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CHARACTERISATION OF CERTAIN TYPES OF SOFT GRAPHS

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ABSTRACT. Soft graph theory offers a parametrized perspective on graphs by classifying the universe's components according to a specified set of parameters. This paper explores soft graph theory, which classifies graph components based on a set of parameters, focusing on the wheel and friendship graph families. By analyzing soft graphs with distance-based parameter sets, the study provides key results on the isomorphic subgraphs formed within these structures. These findings offer important insights into the structure and behavior of soft graphs, enhancing our understanding of soft graph theory.

1. INTRODUCTION

The majority of our conventional modeling, reasoning, and computing tools are characterized by crispness, determinism, and precision. In disciplines such as social science, medicine, engineering, economics, and even environmental studies, there are several complex problems involving facts that are not necessarily clear-cut. Because these challenges involve a variety of uncertainties, we are not always able to apply classical methodologies. Molodtsov [7] introduced an idea on the theory of soft sets in 1999 as a novel mathematical technique for handling ambiguity or uncertainty. Soft set theory is easily applicable to a wide range of domains since it does not face the challenge of determining how to set the membership function. Soft set theory applications to different fields of study and practical issues are starting to gain momentum. In 2002, Maji et al. [6] presented the first example of a practical use of the theory of soft sets in decision-making scenarios. An algorithm and programme for attribute reduction utilizing multiple soft sets are introduced by Reddy et al.[11]. The program's applications to crop selection for farmers are discussed. In 2021, the idea of a regular semiopen soft set with applications was introduced by Elavarasan and Vadivelt[3]. A number of related results are presented, together with an introduction and illustrations of the upper and lower cuts of soft multigroups by Joseph et al.[9] in 2024. Bobin George and Rajesh K Thumbakara [15] have provided a concept of the soft graph, which is important to this subject. Authors have defined terms like soft complete graphs,

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homomorphism as well as isomorphism in soft graphs. Authors Jinta Bose et al. [5] provides in his survey paper about development of soft graphs, beginning with an introduction to the fundamental ideas and history of soft set theory, emphasising its advantages and uses. Additionally, Akram et al. [1, 2] investigated various results related to soft graphs and explored several operations on them. A novel idea of the adjacency as well as incidence matrices for the soft graph was presented by Thenge et al. [12, 14]. Star, comb, bistar, and other soft graphs were presented by Palani et al. [10, 8]. For a variety of graph families, such as wheel and fan graphs, V. Govindan et al.[4] presented the idea of cube sum labeling. Connected soft graphs introduced by Thenge and et al. [13] in 2020. The properties of different soft graphs in relation to the isomorphism of particular kinds of friendship and wheel graphs will be investigated in this study.

2. Preliminaries

The idea of soft sets, first put forth by Molodtsov [7], is presented in this section. A number of helpful terminology pertaining to soft graphs are also included. Here, U, E, and $P(U)$ represent the universal set, set of parameters, and power set respectively.

Definition 2.1 (Soft Set(Molodtsov)[7])**.** A ordered pair (S, X) is referred to as a soft set over U, where S is a function given by $S : X \to P(U)$. Thus, a soft set over U is a parameterized collection of subsets of the universal set U. It is clear that a soft set is not the same as a crisp set.

Definition 2.2 (Soft subset(Molodtsov)[7])**.** For two soft sets (S_1, X) and (S_2, Y) over a common universe $U, (S_1, X)$ is a soft subset of (S_2, Y) if

i. $X \subset Y$

ii. For all $x \in X$, $S_1(x)$ and $S_2(x)$ represent the same approximation. We can write $(S_1, X) \subset (S_2, Y)$.

(S_1, X) is said to be a soft super set of (S_2, Y), if (S_2, Y) is a soft subset of (S_1, X).

Definition 2.3 (Soft graph(Akram) [1])**.** A 4-tuple $G^* = (G, P, Q, X)$ is said to be Soft graph which satisfies following properties,

i. $G(V, E)$ is a simple graph.

ii. $X \neq \phi$, set of parameters.

iii. (P, X) is soft-set on V.

iv. (Q, X) is soft-set on E.

v. $(P(t), Q(t)), \forall t \in X$ is a subgraph of G.

The subgraph $(P(t), Q(t))$ is represented for convenience by $R(t)$ and the corresponding a soft graph can be represented as $G^* = (G, P, Q, X) = \{R(t) \mid t \in X\}$

Here, we denote the Soft graph as (R, X).

