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ON SOME RESULTS ABOUT BIPOLAR SOFT IDEALS OF GAMMA NEAR-RINGS

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ABSTRACT. With the advancement of scientific studies, problems involving uncertainty have emerged. Classical mathematics needed to be more robust to model or make sense of these uncertain problems. Due to this insufficiency, scientists have put forward new theories. One of them is the soft set theory, which Molodtsov first studied. Then, many researchers have done various studies in different fields using soft sets. In this study, bipolar soft ideals of a gamma near-ring were defined. Some basic properties of this algebraic structure have been analyzed. The bipolar soft coset set was determined with the help of bipolar soft ideals, and this set was shown to be a gamma near-ring. Finally, a function was defined with the help of bipolar soft ideals of a gamma near-ring, and this study concluded by showing that this function is a gamma near-ring epimorphism.

1. INTRODUCTION

Mathematics has been needed since the history of humanity has existed. It is an indisputable fact that mathematics plays an important role in making sense of life. The advancement of science requires interdisciplinary studies that are directly related to the improvement of mathematical studies. While certain results were obtained, in which Aristotelian logic was at the forefront, with the development of human history, classical mathematics was insufficient in line with technological needs. Systems that cannot be solved with classical logic and whose uncertainty situations cannot be modeled have introduced concepts such as fuzzy set theory [27], rough set theory [24], soft set theory [20].

A simple mechanism, radio, can be considered to understand Zadeh's fuzzy set work. In addition to the precise expressions of the sound 'off' and 'on' in the radio, a range is obtained in the decision-making mechanism when 'increase and decrease the volume' is added. This range is obtained by the range of real numbers [0,1]. Zadeh's work presents modeling that facilitates decision-making in imprecise statements to the scientific world in real-life situations. The idea, which Zadeh calls "a little simple and a little fun," later

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became the first step in changing the perspective of the scientific world and humanizing the machines in the world of technology. With his work, Zadeh has removed the sharpness of classical logic and opened a new horizon in science. In 1970, fuzzy logic was first applied to a steam engine by Prof. Ebrahim H. Hamdani at Queen Mary College, London. In 1980, it was used to control the kiln of a cement factory in Denmark. They analyzed that the temperature of the chemicals used in the furnace needed to be more or less with fuzzy logic. Thus, they led to precise measurements and prevention of errors.

Due to the difficulties in determining the membership function required for the operations performed in the fuzzy set, scientists wanted to create an uncertainty without a membership function. As a result of the studies, the soft set theory, which Molodstov obtained without needing a membership function, was put forward. Molodstov has applied soft set theory to many fields, such as game theory, probability theory, Riemann integral, and measurement theory [20]. Molodstov's work was followed in the world of science, and Maji et al. [19] first defined the soft set operations by examining the algebraic structures of the soft set.

Aktaş and Çağman [3] defined a soft group and examined the relationship between fuzzy and soft sets. This study carried soft sets over an algebraic structure for the first time. Later, Acar et al. [2] conducted a study on soft rings. Feng et al. [11, 12, 13] studied soft semirings and soft sets of fuzzy and rough sets. Çağman and Enginoğlu [6] redefined soft set operations and made them more functional. Later, algebraic structures of soft intersection groups and rings were investigated by [7] and [8].

While studies continue in the scientific world, Shabir and Naz [26] defined bipolar soft sets as a new perspective on soft sets. Naz and Shabir [22] studied the algebraic structure of bipolar fuzzy soft sets. Karaaslan and Karataş [16] redefined the bipolar soft set concept and bipolar soft set operations from a different perspective. They also presented an applied decision-making method by examining its basic features. Abdullah et al. [1] presented bipolar fuzzy soft sets as a decision-making method. The structure of the fuzzy soft near-gamma ring was investigated by Ersoy et al. [10]. Karaaslan et al. [17] defined the bipolar soft group with their new approach. Karaaslan [18] conducted studies on bipolar soft-rough relations. Hayat and Mahmood [15] examined some algebraic structures of bipolar soft sets. Hakim et al. [14] investigated the structure of fuzzy bipolar soft semiprime ideals in ordered semigroups and examined some of their properties. On the other hand, Çıtak [9] studied the bipolar soft ideal in near-rings. Mubarak et al. [21] investigated the roughness of fuzzy bipolar soft sets with the help of soft binary relations.

In algebra, Nobusawa [23] studied rings, and then Barnes [4] studied Nobusawa's gamma rings. Booth [5] studied gamma near-rings.

In this study, bipolar soft ideals of gamma near-rings are discussed. First, bipolar soft sets, bipolar soft groups, and bipolar soft ideals are investigated. Next, the bipolar soft ideal of a gamma near-ring is defined, and some of its basic properties are examined. With the help of bipolar soft ideals, the bipolar soft coset set is constructed, and it is shown that this set is a gamma near-ring. By defining the bipolar soft set with the help of coset set, it has been proved that this set is a bipolar soft ideal. The study is concluded by defining a mapping using the bipolar soft ideal of gamma near-ring and showing that this mapping is a gamma near-ring epimorphism.

2. PRELIMINARY

Definition 2.1. [25] Let $(\aleph, +)$ be a group (not necessarily abelian) and Γ be a nonempty set. Then, $\aleph$ is said to be a Γ-near-ring (Gamma near-ring) if there exists a mapping $\aleph \times \Gamma \times \aleph \longrightarrow \aleph$ the following requirements:

- $(n_1 + n_2)tn_3 = n_1tn_3 + n_2tn_3$
- $(n_1tn_2)kn_3 = n_1t(n_2kn_3)$

for all $n_1, n_2, n_3 \in \aleph$ and $t, k \in \Gamma$

Definition 2.2. [25] Let $\aleph$ be a Γ-near-ring and I be a normal subgroup of $\aleph$. Then, for all $n_1, n_2 \in \aleph$, $t \in \Gamma$, $i \in I$

- I is said to be a left ideal if $n_1t(n_2 + i) - n_1tn_2 \in I$
- I is said to be a right ideal if $itn_1 \in I$
- I is said to be ideal if it is both a left ideal and a right ideal.

