x)$

$$\begin{aligned} \text{Now } \mu \circ \mu \circ \chi_M(x) &= \sup_{x=uv} \min\{\mu(uv), \mu \circ \chi_M(s)\} \\ &\leq \sup_{x=uv} \min\{\mu(uv), \mu(s)\} \\ &= \mu(x) \end{aligned}$$

Hence μ is a fuzzy right weak- interior ideal of the semiring M . □

Corollary 3.2. Every fuzzy left ideal of a semiring M is a fuzzy left weak- interior ideal of M .

Corollary 3.3. Every fuzzy ideal of a semiring M is a fuzzy weak- interior ideal of M .

Theorem 3.4. Let M be a semiring and μ be a non-empty fuzzy subset of M . A fuzzy subset μ is a fuzzy left weak- interior ideal of a semiring M if and only if the level subset μ_t of μ is a left weak- interior ideal of a semiring M for every $t \in [0, 1]$, where $\mu_t \neq \phi$.

Proof. Let M be a semiring and μ be a non-empty fuzzy subset of M . Suppose μ is a fuzzy left weak- interior ideal of a semiring M , $\mu_t \neq \phi$, $t \in [0, 1]$ and $a, b \in \mu_t$. Then

$$\begin{aligned} \mu(a) &\geq t, \mu(b) \geq t \\ \Rightarrow \mu(a + b) &\geq \min\{\mu(a), \mu(b)\} \geq t \\ \Rightarrow a + b &\in \mu_t. \end{aligned}$$

Let $x \in M\mu_t\mu_t$. Then $x = bad$, where $b \in M, a, c \in \mu_t$, Then

$$\begin{aligned}\chi_M \circ \mu \circ \mu(x) &\geq t \\ \Rightarrow \mu(x) &\geq \chi_M \circ \mu \circ \mu(x) \geq t\end{aligned}$$

Therefore $x \in \mu_t$. Hence μ_t is a left weak- interior ideal of the semiring M .

Conversely suppose that μ_t is a left weak- interior ideal of the semiring M , for all $t \in Im(\mu)$.

Let $x, y \in M, \mu(x) = t_1, \mu(y) = t_2$ and $t_1 \geq t_2$. Then $x, y \in \mu_{t_2}$.

$$\begin{aligned}\Rightarrow x + y &\in \mu_{t_2} \text{ and } xy \in \mu_{t_2} \\ \Rightarrow \mu(x + y) &\geq t_2 = \min\{t_1, t_2\} = \min\{\mu(x), \mu(y)\}\end{aligned}$$

Therefore $\mu(x + y) \geq t_2 = \min\{\mu(x), \mu(y)\}$.

We have $M\mu_l\mu_l \subseteq \mu_t$, for all $l \in Im(\mu)$.

Suppose $t = \min\{Im(\mu)\}$. Then $M\mu_t\mu_t \subseteq \mu_t$. Therefore $\chi_M \circ \mu \circ \mu \subseteq \mu$. Hence μ is a fuzzy left weak- interior ideal of the semiring M . $\square$

Corollary 3.5. *Let M be a semiring and μ be a non-empty fuzzy subset of M . A fuzzy subset μ is a fuzzy right weak- interior ideal ideal of a semiring if and only if the level subset μ_t of μ is a right weak- interior ideal ideal of a semiring M for every $t \in [0, 1]$, where $\mu_t \neq \phi$.*

Theorem 3.6. *Let I be a non-empty subset of a semiring M and χ_I be the characteristic function of I . Then I is a right weak- interior ideal ideal of a semiring M if and only if χ_I is a fuzzy right weak- interior ideal of a semiring M .*

Proof. Let I be a non-empty subset of a semiring M and χ_I be the characteristic function of I . Suppose I is a right weak- interior ideal of the semiring M . Obviously χ_I is a fuzzy subsemiring of M . We have $IIM \subseteq I$. Then

$$\chi_I \circ \chi_I \circ \chi_M = \chi_{IIM} = \chi_{IIM} \subseteq \chi_I$$

. Therefore χ_I is a fuzzy right weak- interior ideal of the semiring M . Conversely suppose that χ_I is a fuzzy right weak- interior ideal of M . Then I is a subsemiring of M . We have

$$\chi_I \circ \chi_I \circ \chi_M \subseteq \chi_I$$

$$\Rightarrow \chi_{IIM} \subseteq \chi_I$$

Therefore $IIM \subseteq I$.

Hence I is a right weak- interior ideal of the semiring M . $\square$

Theorem 3.7. *If μ and λ are fuzzy left weak- interior ideals of a semiring M , then $\mu \cap \lambda$ is a fuzzy left weak- interior ideal of a semiring M .*

Proof. Let μ and λ be fuzzy left weak- interior ideals of the semiring M and $x, y \in M$. Then

$$\begin{aligned}
 \mu \cap \lambda(x+y) &= \min\{\mu(x+y), \lambda(x+y)\} \\
 &\geq \min\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\} \\
 &= \min\{\min\{\mu(x), \lambda(x)\}, \min\{\mu(y), \lambda(y)\}\} \\
 &= \min\{\mu \cap \lambda(x), \mu \cap \lambda(y)\} \\
 \chi_M \circ \mu \cap \lambda(x) &= \sup_{x=ab} \min\{\chi_M(a), \mu \cap \lambda(b)\} \\
 &= \sup_{x=ab} \min\{\chi_M(a), \min\{\mu(b), \lambda(b)\}\} \\
 &= \sup_{x=ab} \min\{\min\{\chi_M(a), \mu(b)\}, \min\{\chi_M(a), \lambda(b)\}\} \\
 &= \min\{\sup_{x=ab} \min\{\chi_M(a), \mu(b)\}, \sup_{x=ab} \min\{\chi_M(a), \lambda(b)\}\} \\
 &= \min\{\chi_M \circ \mu(x), \chi_M \circ \lambda(x)\} \\
 &= \chi_M \circ \mu \cap \chi_M \circ \lambda(x)
 \end{aligned}$$

Therefore $\chi_M \circ \mu \cap \lambda \circ \mu \cap \lambda = \chi_M \circ \mu \circ \mu \cap \chi_M \circ \lambda \circ \lambda$

Hence $\chi_M \circ \mu \cap \lambda \circ \mu \cap \lambda = \chi_M \circ \mu \circ \mu \cap \chi_M \circ \lambda \circ \lambda \subseteq \mu \cap \lambda$.