Example 2.4. Consider graph G in Figure 1,

Let $X = \{v_1, v_3\}$ Define the sets $P(x) = \{y \in V \mid d(x, y) \leq 1\}$, and $Q(x) = \{xz \in E \mid z \in P(x)\}$. We get $P(v_1) = \{v_1, v_2, v_4\}$ and $P(v_3) = \{v_3, v_2, v_4\}$. Also, $Q(v_1) = \{v_1v_2, v_1v_4\}$ and $Q(v_3) = \{v_3v_2, v_3v_4\}$. Let us denote the soft graph $R(x) = (P(x), Q(x)), \forall x \in X$. The subgraph that is induced by $R(v_1), R(v_2)$ in G is connected subgraph of G for every $x \in X$. The graph (R, X) is obviously soft graph.

Now consider Figure 2, Let $Y = \{v_1, v_2\}$. Define $P(x) = \{y \in V \mid d(x, y) = 1\}$, $Q(x) = \{xz \in E \mid z \in P(x)\}$. Then we get $P(v_1) = \{v_2, v_4\}$ and $P(v_2) = \{v_1, v_3\}$. Also $Q(v_1) = \phi, Q(v_2) = \phi$. Denote the soft graph $R(x) = (P(x), Q(x)), \forall x \in Y$. The induced subgraph by $R(v_1), R(v_2)$ in G is disconnected subgraph, for all $x \in Y$. So, (R, Y) is not a soft graph.

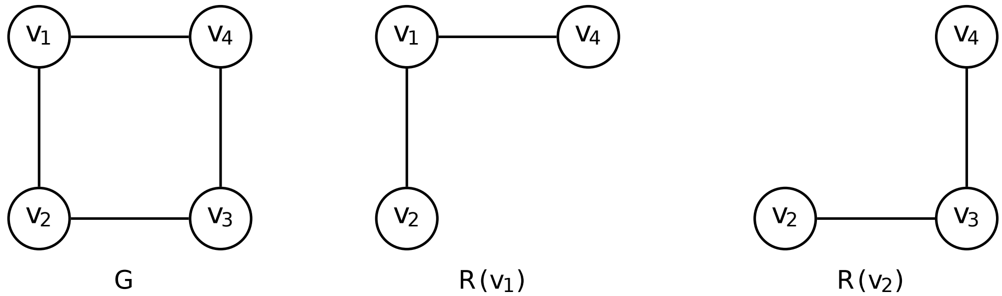

FIGURE 1. (R, X) Soft graph in G

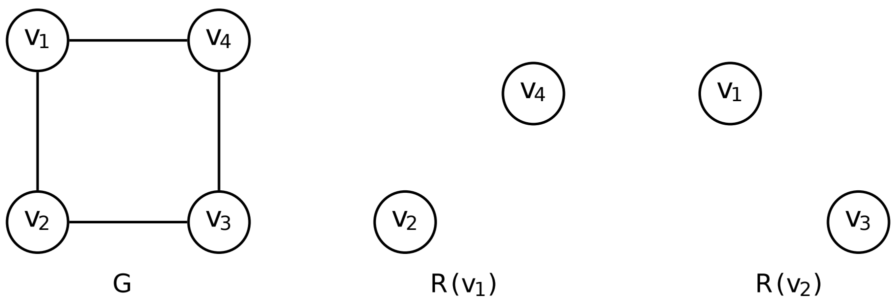

FIGURE 2. (R, Y) not Soft graph in G

3. RESULTS AND DISCUSSIONS

Theorem 3.1. *Let W_n be a wheel graph then (R, X) is soft graph of W_n for $X \subseteq V$ if $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = k\}$ where $k = 1, 2$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$.*

Proof. Consider two cases depend on the set X as follows:

Case (1): If $X = \{v_i\}$ for some i, then there are two cases arises as follows: if $P(x) = \{y \in V \mid d(x, y) = 1\}$, $Q(x) = \{xz \in E \mid z \in P(x)\}$ and $R(x) = (P(x), Q(x))$ $\forall x \in X$ then $R(v_i) = \{v_0, v_j\}$ for some $j, j \neq i$ and $v_0 \in$ centre(G). Therefore, the induced subgraph by $R(v_i)$ is connected subgraph.

Hence (R, X) is soft graph.