Definition 2.3. [20] Let E be a parameter set, $S \subseteq E$, and U be an initial set, $P(U)$ be a power set of U. Then, a mapping $F : S \to P(U)$ is said to be a soft set over U. A soft set over U can be represented by $F_S = \{(m, F(m)) | \forall m \in S\}$.

Definition 2.4. [18] Let E be a parameter set, $S \subseteq E$ and $\Im : S \to E$ be an injective function. Then, $S \cup \Im(S)$ is said to be an extended parameter set of S and denoted by ξ_S.

Definition 2.5. [18] Let E be a parameter set, $S \subseteq E$, U be an initial set and $\Im : S \to \xi_S$ be an injective function such that $S \cup \Im(S) = \xi_S$. If $\Im_1 : S \to P(U)$ and $\Im_2 : \Im(S) \to P(U)$ are two mappings such that $\Im_1(m) \cap \Im_2(\Im(m)) = \emptyset$, then $(\Im_1, \Im_2)$ is said to be a bipolar soft set over U. A bipolar soft set over U can be represented by

$$\Im_S = (\Im_1, \Im_2) = \{(m, \Im_1(m), \Im_2(\Im(m))) | \forall m \in E, \Im_1(m) \cap \Im_2(\Im(m)) = \emptyset\}$$

Remark. [18] *Henceforward, we will denote the sets $\Im_1$ and $\Im_2$ with $\Im_S^+$ and $\Im_S^-$, respectively and these sets will be called positive and negative soft sets of a bipolar soft set $\Im_S$, respectively. A bipolar soft set over U can be represented by*

$$\Im_S = (\Im_S^+, \Im_S^-) = \{(m, \Im_S^+(m), \Im_S^-(m)) | \forall m \in E, \Im_S^+(m) \cap \Im_S^-(m)) = \emptyset\}$$

Definition 2.6. [16] Let $\Im_S = (\Im_S^+, \Im_S^-)$ and $\Im_T = (\Im_T^+, \Im_T^-)$ be two bipolar soft sets over U. Then,

- $\Im_S$ is said to be a null bipolar soft set if $\Im_S^+(m) = \emptyset$ and $\Im_S^-(m) = U$, and, denoted by $(\emptyset, U)$
- $\Im_S$ is said to be a absolute bipolar soft set if $\Im_S^+(m) = U$ and $\Im_S^-(m) = \emptyset$, and, denoted by $(U, \emptyset)$
- $\Im_S$ is said to be a bipolar soft subset of $\Im_T$ if $\Im_S^+(m) \subseteq \Im_T^+(m)$ and $\Im_S^-(m) \subseteq \Im_T^-(m)$, and, denoted by $\Im_S \sqsubseteq \Im_T$
- $\Im_S$ is said to be an equal bipolar soft set if $\Im_S \sqsubseteq \Im_T$ and $\Im_T \sqsubseteq \Im_S$, and denoted by $\Im_S = \Im_T$
- Union of $\Im_S$ and $\Im_T$, denoted by $\Im_S \sqcup \Im_T = \Im_{S \cup T}$, is defined as $\Im_{S\cup T}^+(m) = \Im_S^+(m) \cup \Im_T^+(m)$ and $\Im_{S\cup T}^-(m) = \Im_S^-(m) \cup \Im_T^-(m)$
- Intersection of $\Im_S$ and $\Im_T$, denoted by $\Im_S \sqcap \Im_T = \Im_{S\cap T}$, is defined as $\Im_{S\cap T}^+(m) = \Im_S^+(m) \cap \Im_T^+(m)$ and $\Im_{S\cap T}^-(m) = \Im_S^-(m) \cap \Im_T^-(m)$

for all $m \in E$.

Theorem 2.1. [16] *Let $\Im_S = (\Im_S^+, \Im_S^-)$, $\Im_T = (\Im_T^+, \Im_T^-)$ and $\Im_T = (\Im_K^+, \Im_K^-)$ be bipolar soft sets over U. Then,*

- $(\emptyset, U) \sqsubseteq \Im_S$ *and* $\Im_S \sqsubseteq (U, \emptyset)$
- $\Im_S \sqcup (\emptyset, U) = \Im_S$ *and* $\Im_S \sqcap (\emptyset, U) = \emptyset$
- $\Im_S \sqcup (U, \emptyset) = (U, \emptyset)$ *and* $\Im_S \sqcap (U, \emptyset) = (U, \Im_S)$
- $\Im_S \sqcup \Im_S = \Im_S$ *and* $\Im_S \sqcap \Im_S = \Im_S$
- $\Im_S \sqcup \Im_T = \Im_T \sqcup \Im_S$ *and* $\Im_S \sqcap \Im_T = \Im_T \sqcap \Im_S$
- $\Im_S \sqcup (\Im_T \sqcup \Im_K) = (\Im_S \sqcup \Im_T) \sqcup \Im_K$ *and* $\Im_S \sqcap (\Im_T \sqcap \Im_K) = (\Im_S \sqcap \Im_T) \sqcap \Im_K$
- $\Im_S \sqcup (\Im_T \sqcap \Im_K) = (\Im_S \sqcup \Im_T) \sqcap (\Im_S \sqcup \Im_K)$ *and* $\Im_S \sqcap (\Im_T \sqcup \Im_K) = (\Im_S \sqcap \Im_T) \sqcup (\Im_S \sqcap \Im_K)$

3. Bipolar Soft Ideals of Gamma Near-Rings

Definition 3.1. Let $\aleph$ be a Γ-near-ring and $\Im_\aleph = (\Im_\aleph^+, \Im_\aleph^-)$ be a bipolar soft set over U. Then, $\Im_\aleph = (\Im_\aleph^+, \Im_\aleph^-)$ is said to be a Γ-bipolar soft left ideal over U if

