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APPLICATIONS OF AN ANTI-FUZZY SEMIGROUP

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ABSTRACT. This paper introduces the "anti fuzzy semigroup," a novel algebraic structure that integrates the concepts of semigroups and anti fuzzy sets. We demonstrate, through a specific example, how a Kinship system can be represented as an anti fuzzy semigroup, effectively capturing the relationships between managers and subordinates. This application underscores the potential of anti fuzzy semigroups to model complex hierarchical structures and relationships within organizational settings. Furthermore, we investigate the representation of DNA sequences using this framework. We delve into the algebraic properties of anti fuzzy semigroups, proving that the union of two such semigroups always results in another anti fuzzy semigroup. However, we provide a counterexample to demonstrate that the intersection of two anti fuzzy semigroups may not necessarily preserve the anti fuzzy semigroup property. Finally, the Cartesian product of two anti fuzzy semigroups forms an anti fuzzy semigroup.

1. INTRODUCTION

A mathematical framework called fuzzy sets makes the ability to articulate and deal with ambiguous or inaccurate data. Fuzzy sets are a method for dealing with ambiguity and vagueness in information that was first introduced by Zadeh [12] in 1965. Fuzzy set is a powerful tool deals with the uncertainty or vague information. The fuzzy algebraic structures have been evolved based on on fuzzy group concepts evolved by Rosenfeld [8]. In semigroups, Kuroki [5, 6, 7] described and characterized a fuzzy semigroup and various types of fuzzy ideals. In 1990, Biswas[2] first put forward the concept of anti-fuzzy subgroup. The anti-fuzzy bi-Γ-ideals of Γ-semigroup concept was introduced by Yassein and Mohammed [3]. A particular type of a traditional semigroup that contains the idea of uncertainty or fuzziness is a negative fuzzy semigroup, often referred to as an anti-fuzzy semigroup. Anti-fuzzy semigroups allow for the encoding and manipulation of ambiguous or imprecise data, whereas traditional semigroups operate on crisp or precise parts. The degree of membership or non-membership of each element in the anti-fuzzy semigroup is represented by membership values, which are associated with the elements in the semigroup. These membership values, which range from 0 to 1, represent the degree of ambiguity or uncertainty associated to the objects. The idea of anti fuzzy M-semigroup

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has been evolved by Vijayabalaji and Sivaramakrishnan [10].

Understanding relationships between families provides an understanding of the social structure, principles, and dynamics of various cultures and societies. The identities, roles, and relationships of people within their families and social groups are fundamentally shaped by it. In this paper, we provided a canonical example for an anti fuzzy semigroup (AFS) using Kinship relationships (KR) and biological implications of AFS, examining their potential applications in these domains. It can be affirmed that the intersection of two $AFSs$ need not be an AFS, although the union of two anti fuzzy semigroups $AFSs$ could an AFS. It has been illustrated to display few results on it. Additionally, we investigate the cartesian product of this structure and present some related results.

2. Preliminary Concepts

Some of the notations and definitions used in this paper are summarized in this section.

Definition 2.1 (3)**.** Let **S** be a semigroup. If Ξ is mapping from **S** to the unit interval then $(\mathbf{S}, \Xi)$ is called an AFS and satisfies the condition $\Xi(s_1 s_2) \leq \mathbf{max}\{\Xi(s_1), \Xi(s_2)\}$, $\forall s_1, s_2 \in \mathbf{S}$.

Example 2.2. Let $\mathbf{S} = \{e, r, v, w\}$ be a semigroup with the following operation $\cdot$

Define the fuzzy set $\Xi : \mathbf{S} \to [0, 1]$ by

$$\Xi(s) = \begin{cases} 0, & \text{if } s = e, r \\ 0.92, & \text{otherwise.} \end{cases}$$

Then $(\mathbf{S}, \Xi)$ is an AFS.

Definition 2.3 (9)**.** A Kinship system (KS) is a semigroup $\mathbf{S} = [\Im, \wp]$, where:

(1) $\Im$ is a set of " elementary KR ",

(2) $\wp$ is a relation on $\Im^*$, which expresses equality of KR.

Remark. *S is defined as Subordinate Set and M is defined as Manager Set. Subordinate means " Subject to or under the authority of superior "(Dictionary.com). Hence it is understood that subordinate works under the authority of the Superior. Here the superior has been referred as Manager.*

A person who is assigned with the work can be subordinate (www.dictionary.cambridge.org). Here the person who assigns the work has been referred as manager.