Now, if $P(x) = \{y \in V \mid d(x, y) = 2\}$, $Q(x) = \{xz \in E \mid z \in P(x)\}$ and $R(x) = (P(x), Q(x))$ $\forall x \in X$ then, $F(v_i) = F(v_j)$ for some j and $i \neq j$. Therefore, the induced subgraph by $F(v_i)$ is connected subgraph. Hence (R, X) is soft graph.

Case (2): If $X = \{v_0\}$ where $v_0 \in$ centre(G), then $P(x) = \{y \in V \mid d(x, y) = 1\}$, $Q(x) = \{xz \in E \mid z \in P(x)\}$ and $R(x) = (P(x), Q(x))$, $\forall x \in X$, $R(v_0) = \{v_i, i = 1, 2, \ldots, n\}$. Therefore, the induced subgraph by $R(v_i)$ is connected subgraph.

Hence, (R, X) is soft graph. □

Theorem 3.2. *Let W_n be a wheel graph. If (R, X) is soft graph of W_n for $X \subset V(W_n)$ be any one parameter set with $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = 1\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then each R_i in soft graph (R, X) isomorphic to cycle graph C_{n-1} or path graph P_3 if $n > 4$.*

Proof. Let W_n be a wheel graph. Consider two cases depend on the n vertices as follows:

Case (1) When $n = 4$:

Consider $X \subset V(W_n)$ to be a set containing exactly one vertex. Assume that $X = \{v_0\}$, be a central vertex.

Now, $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = 1\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$. Thus for central vertex v_0, $P(x) = \{y \in V \mid d(v_0, y) = 1\}$ which implies that v_0 is adjacent to y.

In the soft graph (R, X) of W_n for $X \subset V(W_n)$, each R_i represents a subgraph induced by the vertices related to a specific element in X and given $X = \{v_0\}$, the subgraph induced by v_0 will include all vertices adjacent to central vertex v_0. In W_n, the central vertex v_0 is adjacent to all other vertices in the cycle C_{n-1}. Therefore, the subgraph induced by the soft relation when $X = \{v_0\}$ includes the central vertex and all vertices of the cycle C_{n-1}.

Case (2) When $n > 4$:

If we consider the subgraph induced by v_0 and its adjacent vertices, we basically have the entire wheel graph W_n. The structure of the subgraph R_i can be considered as C_{n-1} along with the central vertex v_0. However, the nature of the subgraph depends on how we partitioned the vertices. Here the vertices form a cycle C_{n-1}, and the central vertex v_0 is connected to all vertices in this cycle. So, if we remove the central vertex, the remaining graph is a cycle C_{n-1}.

Now, for specific partitions or different selections of R_i, if we consider subgraphs that include the central vertex v_0 and just two adjacent vertices of the cycle, we get a path graph P_3.

Therefore, in the soft graph (R, X) with $X = \{v_0\}$ and $n > 4$, each R_i is isomorphic to C_{n-1} if the subgraph includes all vertices of the cycle or each R_i can also be isomorphic to P_3. □

Theorem 3.3. *Let W_n be a wheel graph. If (R, X) is soft graph of W_n for $X \subset V(W_n)$ be any one parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = 2\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then in soft graph (R, X), each R_i is path graph P_{n-4} if exist for $n > 4$.*

Proof. Let W_n be a wheel graph with $n > 4$. Consider $X \subset V(W_n)$ to be a set containing exactly one vertex. Assuming $X = \{v_0\}$ with v_0 as the wheel graph's central vertex.

Now, $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = 2\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$. Thus for the central vertex v_0, $P(x) = \{y \in V \mid d(v_0, y) = 2\}$ which implies that y is at a distance of 2 from the central vertex v_0.

In the soft graph (R, X) with $X = \{v_0\}$ and $n > 4$, each R_i corresponds to a subgraph induced by the vertices related to a specific element in X. If we remove central vertex v_0 from the wheel graph W_n, we get a cycle C_{n-1}. So the induced subgraph R_i corresponding to v_0 and the relation in (R, X) produces a path P_{n-4} in C_{n-1}. This path includes $n - 4$ vertices, starting from v_0 and spanning outwards to vertices at a distance of 2 from v_0. Therefore, in (R, X), each R_i isomorphic to P_{n-4} if $X = \{v_0\}$ and $n > 4$. □

Remark. *Let W_n be a wheel graph. If (R, X) is soft graph of W_n for $X \subset V(W_n)$ be any one parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = k\}$ where $k > 2$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then soft graph does not exist.*

Theorem 3.4. *Consider a wheel graph W_n. If (R, X) is soft graph of W_n for $X \subset V(W_n)$ be any one parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) \leq 1\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then for soft graph (R, X), each R_i isomorphic to wheel graph W_n or graph $K_4 - \{e\}$, K_4 is complete graph and e is any edge.*

Proof. Let W_n be a wheel graph with $n > 4$. Consider $X \subset V(W_n)$ to be a set containing exactly one vertex. Assuming $X = \{v_0\}$ with v_0 as the wheel graph's central vertex.