- $\Im_\aleph^+(n_1 - n_2) \supseteq \Im_\aleph^+(n_1) \cap \Im_\aleph^+(n_2)$, $\Im_\aleph^-(n_1 - n_2) \subseteq \Im_\aleph^-(n_1) \cup \Im_\aleph^-(n_2)$
- $\Im_\aleph^+(n_2 + n_1 - n_2) \supseteq \Im_\aleph^+(n_1)$, $\Im_\aleph^-(n_2 + n_1 - n_2) \subseteq \Im_\aleph^-(n_1)$
- $\Im_\aleph^+(n_1 t(n_2 + n_3) - n_1 t n_2) \supseteq \Im_\aleph^+(n_3)$, $\Im_\aleph^-(n_1 t(n_2 + n_3) - n_1 t n_2) \subseteq \Im_\aleph^-(n_3)$
  for all $n_1, n_2, n_3 \in \aleph$ and $t \in \Gamma$. Then, $\Im_\aleph = (\Im_\aleph^+, \Im_\aleph^-)$ is called a Γ-bipolar soft right ideal over U if
- $\Im_\aleph^+(n_1 t n_2) \supseteq \Im_\aleph^+(n_1)$, $\Im_\aleph^-(n_1 t n_2) \subseteq \Im_\aleph^-(n_1)$
  for all $n_1, n_2, n_3 \in \aleph$ and $t \in \Gamma$.

Remark. *If $\Im_\aleph$ is both a Γ-bipolar soft left ideal and a Γ-bipolar soft right ideal, then $\Im_\aleph$ is said to be a Γ-bipolar soft ideal over U.*

Proposition 3.1. *Let $\Im_\aleph = (\Im_\aleph^+, \Im_\aleph^-)$ be a Γ-bipolar soft ideal over U.*

- $\Im_\aleph^+(e_\aleph) \supseteq \Im_\aleph^+(n)$, $\Im_\aleph^-(e_\aleph) \subseteq \Im_\aleph^-(n)$
- $\Im_\aleph^+(n) = \Im_\aleph^+(-n)$, $\Im_\aleph^-(n) = \Im_\aleph^-(-n)$
  for all $n \in \aleph$.

Proof.

- For all $n \in \aleph$,
  $$\Im_\aleph^+(e_\aleph) = \Im_\aleph^+(n - n) \supseteq \Im_\aleph^+(n) \cap \Im_\aleph^+(n) = \Im_\aleph^+(n)$$
  Thus, $\Im_\aleph^+(e_\aleph) \supseteq \Im_\aleph^+(n)$. Similarly, it is shown that $\Im_\aleph^-(e_\aleph) \subseteq \Im_\aleph^-(n)$.
- For all $n \in \aleph$
  $$\Im_\aleph^+(-n) = \Im_\aleph^+(e_\aleph - n) \supseteq \Im_\aleph^+(e_\aleph) \cap \Im_\aleph^+(n) = \Im_\aleph^+(n)$$
  Thus, $\Im_\aleph^+(-n) \supseteq \Im_\aleph^+(n)$. Also,
  $$\Im_\aleph^+(n) = \Im_\aleph^+(-(-n)) = \Im_\aleph^+(e_\aleph - (-n)) \supseteq \Im_\aleph^+(e_\aleph) \cap \Im_\aleph^+(-n) = \Im_\aleph^+(-n)$$
  Thus, $\Im_\aleph^+(n) \supseteq \Im_\aleph^+(-n)$. Therefore, $\Im_\aleph^+(n) = \Im_\aleph^+(-n)$ is obtained. Similarly, it is shown that $\Im_\aleph^-(n) = \Im_\aleph^-(-n)$.

□

Definition 3.2. Let $\Im_\aleph = (\Im_\aleph^+, \Im_\aleph^-)$ be a Γ-bipolar soft ideal over U and $n_1 \in \Im_\aleph$. Then, $n_1 + \Im_\aleph$ is said to be a Γ-bipolar soft coset of $\Im_\aleph$ defined by $(n_1 + \Im_\aleph^+)(n_2) = \Im_\aleph^+(n_2 - n_1)$ and $(n_1 + \Im_\aleph^-)(n_2) = \Im_\aleph^-(n_2 - n_1)$ for all $n_2 \in \Im_\aleph$.

Proposition 3.2. *Let $\Im_\aleph = (\Im_\aleph^+, \Im_\aleph^-)$ be a Γ-bipolar soft ideal over U. Then,*

- $\Im_\aleph(n_1 - n_2) = \Im_\aleph(e_\aleph)$ *iff* $n_1 + \Im_\aleph = n_2 + \Im_\aleph$
- $\Im_\aleph(n_1) = \Im_\aleph(n_2)$ *if* $n_1 + \Im_\aleph = n_2 + \Im_\aleph$

- $\Im_{\aleph}(n_1+n_2)=\Im_{\aleph}(n_2+n_1)$

for all $n_1, n_2 \in \aleph$.