Manager is someone who controls and Organizes someone (www.dictionary.cambridge.org). Here "someone " has been referred as Subordinate.

3. Application of an anti-fuzzy semigroup

We have proved that Kinship system (KS) is associated with AFS by means of an example.

Example 3.1. Consider KS is a semigroup $\mathbf{S} = [\Im, \wp]$ then

(1) $\Im$ is a set of KR,

(2) $\wp$ is a relation on $\Im^* L$ expresses equality of KR.

We take manager and subordinate in its term meaning defined in the Dictionary. In a Department S and M Jointly works for attaining the defined objectives. M and M works together the outcome or product would result relationship set with the subordinate set as they arrive at set of instruction to be given for subordinates. MS is Equal to SM, as they join as a team to work for the common Goal. In the organization both Top down as well as Bottom up approach is existing. The product decision arises in either way and remains neutral. S and S work together for the attainment of team (group of subordinates) targets as subordination of individual interests with group interest in applicable (As per Mc Kinsey $7S$ Model). In general, the defined relationships between the Manager and the subordinate would be formal and directed toward the attainment of common objective. Hence forth, Manager Set and Subordinate can be identified as the Kinship set with defined relationships.

Let M:= " is manager of ", S:= "is subordinate of", MS:= "is manager of the subordinate", SM:= "is subordinate of the manager".

Let $\mathbf{S} = \{M, S, MS, SM\}$. The set of equality of $KR = \wp = \{(MM, M), (SS, S), (MS, SM)\}$.

Let the symbol $\diamond$ denote the operation of relation product. In the first pair of L, we have (MM, M) which means that the relationship "manager of the manager" is same as the relationship "manager". That is, $M \nabla M = M$.

TABLE 1. An illustration of AFS can be represented by Cayley table with Kinship relationship under the operation ∇

∇	M	S	MS	SM
M	M	MS	MS	SM
S	SM	S	MS	SM
MS	SM	MS	MS	SM
SM	SM	SM	MS	SM

It is obvious that $\mathbf{S} = [\Im, \wp]$ is a semigroup.

Now we define a fuzzy subset $\Xi : \mathbf{S} \to [0, 1]$ by

$\Xi(S) = 0,$

$\Xi(M) = \Xi(MS) = \Xi(SM) = 0.8.$

Clearly, Ξ is a AFS of **S**.

Example 3.2. Zhang et al have defined $f(s)$ as a function that maps the set

$f(s) : \{A_{\mathbf{S}}, G_{\mathbf{S}}, T_{\mathbf{S}}, C_{\mathbf{S}}\}$ to the set $\{1, -1, i, -i\}$ as

$$f(s) = \begin{cases} 1, & \text{if } s = G_{\mathbf{S}} \\ -1, & \text{if } s = T_{\mathbf{S}} \\ i, & \text{if } s = A_{\mathbf{S}} \\ -i, & \text{if } s = C_{\mathbf{S}} \end{cases}$$

where $A_{\mathbf{S}}$-Adenine, $G_{\mathbf{S}}$-Guanine, $C_{\mathbf{S}}$-Cytosine and $T_{\mathbf{S}}$-Thymine and s is the one of the four nucleotides.

We consider

$A_S = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};$
$T_{\mathbf{S}} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix};$
$G_{\mathbf{S}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$
$C_{\mathbf{S}} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$

Let $\mathbf{S} = \{A_{\mathbf{S}}, G_{\mathbf{S}}, T_{\mathbf{S}}, C_{\mathbf{S}}\}$ represent a semigroup with the following operation $\cdot$

TABLE 2. An illustration for an AFS: Cayley table with DNA Sequences under the Operation $\cdot$

$\cdot$	$A_{\mathbf{S}}$	$G_{\mathbf{S}}$	$T_{\mathbf{S}}$	$C_{\mathbf{S}}$
$A_{\mathbf{S}}$	$T_{\mathbf{S}}$	$A_{\mathbf{S}}$	$C_{\mathbf{S}}$	$G_{\mathbf{S}}$
$G_{\mathbf{S}}$	$A_{\mathbf{S}}$	$G_{\mathbf{S}}$	$T_{\mathbf{S}}$	$C_{\mathbf{S}}$
$T_{\mathbf{S}}$	$C_{\mathbf{S}}$	$T_{\mathbf{S}}$	$G_{\mathbf{S}}$	$A_{\mathbf{S}}$
$C_{\mathbf{S}}$	$G_{\mathbf{S}}$	$C_{\mathbf{S}}$	$A_{\mathbf{S}}$	$T_{\mathbf{S}}$

Define the mapping $\Xi : \mathbf{S} \to [0,1]$ by

$$\Xi(s) = \begin{cases} 0, & \text{if } s = G_{\mathbf{S}} \\ \alpha, & \text{otherwise}, 0 < \alpha \leq 1. \end{cases}$$

We see that, Ξ is a AFS of **S**.