Now, $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) \leq 1\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$. Thus for the central vertex v_0, $P(x) = \{y \in V \mid d(v_0, y) \leq 1\}$ which implies that y is adjacent to v_0.

In soft graph (R, X), each R_i corresponds to a subgraph induced by the vertices related to a specific element in X.

The relation $d(v_0, y) \leq 1$ includes all vertices adjacent to v_0, which forms a subgraph that is isomorphic to the wheel graph W_n. Also, R_i can also be isomorphic to graph $K_4 - \{e\}$, where e is any edge in K_4, because if $X = \{v_0\}$, then the induced subgraph R_i include v_0 and its adjacent vertices, which can form, a subgraph of $K_4 - \{e\}$.

This proves that each R_i in the soft graph (R, X) isomorphic to either wheel graph W_n or graph $K_4 - \{e\}$. □

Theorem 3.5. *Let W_n be a wheel graph and if (R, X) is soft graph of W_n for $X \subset V(W_n)$ be any one parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) \leq k\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then each R_i in Soft graph (R, X) isomorphic to the wheel graph W_n when $k > 1$.*

Proof. Let W_n be a wheel graph with $n > 4$. Consider $X \subset V(W_n)$ to be a set containing exactly one vertex. Assuming $X = \{v_0\}$ with v_0 as the wheel graph's central vertex.

Now, $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) \leq k\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$. Thus for the central vertex v_0, $P(x) = \{y \in V \mid d(v_0, y) \leq k\}$ which implies that y is within a distance k from the central vertex v_0.

In soft graph (R, X), each R_i corresponds to a subgraph induced by the vertices related to a specific element in X.

Now, considering $X = \{v_c\}$, the relation $d(v_0, y) \leq k$ includes all vertices within a distance k from v_0, forming a subgraph that is isomorphic to the wheel graph W_n. Since $k > 1$, the induced subgraph R_i includes all vertices of W_n because every vertex in W_n is within distance k from v_0.

This shows that each R_i in Soft graph (R, X) isomorphic to the wheel graph W_n when $k > 1$. □

Definition 3.1. A friendship graph $\mathbb{F}_n$ in which every two distinct vertices has exactly one neighbour in common. Specifically, if $\mathbb{F}_n$ has n triangles, it has $2n + 1$ vertices and $3n$ edges. Following are $\mathbb{F}_2$ and $\mathbb{F}_3$ friendship graphs respectively.

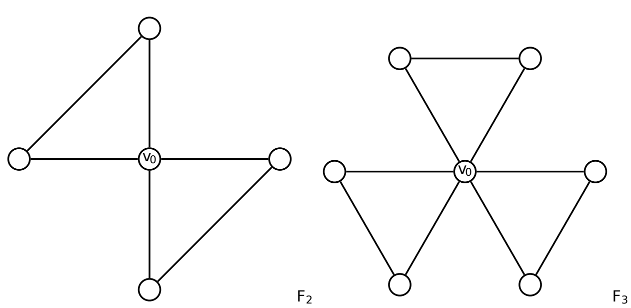

FIGURE 3. Friendship Graphs

Theorem 3.6. *Consider a friendship graph $\mathbb{F}_n$. If (R, X) is soft graph of $\mathbb{F}_n$, for $X \subset V(\mathbb{F}_n)$ be any one-parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = 1\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then each R_i in Soft graph (R, X) isomorphic to nK_2 or K_2.*

Proof. Let $\mathbb{F}_n$ be a friendship graph . Let $X \subset V(\mathbb{F}_n)$ be parameter set contains one element.

Now $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = 1\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$ Since, the friendship graph $\mathbb{F}_n$ consists of n triangles that share a common central vertex, so it has has a central vertex say v_0.

Each of the n triangles consists of central vertex v_0 and two other vertices. The vertex-set of graph $\mathbb{F}_n$, written as $V(\mathbb{F}_n) = \{v_0, v_{i1}, v_{i2} \mid i = 1, 2, \ldots, n\}$.