Proof. For all $n_1, n_2 \in \aleph$,

- Let $\Im_{\aleph}(n_1-n_2)=\Im_{\aleph}(e_{\aleph})$. For all $n_3 \in \aleph$
$$(n_1+\Im^+_{\aleph})(n_3)=\Im^+_{\aleph}(n_3-n_1)=\Im^+_{\aleph}(e_{\aleph})$$
and
$$(n_2+\Im^+_{\aleph})(n_3)=\Im^+_{\aleph}(n_3-n_2)=\Im^+_{\aleph}(e_{\aleph})$$
Thus, $(n_1+\Im^+_{\aleph})(n_3)=(n_2+\Im^+_{\aleph})(n_3)$. It follows that $n_1+\Im^+_{\aleph}=n_2+\Im^+_{\aleph}$. Similarly, it is shown that $n_1+\Im^-_{\aleph}=n_2+\Im^-_{\aleph}$. Therefore, $n_1+\Im_{\aleph}=n_2+\Im_{\aleph}$. On the other hand, let $n_1+\Im_{\aleph}=n_2+\Im_{\aleph}$. For all $n_4 \in \aleph$
$$(n_1+\Im^+_{\aleph})(n_4)=(n_2+\Im^+_{\aleph})(n_4)$$
$$\Im^+_{\aleph}(n_4-n_1)=\Im^+_{\aleph}(n_4-n_2)$$
By choosing $n_4=n_1$, it follows that
$$\Im^+_{\aleph}(e_{\aleph})=\Im^+_{\aleph}(n_1-n_2).$$
Similarly, it is shown that $\Im^-_{\aleph}(e_{\aleph})=\Im^-_{\aleph}(n_1-n_2)$. Therefore, $\Im_{\aleph}(n_1-n_2)=\Im_{\aleph}(e_{\aleph})$.
- Let $\Im_{\aleph}(n_1)=\Im_{\aleph}(n_2)$. For all $n_3 \in \aleph$,
$$(n_1+\Im^+_{\aleph})(n_3)=(n_2+\Im^+_{\aleph})(n_3)$$
$$\Im^+_{\aleph}(n_3-n_1)=\Im^+_{\aleph}(n_3-n_2).$$
Choosing $n_3=e_{\aleph}$, it follows that
$$\Im^+_{\aleph}(e_{\aleph}-n_1)=\Im^+_{\aleph}(e_{\aleph}-n_2)$$
$$\Im^+_{\aleph}(-n_1)=\Im^+_{\aleph}(-n_2)$$
$$\Im^+_{\aleph}(n_1)=\Im^+_{\aleph}(n_2)$$
Similarly, it is shown that $\Im^-_{\aleph}(n_1)=\Im^-_{\aleph}(n_2)$. Thus, it is obtained that $\Im_{\aleph}(n_1)=\Im_{\aleph}(n_2)$ if $n_1+\Im_{\aleph}=n_2+\Im_{\aleph}$.
- For all $n_3 \in \aleph$,
$$\Im^+_{\aleph}(e_{\aleph})=\Im^+_{\aleph}(n_1+n_2-n_2-n_1)=\Im^+_{\aleph}(n_1+n_2-(n_2+n_1)).$$
Moreover, $n_1+n_2+\Im^+_{\aleph}=n_2+n_1+\Im^+_{\aleph}$. And, it is found that $\Im^+_{\aleph}(n_1+n_2)=\Im^+_{\aleph}(n_2+n_1)$. Similarly, it is shown that $\Im^-_{\aleph}(n_1+n_2)=\Im^-_{\aleph}(n_2+n_1)$. Hence, it is seen to be $\Im_{\aleph}(n_1+n_2)=\Im_{\aleph}(n_2+n_1)$.

□

Remark. *Let* $\Im_{\aleph}=(\Im^+_{\aleph},\Im^-_{\aleph})$ *be a* Γ*-bipolar soft ideal over* U*. The set of all soft cosets is denoted by* $\aleph/\Im_{\aleph}=\{n_1+\Im_{\aleph}: n_1 \in \aleph\}$.

Theorem 3.3. *Let* $\Im_{\aleph}=(\Im^+_{\aleph},\Im^-_{\aleph})$ *be a* Γ*-bipolar soft ideal over* U*.* $\aleph/\Im_{\aleph}$ *is a* Γ*-near-ring with respect to the* $'+'$ *and* $'t'$ *operations where* $(n_1+\Im_{\aleph})+(n_2+\Im_{\aleph})=(n_1+n_2)+\Im_{\aleph}$ *and* $(n_1+\Im_{\aleph})t(n_2+\Im_{\aleph})=(n_1tn_2)+\Im_{\aleph}$ *for all* $n_1,n_2 \in \aleph$ *and* $t \in \Gamma$.

Proof. Firstly, we will investigate that $'+'$ is well defined. Let $n_1+\Im_{\aleph}=n_3+\Im_{\aleph}$ and $n_2+\Im_{\aleph}=n_4+\Im_{\aleph}$ for all $n_1+\Im_{\aleph}, n_2+\Im_{\aleph}, n_3+\Im_{\aleph}, n_4+\Im_{\aleph} \in \aleph/\Im_{\aleph}$. From Proposition

3.2, it is obtained that $\Im_{\aleph}^{+}(n_1 - n_3) = \Im_{\aleph}^{+}(n_2 - n_4) = \Im_{\aleph}^{+}(e_{\aleph})$. Then,

$$\begin{aligned} \Im_{\aleph}^{+}((n_1+n_2)-(n_3+n_4)) &= \Im_{\aleph}^{+}((n_3+n_4)-(n_1+n_2)) \\ &= \Im_{\aleph}^{+}((n_3+n_4)-(n_4+n_1)+(n_4+n_1)-(n_1+n_2)) \\ &= \Im_{\aleph}^{+}((n_3+n_4-n_4-n_1+n_4+n_1-n_1-n_2)) \\ &= \Im_{\aleph}^{+}((n_3-n_1)+(n_4-n_2)) \\ &\supseteq \Im_{\aleph}^{+}(n_3-n_1)\cap\Im_{\aleph}^{+}(n_4-n_2) \\ &= \Im_{\aleph}^{+}(e_{\aleph})\cap\Im_{\aleph}^{+}(e_{\aleph}) \\ &= \Im_{\aleph}^{+}(e_{\aleph}) \end{aligned}$$