Hence $\mu \cap \lambda$ is a left fuzzy weak- interior ideal of M . Hence the theorem. $\square$

Corollary 3.8. *If μ and λ are fuzzy right weak- interior ideals of semiring M then $\mu \cap \lambda$ is a fuzzy right weak- interior ideal of semiring M .*

Corollary 3.9. *Let μ and λ be fuzzy right ideal and fuzzy left ideal of a semiring M respectively. Then $\mu \cap \lambda$ is a fuzzy weak- interior ideal of a semiring M .*

Corollary 3.10. *Let μ and λ be fuzzy right ideal and fuzzy left ideal of semiring M respectively. Then $\mu \cap \lambda$ is a right fuzzy weak- interior ideal of semiring M .*

Proof of the following theorems are similar to theorems in [14]. So we omit the proofs.

Theorem 3.11. *If μ is a fuzzy tri-ideal of regular semiring M then μ is a fuzzy ideal of M .*

Theorem 3.12. *A semiring M is a regular if and only if $\lambda \circ \mu = \lambda \cap \mu$, for any fuzzy right ideal λ and fuzzy left ideal μ of M .*

Theorem 3.13. *Let M be a regular semiring. Then μ is a fuzzy left weak- interior ideal of M if and only if μ is a fuzzy ideal of M .*

Proof. Let μ be a fuzzy left weak- interior ideal of the semiring M and $x \in M$. Then

$$\chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu$$

Suppose $\chi_M \circ \mu(x) > \mu(x)$ and $\mu \circ \chi_M(x) > \mu(x)$

Since M is a regular, there exist $y \in M$ such that $x = xyx$.

$$\begin{aligned}
 \mu \circ \chi_M(x) &= \sup_{x=xyx} \min\{\mu(x), \chi_M(yx)\} \\
 &= \sup_{x=xyx} \min\{\mu(x), 1\} \\
 &= \sup_{x=xyx} \mu(x) \\
 &> \mu(x).
 \end{aligned}$$

$$\begin{aligned}
 \mu \circ \chi_M \circ \mu \circ \chi_M(x) &= \sup_{x=xyx} \min\{\mu \circ \chi_M(x), \mu \circ \chi_M(yx)\} \\
 &> \sup_{x=xyx} \min\{\mu(x), \mu(yx)\} \\
 &= \mu(x)
 \end{aligned}$$

Which is a contradiction. Hence μ is a fuzzy ideal of M . By Theorem [4.15], converse is true $\square$

Corollary 3.14. *Let M be a regular semiring. Then μ is a fuzzy right weak interior ideal of M if and only if μ is a fuzzy ideal of M .*

Theorem 3.15. *Let M be a semiring. Then M is a regular if and only if $\mu = \chi_M \circ \mu \circ \mu$, for any fuzzy left weak-interior ideal μ of a semiring M .*

Proof. Let μ be a fuzzy left weak-interior ideal of the regular semiring M and $x, y \in M$. Then $\chi_M \circ \mu \circ \mu \subseteq \mu$.

$$\begin{aligned}\chi_M \circ \mu \circ \mu(x) &= \sup_{x=xyx} \{ \min\{\chi_M \circ \mu(x), \mu(yx)\} \} \\ &\geq \sup_{x=xyx} \{ \min\{\mu(x), \mu(yx)\} \} \\ &= \mu(x).\end{aligned}$$

Therefore $\mu \subseteq \chi_M \circ \mu \circ \mu$. Hence $\chi_M \circ \mu \circ \mu = \mu$.

Conversely suppose that $\mu = \chi_M \circ \mu \circ \mu$, for any fuzzy weak-interior ideal μ of the semiring M . Let B be a weak-interior ideal of the semiring M .

Then by Theorem 3.8, χ_B be a fuzzy weak-interior ideal of the semiring M .

$$\begin{aligned}\text{Therefore } \chi_B &= \chi_M \circ \chi_B \circ \chi_B \\ &= \chi_{MBB} \\ B &= MBB\end{aligned}$$

Hence M is a regular semiring $\square$

4. CONCLUSIONS

As a further generalization of ideals, Marapureddy Murali Krishna Rao⁰ introduced the notions of bi quasi ideals, quasi interior ideals, bi interior ideals, tri-quasi ideals, tri ideals and weak interior ideals of semirings and semigroups as a generalization of an ideal, a (right) ideal, a quasi ideal, a bi ideal and an interior ideal and studied some of their properties. In this paper, we introduced fuzzy weak interior ideals of a semiring and studied some of their properties. In continuity of this paper, we study the properties of fuzzy prime weak interior ideals, fuzzy maximal weak interior ideals and fuzzy soft weak interior ideals of algebraic structures.

5. ACKNOWLEDGEMENTS

The authors are thankful to the referee for useful comments and suggestions which have definitely improved the paper.

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