Now let $X = \{a\}$, where $a \in V(\mathbb{F}_n)$. Since A has only one element, the condition $d(x, y) = 1$ is based on adjacency in the original graph $\mathbb{F}_n$. We need to determine the structure of each R_i, which represents the subgraphs induced by the adjacency relation.

Case 1: $a = v_0$ (central vertex)

The central vertex v_0 is adjacent to all other vertices in $\mathbb{F}_n$. Therefore, each R_i consists of pairs (v_0, v_{i1}) and (v_0, v_{i2}) for $i = 1, 2, \ldots, n$ and forms n copies of K_2, i.e., nK_2.

Case 2: $a \neq v_0$ (non-central vertex)

Let $a = v_{ij}$ for some i and j. v_{ij} is adjacent to the central vertex v_0 and the other vertex in the same triangle, $v_{i(3-j)}$. So, in this case, R_i is isomorphic to K_2 because it consists of the edge (v_{ij}, v_0) and $(v_{ij}, v_{i(3-j)})$

Hence in the (R, X) soft graph, each R_i in Soft graph (R, X) isomorphic to nK_2 or K_2. □

Theorem 3.7. *Consider a friendship graph $\mathbb{F}_n$. If (R, X) is soft graph of $\mathbb{F}_n$ for $X \subset V(\mathbb{F}_n)$ be any one parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = 2\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then each R_i in soft graph (R, X) is either isomorphic to $(n-1)K_2$ or K_2, if exist.*

Proof. Given a friendship graph $\mathbb{F}_n$ and a subset $A \subset V(\mathbb{F}_n)$, we define the soft graph (R, X) with the sets $P(x) = \{y \in V(\mathbb{F}_n) \mid d(x, y) = 2\}$ and $Q(x) = \{xu \in E(\mathbb{F}_n) \mid u \in P(x)\}$

In the friendship graph $\mathbb{F}_n$, the common central vertex v_0 is connected to all other vertices. Each other vertex $(v_i) \neq (v_0)$ is part of exactly one triangle, thus it is connected to v_0 and one other vertex v_j.

Case 1:

For any vertex v_i, $P(v_i)$ would be the vertex-set at distance 2 from v_i, which are other peripheral vertices not directly connected to v_i and $Q(v_i)$ would be the edges connecting v_i to vertices in $P(v_i)$.

So, for any peripheral vertex v_i, $P(v_i)$ would include other peripheral vertices at distance 2 and $Q(v_i)$ would include edges between v_i and the vertices in $P(v_i)$. This will forms a series of K_2 subgraphs, since v_i is directly connected to the central vertex and one other vertex, and indirectly connected to all other peripheral vertices through the central vertex.

Case 2:

For the central vertex v_0, $P(v_0)$ would include all peripheral vertices, and $Q(v_0)$ would include all edges connecting v_0 to each peripheral vertex. Clearly, it does not form K_2 or $(n-1)K_2$.

This prove that each R_i in soft graph (R, X) of the friendship graph $\mathbb{F}_n$, is either isomorphic to $(n-1)K_2$ or K_2, if exist.

□

Theorem 3.8. *Consider a friendship graph $\mathbb{F}_n$. If (R, X) is soft graph of F_n for $X \subset V(\mathbb{F}_n)$ be any one parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) = k\}$ where $k > 2$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then a soft graph (R, X) does not exist.*

Proof. Friendship graph $\mathbb{F}_n$ contains n-triangles that shares a common vertex with $2n+1$ vertices and $3n$ edges. Now for $d(x, y) = k$, where $k > 2$ in $\mathbb{F}_n$, the maximum distance between any two vertices is 2, the central vertex v_0 is at distance 1 from all other peripheral vertices. Any peripheral vertex v_i is at distance 1 from the central vertex v_0 and at distance 2 from any other peripheral vertex v_j.