Also, it is followed that $\Im_{\aleph}^{+}(e_{\aleph}) \supseteq \Im_{\aleph}^{+}((n_1+n_2)-(n_3+n_4))$ from Proposition 3.1. Therefore, $\Im_{\aleph}^{+}((n_1+n_2)-(n_3+n_4)) = \Im_{\aleph}^{+}(e_{\aleph})$. Thus, $(n_1+n_2)+\Im_{\aleph}^{+} = (n_3+n_4)+\Im_{\aleph}^{+}$, and it is obtained that $(n_1+\Im_{\aleph}^{+})+(n_2+\Im_{\aleph}^{+}) = (n_3+\Im_{\aleph}^{+})+(n_4+\Im_{\aleph}^{+})$. Similarly, it is shown that $(n_1+\Im_{\aleph}^{-})+(n_2+\Im_{\aleph}^{-}) = (n_3+\Im_{\aleph}^{-})+(n_4+\Im_{\aleph}^{-})$. Now, we will investigate that $'t'$ is well defined. Let $(n_1+\Im_{\aleph}) = (n_3+\Im_{\aleph})$ and $(n_2+\Im_{\aleph}) = (n_4+\Im_{\aleph})$ for all $n_1+\Im_{\aleph}, n_2+\Im_{\aleph}, n_3+\Im_{\aleph}, n_4+\Im_{\aleph} \in \aleph/\Im_{\aleph}$ and $t \in \Gamma$. Then,

$$\begin{aligned} \Im_{\aleph}^{+}(n_1tn_2-n_3tn_4) &= \Im_{\aleph}^{+}(n_3tn_4-n_1tn_2) \\ &= \Im_{\aleph}^{+}(n_3tn_4-n_1tn_4+n_1tn_4-n_1tn_2) \\ &= \Im_{\aleph}^{+}((n_3-n_1)tn_4+n_1t(n_2+(-n_2+n_4)-n_1tn_2)) \\ &= \Im_{\aleph}^{+}((n_3-n_1)tn_4-(-n_1t(n_2+(-n_2+n_4)+-n_1tn_2))) \\ &\supseteq \Im_{\aleph}^{+}((n_3-n_1)tn_4)\cap\Im_{\aleph}^{+}(-n_1t(n_2+(-n_2+n_4))+n_1tn_2) \\ &= \Im_{\aleph}^{+}((n_3-n_1)tn_4)\cap\Im_{\aleph}^{+}(n_1t(n_2+(-n_2+n_4))-n_1tn_2) \\ &\supseteq \Im_{\aleph}^{+}(n_3-n_1)\cap\Im_{\aleph}^{+}(-n_2+n_4) \\ &= \Im_{\aleph}^{+}(e_{\aleph})\cap\Im_{\aleph}^{+}(e_{\aleph}) \\ &= \Im_{\aleph}^{+}(e_{\aleph}) \end{aligned}$$

Also, it is followed that $\Im_{\aleph}^{+}(e_{\aleph}) \supseteq \Im_{\aleph}^{+}(n_1tn_2-n_3tn_4)$ from Proposition 3.1. Therefore, $\Im_{\aleph}^{+}(n_1tn_2-n_3tn_4) = \Im_{\aleph}^{+}(e_{\aleph})$. Thus, $(n_1tn_2)+\Im_{\aleph}^{+} = (n_3tn_4)+\Im_{\aleph}^{+}$, and it is obtained that $(n_1+\Im_{\aleph}^{+})t(n_2+\Im_{\aleph}^{+}) = (n_3+\Im_{\aleph}^{+})t(n_4+\Im_{\aleph}^{+})$. Similarly, it is shown that $(n_1+\Im_{\aleph}^{-})t(n_2+\Im_{\aleph}^{-}) = (n_3+\Im_{\aleph}^{-})t(n_4+\Im_{\aleph}^{-})$. Hence, $'+'$ and $'t'$ are well defined.
For all $n_1+\Im_{\aleph}, n_2+\Im_{\aleph}, n_3+\Im_{\aleph} \in \aleph/\Im_{\aleph}$ and $t, k \in \Gamma$,

$$\begin{aligned} [(n_1+\Im_{\aleph}^{+})+(n_2+\Im_{\aleph}^{+})]t(n_3+\Im_{\aleph}^{+}) &= ((n_1+n_2)+\Im_{\aleph}^{+})t(n_3+\Im_{\aleph}^{+}) \\ &= (n_1+n_2)tn_3+\Im_{\aleph}^{+} \\ &= (n_1tn_3+n_2tn_3)+\Im_{\aleph}^{+} \\ &= (n_1tn_3+\Im_{\aleph}^{+})+(n_2tn_3+\Im_{\aleph}^{+}) \\ &= (n_1+\Im_{\aleph}^{+})t(n_3+\Im_{\aleph}^{+})+(n_2+\Im_{\aleph}^{+})t(n_3+\Im_{\aleph}^{+}) \end{aligned}$$

It is obtained that $[(n_1+\Im_{\aleph}^{+})+(n_2+\Im_{\aleph}^{+})]t(n_3+\Im_{\aleph}^{+}) = (n_1+\Im_{\aleph}^{+})t(n_3+\Im_{\aleph}^{+}) + (n_2+\Im_{\aleph}^{+})t(n_3+\Im_{\aleph}^{+})$ Similarly, it is shown that $[(n_1+\Im_{\aleph}^{-})+(n_2+\Im_{\aleph}^{-})]t(n_3+\Im_{\aleph}^{-}) = (n_1+\Im_{\aleph}^{-})t(n_3+\Im_{\aleph}^{-})+(n_2+\Im_{\aleph}^{-})t(n_3+\Im_{\aleph}^{-})$. Also,

$$\begin{aligned} [(n_1+\Im_{\aleph}^{+})t(n_2+\Im_{\aleph}^{+})]k(n_3+\Im_{\aleph}^{+}) &= (n_1tn_2+\Im_{\aleph}^{+})k(n_3+\Im_{\aleph}^{+}) \\ &= (n_1tn_2)kn_3+\Im_{\aleph}^{+} \\ &= n_1t(n_2kn_3)+\Im_{\aleph}^{+} \\ &= (n_1+\Im_{\aleph}^{+})t(n_2kn_3+\Im_{\aleph}^{+}) \\ &= (n_1+\Im_{\aleph}^{+})t[(n_2+\Im_{\aleph}^{+})k(n_3+\Im_{\aleph}^{+})] \end{aligned}$$