From the illustration below, we can confirm that the union of two $AFSs$ is also a AFS.

Theorem 3.1. *Let* $(\boldsymbol{S}, \Xi_1)$ *and* $(\boldsymbol{S}, \Xi_2)$ *be two* $AFSs$*. Then* $(\boldsymbol{S}, \Xi_1 \cup \Xi_2)$ *is an* AFS*.*

Proof. Consider Ξ_1 and Ξ_2 be two $AFSs$.

Define $\Xi_1 \cup \Xi_2 : \mathbf{S} \to [0,1]$ by

$(\Xi_1 \cup \Xi_2)(s) = \mathbf{max}\{\Xi_1(s), \Xi_2(s)\}, \forall s \in \mathbf{S}.$

$$\begin{aligned}(\Xi_1 \cup \Xi_2)(s_1 s_2) &= \mathbf{max}\{\Xi_1(s_1 s_2), \Xi_2(s_1 s_2)\} \\ &\leq \mathbf{max}\{\mathbf{max}[\Xi_1(s_1), \Xi_1(s_2)], \mathbf{max}[\Xi_2(s_1), \Xi_2(s_2)]\} \\ &= \mathbf{max}\{\mathbf{max}[\Xi_1(s_1), \Xi_2(s_1)], \mathbf{max}[\Xi_1(s_2), \Xi_2(s_2)]\} \\ &= \mathbf{max}\{(\Xi_1 \cup \Xi_2)(s_1), (\Xi_1 \cup \Xi_2)(s_2)\}\end{aligned}$$

$\Rightarrow (\Xi_1 \cup \Xi_2)(s_1 s_2) \leq \mathbf{max}\{(\Xi_1 \cup \Xi_2)(s_1), (\Xi_1 \cup \Xi_2)(s_2)\}$

This shows that the union of two $AFSs$ have been defined to form another AFS. □

Given two $AFSs$ then their intersection need not be an AFS.

Example 3.3. Consider $\mathbf{S} = \{e, p, q, pq\}$ be a semigroup, note that $p^2 = e = q^2$ and $pq = qp$.

TABLE 3. An illustration of the Cayley table for a AFS under the operation $\cdot$

$\cdot$	p	q	pq	e
p	e	pq	q	p
q	pq	e	p	q
pq	q	p	e	pq
e	p	q	pq	e

Define the fuzzy sets $\Xi_1(x), \Xi_2(s) : \mathbf{S} \to [0,1]$ as follows:

$$\Xi_1(s) = \begin{cases} 0 & \text{if } s = e \\ 0.39 & \text{if } s = p \\ 0.95 & \text{if } s = q, pq. \end{cases}$$

$$\Xi_2(s) = \begin{cases} 0 & \text{if } s = e \\ 0.79 & \text{if } s = p, pq \\ 0.61 & \text{if } s = q. \end{cases}$$

Notice that Ξ_1, Ξ_2 are $AFSs$.

Define $(\Xi_1 \cap \Xi_2)(s) = \textbf{min}\{\Xi_1(s), \Xi_2(s)\}, \forall s \in \mathbf{S}$.

Now,

$$(\Xi_1 \cap \Xi_2)(s) = \begin{cases} 0 & \text{if } s = e \\ 0.39 & \text{if } s = p \\ 0.61 & \text{if } s = q \\ 0.79 & \text{if } s = pq. \end{cases}$$

But $(\Xi_1 \cap \Xi_2)(pq) \leq \textbf{max}\{(\Xi_1 \cap \Xi_2)(p), (\Xi_1 \cap \Xi_2)(q)\}$

$$= \textbf{max}\{0.39, 0.61\} = 0.61.$$

Therefore, $(\Xi_1 \cap \Xi_2)(pq) = 0.79$. So $0.79 \leq 0.61$. This is absurd.