So, for any $x \in V(\mathbb{F}_n)$, there are no vertices t such that $d(x, t) = k$, for $k > 2$ because in $\mathbb{F}_n$, the distance between two vertices is 2. Hence, $P(x)$ will always be an empty set for $k > 2$. Since $P(x)$ is empty for all $x \in V(\mathbb{F}_n)$, $Q(x)$ will also be empty. Thus, for $k > 2$, $P(x)$ and $Q(x)$ will always be empty for all vertices x in $\mathbb{F}_n$, as a result, the soft graph (R, X) does not exist. □

Theorem 3.9. *Consider a friendship graph $\mathbb{F}_n$. If (R, X) is soft graph of F_n for $X \subset V(\mathbb{F}_n)$ be any one parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) \leq 1\}$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then for each R_i in Soft graph (R, X), isomorphic to $\mathbb{F}_n$ or C_3.*

Proof. Given a friendship graph $\mathbb{F}_n$ and a subset $X \subset V(\mathbb{F}_n)$, we define the soft graph (R, X) with the sets $P(x) = \{y \in V(\mathbb{F}_n) \mid d(x, y) \leq 1\}$ and $Q(x) = \{xz \in E(\mathbb{F}_n) \mid z \in P(x)\}$

Now we discuss the elements of $P(x)$ and $Q(x)$.

$P(x)$ contains the vertex x and all vertices that are adjacent to x. In the friendship graph $\mathbb{F}_n$, since every vertex y (where $y \neq x$) is part of some triangle having x as the common vertex, $P(x)$ includes x and all vertices directly connected to x.

$Q(x)$ includes all edges xu where $u \in P(x)$. In the friendship graph $\mathbb{F}_n$, this means all edges from x to its neighbours.

Case 1: If x is the central vertex of $\mathbb{F}_n$:

If $P(x)$ includes x and all other $2n$ vertices, then $Q(x)$ includes all edges connecting x to these $2n$ vertices, and any edges between these $(2n)$ vertices that form part of the n triangles. Hence the resulting induced subgraph R_i isomorphic to $\mathbb{F}_n$, since it includes all the vertices and edges of the original friendship graph.

Case 2: If x is not the central vertex:

Suppose x is part of some triangle. Then, $P(x)$ includes x, the central vertex, and the other vertex of that triangle and $Q(x)$ includes the edges of the triangle. Hence the resulting induced subgraph R_i is isomorphic to cycle C_3, since it forms a triangle.

Hence each R_i in the soft graph (R, X) formed from the friendship graph $\mathbb{F}_n$ is either isomorphic to $\mathbb{F}_n$ (if x is the central vertex), or isomorphic to C_3 if x is a non-central vertex part of some triangle. □

Theorem 3.10. *Consider a friendship graph $\mathbb{F}_n$. If (R, X) is soft graph of F_n for $X \subset V(\mathbb{F}_n)$ be any one-parameter set where $R(x) = (P(x), Q(x))$, $P(x) = \{y \in V \mid d(x, y) \leq k\}$ where $k > 2$ and $Q(x) = \{xz \in E \mid z \in P(x)\}$, then each R_i in soft graph (R, X), isomorphic to $\mathbb{F}_n$.*

Proof. Given a friendship graph $\mathbb{F}_n$ and a subset $X \subset V(\mathbb{F}_n)$, we define the soft graph (R, X) with the sets $P(x) = \{y \in V(\mathbb{F}_n) \mid\leq k\}$ where $k > 2$ and $Q(x) = \{xz \in E(\mathbb{F}_n) \mid z \in P(x)\}$

Lets analyze the structure of $P(x)$ and $Q(x)$ with $k > 2$:

Since $P(x)$ contains the vertex x and all vertices that are within distance $k > 2$ from x, in the friendship graph $\mathbb{F}_n$, since $k > 2$, $P(x)$ will include the central vertex and all vertices in $\mathbb{F}_n$, because every vertex is within distance 2 from the central vertex.

Also $Q(x)$ includes all edges xu where $u \in P(x)$. In the friendship graph $\mathbb{F}_n$, since $P(x)$ includes all vertices, $Q(x)$ will include all edges of $\mathbb{F}_n$.

The induced subgraph by $P(x)$ and $Q(x)$ is the entire friendship graph $\mathbb{F}_n$. Thus, each R_i in soft graph (R, X) isomorphic to $\mathbb{F}_n$. □

4. Conclusions

Soft graphs from two significant graph families wheel graphs and friendship graphs have been described in this study. We investigated the structure and isomorphism features of subgraphs within these soft graphs using different distance-based parameter sets. In particular, the results highlight how soft graph structures change depending on the parameter set used, and provide more details into the interaction between soft sets and graph theory. This work contributes to a more comprehensive understanding of soft graphs, their properties, and their potential applications in theoretical and applied graph theory. Future studies could use soft graph models with a wider range of parameter sets to expand this analysis to other graph families, such as hypercubes, trees, or particular graphs. Developing computational tools to automate the construction and analysis of soft graphs would also be a valuable step forward in the field.

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