It is obtained that $[(n_1+\Im_{\aleph}^{+})t(n_2+\Im_{\aleph}^{+})]k(n_3+\Im_{\aleph}^{+}) = (n_1+\Im_{\aleph}^{+})t[(n_2+\Im_{\aleph}^{+})k(n_3+\Im_{\aleph}^{+})]$. Similarly, it is shown that $[(n_1+\Im_{\aleph}^{-})t(n_2+\Im_{\aleph}^{-})]k(n_3+\Im_{\aleph}^{-}) = (n_1+\Im_{\aleph}^{-})t[(n_2+\Im_{\aleph}^{-})k(n_3+\Im_{\aleph}^{-})]$. Finally, $\aleph/\Im_{\aleph}$ is a Γ-near-ring. $\square$

Theorem 3.4. *Let* $\Im_{\aleph} = (\Im_{\aleph}^{+}, \Im_{\aleph}^{-})$ *be a* Γ*-bipolar soft ideal over* U*. Let* $\Omega_{\aleph/\Im_{\aleph}} = (\Omega^{+}_{\aleph/\Im^{+}_{\aleph}}, \Omega^{-}_{\aleph/\Im^{-}_{\aleph}})$ *be a bipolar soft set over* U *defined by* $\Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_1 + \Im^{+}_{\aleph}) = \Im^{+}_{\aleph}(n_1)$ *and* $\Omega^{-}_{\aleph/\Im^{-}_{\aleph}}(n_1 + \Im^{-}_{\aleph}) = \Im^{-}_{\aleph}(n_1)$ *for all* $n_1 \in \aleph$*. Then,* $\Omega_{\aleph/\Im_{\aleph}} = (\Omega^{+}_{\aleph/\Im^{+}_{\aleph}}, \Omega^{-}_{\aleph/\Im^{-}_{\aleph}})$ *is a* Γ*-bipolar soft ideal over* U*.*

Proof. Firstly, we will investigate that $\Omega_{\aleph/\Im_{\aleph}}$ is well defined. Let $n_1 + \Im_{\aleph} = n_2 + \Im_{\aleph}$ for all $n_1 + \Im_{\aleph}, n_2 + \Im_{\aleph} \in \aleph/\Im_{\aleph}$. From Proposition 3.2, it is obtained that $\Im^{+}_{\aleph}(n_1) = \Im^{+}_{\aleph}(n_2)$. Thus, it follows that $\Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_1 + \Im^{+}_{\aleph}) = \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_2 + \Im^{+}_{\aleph})$. Similarly, it is shown that $\Omega^{-}_{\aleph/\Im^{-}_{\aleph}}(n_1 + \Im^{-}_{\aleph}) = \Omega^{-}_{\aleph/\Im^{-}_{\aleph}}(n_2 + \Im^{-}_{\aleph})$. Now, we will prove that $\Omega_{\aleph/\Im_{\aleph}} = (\Omega^{+}_{\aleph/\Im^{+}_{\aleph}}, \Omega^{-}_{\aleph/\Im^{-}_{\aleph}})$ is a Γ-bipolar soft ideal over U. For all $n_1 + \Im_{\aleph}, n_2 + \Im_{\aleph}, n_3 + \Im_{\aleph} \in \aleph/\Im_{\aleph}$ and $t \in \Gamma$,

- $$\begin{aligned}\Omega^{+}_{\aleph/\Im^{+}_{\aleph}}((n_1 + \Im^{+}_{\aleph}) + (n_2 + \Im^{+}_{\aleph})) &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_1 + n_2 + \Im^{+}_{\aleph}) \\ &= \Im^{+}_{\aleph}(n_1 + n_2) \\ &= \Im^{+}_{\aleph}(n_1 - (-n_2)) \\ &\supseteq \Im^{+}_{\aleph}(n_1) \cap \Im^{+}_{\aleph}(n_2) \\ &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_1 + \Im^{+}_{\aleph}) \cap \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_2 + \Im^{+}_{\aleph})\end{aligned}$$

  Similarly, it is shown that $\Omega^{-}_{\aleph/\Im^{-}_{\aleph}}((n_1 + \Im^{-}_{\aleph}) + (n_1 + \Im^{-}_{\aleph})) \subseteq \Omega^{-}_{\aleph/\Im^{-}_{\aleph}}(n_1 + \Im^{-}_{\aleph}) \cap \Omega^{-}_{\aleph/\Im^{-}_{\aleph}}(n_2 + \Im^{-}_{\aleph})$.
- $$\begin{aligned}\Omega^{+}_{\aleph/\Im^{+}_{\aleph}}((n_2 + \Im^{+}_{\aleph}) + (n_1 + \Im^{+}_{\aleph}) - (n_2 + \Im^{+}_{\aleph})) &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_2 + n_1 - n_2 + \Im^{+}_{\aleph}) \\ &= \Im^{+}_{\aleph}(n_2 + n_1 - n_2) \\ &\supseteq \Im^{+}_{\aleph}(n_1) \\ &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_1 + \Im^{+}_{\aleph}).\end{aligned}$$

  Similarly, it is shown that $\Omega^{-}_{\aleph/\Im^{-}_{\aleph}}((n_2 + \Im^{-}_{\aleph}) + (n_1 + \Im^{-}_{\aleph}) - (n_2 + \Im^{-}_{\aleph})) \subseteq \Omega^{-}_{\aleph/\Im^{-}_{\aleph}}(n_1 + \Im^{-}_{\aleph})$.
- $$\begin{aligned}&\Omega^{+}_{\aleph/\Im^{+}_{\aleph}}((n_1 + \Im^{+}_{\aleph})t((n_2 + \Im^{+}_{\aleph}) + (n_3 + \Im^{+}_{\aleph})) - (n_1 + \Im^{+}_{\aleph})t(n_2 + \Im^{+}_{\aleph})) \\ &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}((n_1 + \Im^{+}_{\aleph})t((n_2 + n_3) + \Im^{+}_{\aleph}) - (n_1 + \Im^{+}_{\aleph})t(n_2 + \Im^{+}_{\aleph})) \\ &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}((n_1t(n_2 + n_3) + \Im^{+}_{\aleph}) - (n_1tn_2 + \Im^{+}_{\aleph})) \\ &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}((n_1t(n_2 + n_3) - n_1tn_2) + \Im^{+}_{\aleph}) \\ &= \Im^{+}_{\aleph}(n_1t(n_2 + n_3) - (n_1tn_2)) \\ &\supseteq \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_3 + \Im^{+}_{\aleph}).\end{aligned}$$