Ξ_1 and Ξ_2 are $AFSs$, whereas $(\Xi_1 \cap \Xi_2)$ is not an AFS.

Definition 3.4. If Ξ_1 and Ξ_2 are semigroups of $\mathbf{S}_1$ and $\mathbf{S}_2$ respectively, then the cartesian product of two semigroups of $\mathbf{S}_1$ and $\mathbf{S}_2$ is defined by $(\Xi_1 \times \Xi_2)(s,t) \leq \textbf{max}\{(\Xi_1)(s), (\Xi_2)(t)\}$, where $s \in \mathbf{S}_1, t \in \mathbf{S}_2$.

Corollary 3.2. *If Ξ_1 and Ξ_1 are semigroups of $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ respectively, then the cartesian product of two semigroups of $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ is also a semigroup of $\boldsymbol{S}$.*

Proof. Let $(s,t), (u,v) \in \mathbf{S}_1 \times \mathbf{S}_2$

$$\begin{aligned}(\Xi_1 \times \Xi_2)((s,t),(u,v)) &= (\Xi_1 \times \Xi_2)(su, tv) \\ &= \textbf{max}\{\Xi_1(su), \Xi_2(tv)\} \\ &\leq \textbf{max}\{\textbf{max}[\Xi_1(s), \Xi_1(u)], \textbf{max}[\Xi_2(t), \Xi_2(v)]\} \\ &\leq \textbf{max}\{\textbf{max}[\Xi_1(s), \Xi_2(t)], \textbf{max}[\Xi_1(u), \Xi_2(v)]\} \\ &= \textbf{max}\{(\Xi_1 \times \Xi_2)(s,t), (\Xi_1 \times \Xi_2)(u,v)\}\end{aligned}$$

□

4. MERITS AND LIMITATIONS OF THE PROPOSED WORK

MERITS OF THE PROPOSED WORK

(1) The kinship relationship between managers and subordinates provides a structured framework for facilitating hierarchical structuring and restructuring within organizations.

(2) It supports systematic team formation by offering a mathematical basis for organizing roles and relationships.

(3) The method enables the prediction of the flow of command and task completion between subordinates and managers, promoting operational efficiency.

LIMITATIONS OF THE PROPOSED WORK

(1) The framework does not account for the influence of varying levels of superiors and subordinates within the organization, which could introduce additional complexity.

(2) Human factors present a significant limitation, as interpersonal relationships are inherently unpredictable. For instance, formal kinship relationships established through the framework may not align with casual or informal interactions.

(3) Informal relationships may fail to satisfy the conditions of an anti fuzzy semigroup, and their uncertain nature could disrupt or influence the formal kinship relationships defined in this study.

5. CONCLUSION

This research introduces Anti-Fuzzy Semigroups ($AFSs$) as a novel framework for modeling hierarchical relationships. We have demonstrated that the union of two $AFSs$ always results in another AFS, while the intersection does not necessarily preserve this property. This foundational result provides crucial insights into the algebraic behavior of $AFSs$.

Unique Contributions:

- Defined the fundamental properties of $AFSs$: We established the algebraic behavior of $AFSs$ under union and intersection operations.
- Applied $AFSs$ to model hierarchical relationships: We successfully applied $AFSs$ to represent superior-subordinate relationships in organizational structures, providing a mathematical framework for analyzing managerial hierarchies.

- Explored the connection between $AFSs$ and biological systems: We investigated how DNA sequences can be represented and analyzed using the mathematical framework of $AFSs$.

Potential Directions for Future Work:

- Extend the algebraic framework: Investigate additional properties of $AFSs$ under operations like complement and symmetric difference, further enriching their theoretical foundation.

- Broaden the scope of applications: Explore the use of $AFSs$ in diverse fields such as network theory, linguistics, and social sciences to model hierarchical or relational systems.

- Integrate with emerging fields: Study the interaction of $AFSs$ and fuzzy systems with cutting-edge disciplines like quantum computing and artificial intelligence to propose innovative problem-solving techniques.

- Pythagorean fuzzy semigroups and Pythagorean anti-fuzzy semigroups have been employed to model the relationships between superiors and subordinates, offering a mathematical perspective on managerial hierarchies.

- RNA sequences derived from $AFSs$ and Pythagorean fuzzy semigroups have been analyzed and applied to decision-making problems, showcasing their potential in biological and computational contexts.

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