  Similarly, it is shown that $\Omega^{-}_{\aleph/\Im^{-}_{\aleph}}((n_1 + \Im^{-}_{\aleph})t((n_2 + \Im^{-}_{\aleph}) + (n_3 + \Im^{-}_{\aleph})) - (n_1 + \Im^{-}_{\aleph})t(n_2 + \Im^{-}_{\aleph})) \subseteq \Omega^{-}_{\aleph/\Im^{-}_{\aleph}}(n_3 + \Im^{-}_{\aleph})$.

  In this way, $\Omega_{\aleph/\Im_{\aleph}}$ is a Γ-bipolar soft left ideal over U.

- $$\begin{aligned} \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}((n_1+\Im^{+}_{\aleph})t(n_2+\Im^{+}_{\aleph})) &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_1tn_2+\Im^{+}_{\aleph}) \\ &= \Im^{+}_{\aleph}(n_1tn_2) \\ &\supseteq \Im^{+}_{\aleph}(n_1) \\ &= \Omega^{+}_{\aleph/\Im^{+}_{\aleph}}(n_1+\Im^{+}_{\aleph}) \end{aligned}$$

  Similarly, it is shown that $\Omega^{-}_{\aleph/\Im^{-}_{\aleph}}((n_1+\Im^{-}_{\aleph})t(n_2+\Im^{-}_{\aleph})) \subseteq \Omega^{-}_{\aleph/\Im^{-}_{\aleph}}(n_1+\Im^{-}_{\aleph})$.
  In this way, $\Omega_{\aleph/\Im_{\aleph}}$ is a Γ-bipolar soft right ideal over U.

Last of all, $\Omega_{\aleph/\Im_{\aleph}}$ is a Γ-bipolar soft ideal over U. □

Definition 3.3. Let $\aleph_1$ and $\aleph_2$ be Γ-near-rings. The function $\theta : \aleph_1 \to \aleph_2$ is said to be a Γ-near-ring homomorphism if $\theta(n_1+n_2) = \theta(n_1)+\theta(n_2)$ and $\theta(n_1tn_2) = \theta(n_1)t\theta(n_2)$ for all $n_1, n_2 \in \aleph_1, t \in \Gamma$. Moreover, if θ is one-to-one, then θ is said to be a monomorphism. If θ is surjective, then θ is said to be an epimorphism.

Proposition 3.5. *Let $\Im_{\aleph} = (\Im^{+}_{\aleph}, \Im^{-}_{\aleph})$ be a Γ-bipolar soft ideal over U. The set $\aleph_{\Im_{\aleph}} = \{n \in \aleph : \Im^{+}_{\aleph}(n) = \Im^{+}_{\aleph}(e_{\aleph})$ and $\Im^{-}_{\aleph}(n) = \Im^{-}_{\aleph}(e_{\aleph})\}\}$ is an ideal of Γ-near-ring $\aleph$.*

Proof. Let $n_1, n_2 \in \aleph_{\Im_{\aleph}}$. Then, $\Im^{+}_{\aleph}(n_1) = \Im^{+}_{\aleph}(e_{\aleph})$ and $\Im^{+}_{\aleph}(n_2) = \Im^{+}_{\aleph}(e_{\aleph})$. And,

$$\begin{aligned} \Im^{+}_{\aleph}(n_1-n_2) &\supseteq \Im^{+}_{\aleph}(n_1) \cap \Im^{+}_{\aleph}(n_2) \\ &= \Im^{+}_{\aleph}(e_{\aleph}) \cap \Im^{+}_{\aleph}(e_{\aleph}) \\ &= \Im^{+}_{\aleph}(e_{\aleph}) \end{aligned}$$

Moreover, it is known that $\Im^{+}_{\aleph}(e_{\aleph}) \supseteq \Im^{+}_{\aleph}(n_1-n_2)$. Thus, it is obtained $\Im^{+}_{\aleph}(n_1-n_2) = \Im^{+}_{\aleph}(e_{\aleph})$. Similarly, it is shown that $\Im^{-}_{\aleph}(n_1-n_2) = \Im^{-}_{\aleph}(e_{\aleph})$. Then, $n_1-n_2 \in \aleph_{\Im_{\aleph}}$, and $\aleph_{\Im_{\aleph}}$ is a subgroup of Γ-near-ring $\aleph$.
Let $n_1 \in \aleph_{\Im_{\aleph}}$ and $n_2 \in \aleph$. Since $n_1 \in \aleph_{\Im_{\aleph}}$, then $\Im^{+}_{\aleph}(n_1) = \Im^{+}_{\aleph}(e_{\aleph})$ and $\Im^{-}_{\aleph}(n_1) = \Im^{-}_{\aleph}(e_{\aleph})$. Hence,

$$\Im^{+}_{\aleph}(n_2+n_1+(-n_2)) \supseteq \Im^{+}_{\aleph}(n_1) = \Im^{+}_{\aleph}(e_{\aleph})$$

Also, it is known that $\Im^{+}_{\aleph}(e_{\aleph}) \supseteq \Im^{+}_{\aleph}(n_2+n_1+(-n_2))$. Consequently, $\Im^{+}_{\aleph}(n_2+n_1+(-n_2)) = \Im^{+}_{\aleph}(e_{\aleph})$. Similarly, $\Im^{-}_{\aleph}(n_2+n_1+(-n_2)) = \Im^{-}_{\aleph}(e_{\aleph})$. Thence, it is followed that $n_2+n_1+(-n_2) \in \aleph_{\Im_{\aleph}}$. Thus, $\aleph_{\Im_{\aleph}}$ is a normal subgroup of Γ-near-ring $\aleph$.
Let $n_1, n_2 \in \aleph$, $t \in \Gamma$ and $n \in \aleph_{\Im_{\aleph}}$. Then,

$$\Im^{+}_{\aleph}(n_1t(n_2+n)-n_1tn_2) \supseteq \Im^{+}_{\aleph}(n) = \Im^{+}_{\aleph}(e_{\aleph})$$

Also, it known that $\Im^{+}_{\aleph}(e_{\aleph}) \supseteq \Im^{+}_{\aleph}(n_1t(n_2+n)-n_1tn_2)$. As a result $\Im^{+}_{\aleph}(n_1t(n_2+n)-n_1tn_2) = \Im^{+}_{\aleph}(e_{\aleph})$. Similarly, $\Im^{-}_{\aleph}(n_1t(n_2+n)-n_1tn_2) = \Im^{-}_{\aleph}(e_{\aleph})$. Thence, it is followed that $n_1t(n_2+n)-n_1tn_2 \in \aleph_{\Im_{\aleph}}$. Thus, $\aleph_{\Im_{\aleph}}$ is a left ideal of Γ-near-ring $\aleph$.
Let $n_1 \in \aleph$, $t \in \Gamma$ and $n \in \aleph_{\Im_{\aleph}}$. Then,

$$\Im^{+}_{\aleph}(ntn_1) \supseteq \Im^{+}_{\aleph}(n) = \Im^{+}_{\aleph}(e_{\aleph})$$

Also, it known that $\Im^{+}_{\aleph}(e_{\aleph}) \supseteq \Im^{+}_{\aleph}(ntn_1)$. As a result $\Im^{+}_{\aleph}(ntn_1) = \Im^{+}_{\aleph}(e_{\aleph})$. Similarly, $\Im^{-}_{\aleph}(ntn_1) = \Im^{-}_{\aleph}(e_{\aleph})$. Thence, it is followed that $ntn_1 \in \aleph_{\Im_{\aleph}}$. Thus, $\aleph_{\Im_{\aleph}}$ is a right ideal of Γ-near-ring $\aleph$.
Consequently, $\aleph_{\Im_{\aleph}}$ is an ideal of Γ-near-ring $\aleph$. □

Theorem 3.6. *Let $\Im_{\aleph} = (\Im^{+}_{\aleph}, \Im^{-}_{\aleph})$ be a Γ-bipolar soft ideal over U. Define the function $\theta : \aleph \to \aleph/\Im_{\aleph}$ where $\theta(n) = n+\Im^{+}_{\aleph}$ and $\theta(n) = n+\Im^{-}_{\aleph}$ for all $n \in \aleph$. θ, which $\aleph_{\Im_{\aleph}}$ is kernel, is a Γ-near-ring epimorphism. Moreover, $\aleph/\Im_{\aleph}$ and $\aleph/\aleph_{\Im_{\aleph}}$ are isomorphic.*

Proof. For all $n_1, n_2 \in \aleph$ and $t \in \Gamma$

$$\theta(n_1 + n_2) = (n_1 + n_2) + \Im_{\aleph}^{+} = (n_1 + \Im_{\aleph}^{+}) + (n_2 + \Im_{\aleph}^{+}) = \theta(n_1) + \theta(n_2)$$

and

$$\theta(n_1 + n_2) = (n_1 + n_2) + \Im_{\aleph}^{-} = (n_1 + \Im_{\aleph}^{-}) + (n_2 + \Im_{\aleph}^{-}) = \theta(n_1) + \theta(n_2)$$

Also,

$$\theta(n_1 t n_2) = (n_1 t n_2) + \Im_{\aleph}^{+} = (n_1 + \Im_{\aleph}^{+}) t (n_2 + \Im_{\aleph}^{+}) = \theta(n_1) t \theta(n_2)$$

and

$$\theta(n_1 t n_2) = (n_1 t n_2) + \Im_{\aleph}^{-} = (n_1 + \Im_{\aleph}^{-}) t (n_2 + \Im_{\aleph}^{-}) = \theta(n_1) t \theta(n_2)$$

Also, it is clear that θ is a surjective. Thus, θ is a Γ-near-ring epimorphism. Now,

$$\begin{aligned} ker\theta &= \{n \in \aleph | \theta(n) = e_{\aleph/\Im_{\aleph}}\} \\ &= \{n \in \aleph | n + \Im_{\aleph}^{+} = e_{\aleph} + \Im_{\aleph}^{+} \text{ and } n + \Im_{\aleph}^{-} = e_{\aleph} + \Im_{\aleph}^{-}\} \\ &= \{n \in \aleph | \Im_{\aleph}^{+}(n) = \Im_{\aleph}^{+}(e_{\aleph}) \text{ and } \Im_{\aleph}^{-}(n) = \Im_{\aleph}^{-}(e_{\aleph})\} \\ &= \{n \in \aleph | \Im_{\aleph}^{(}n) = \Im_{\aleph}^{(}e_{\aleph})\} \\ &= \{n \in \aleph | n \in \aleph/\Im_{\aleph}\} \\ &= \aleph_{\Im_{\aleph}} \end{aligned}$$

Moreover, it is easily shown that $\aleph/\Im_{\aleph}$ and $\aleph/\aleph_{\Im_{\aleph}}$ are isomorphic. □

4. Conclusions

In this study, bipolar soft ideals of the gamma near-ring were defined. With the help of bipolar soft ideals, a bipolar soft coset was created and it was shown that this set is a gamma near-ring. A bipolar soft set was defined and proved that it is a bipolar soft ideal. Finally, by defining a mapping with the bipolar soft ideal of the gamma near-ring, the kernel of this mapping was shown, and it was shown that it is a gamma near-ring epimorphism. Using the results obtained from the study, bipolar soft bi-ideal, bipolar soft interior ideals and bipolar soft quasi-ideals of the gamma near-ring can be defined. Moreover, various algebraic properties of these structures can be examined.

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