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# ON SOMEWHAT NEUTROSOPHIC REGULAR SEMI CONTINUOUS FUNCTIONS

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ABSTRACT. In this paper the concept of somewhat neutrosophic regular semi continuous functions, somewhat neutrosophic regular semi-open functions are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are also given. More examples are given to illustrate the concepts introduced in this paper.

## 1. INTRODUCTION

The study of fuzzy set was initiated by Zadeh [?] in 1965. Thereafter the paper of Chang [?] paved the way for the subsequent tremendous growth of the numerous fuzzy topology concepts. Currently Fuzzy Topology has been observed to be very beneficial in fixing many realistic problems. Several mathematicians have tried almost all the pivotal concepts of General Topology for extension to the fuzzy settings. In 1983, Atanassov [?] introduced the concept of intuitionistic fuzzy set which was generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. Later, Coker [?] introduced the concept of intuitionistic fuzzy topological spaces, by using the notion of the intuitionitic fuzzy set. Smarandache [?, ?, ?] introduced the concept of Neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminancy and non membership function that are independently related. In 2012, Salama and Alblowi [?, ?, ?] introduced the concept of Neutrosophic topology. Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology. In 2014, Salama et. al., [?] introduced the concept of Neutrosophic closed sets and Neutrosophic continuous functions.

Gentry and Hoyle [?] introduced and studied the concept of somewhat open functions which are Frolik functions with some conditions being dropped. These ideas are also closely related to the idea of weakly equivalent topologies which was first introduced by Yougslova [?]. Hewitt [?] introduced the concepts of resolvability and irresolvable in topological spaces. The concept of regular semiopen set was introduced by Cameron [?] in

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1978. In 2021, Vijayalakshmi and Praveena [**?**, **?**, **?**] introduced the concept of neutrosophic regular semiopen, neutrosophic regular semiclosed, neutrosophic regular semi continuous, neutrosophic regular semi irresolute, neutrosophic regular semi homeomorphisms and neutrosophic regular semi $C$-homeomorphisms in neutrosophic topological spaces. In this connection we have introduced the concept of somewhat neutrosophic regular semi continuous functions and somewhat neutrosophic regular semi-open functions and studied their properties. Also we have introduced the concept of neutrosophic regular semi resolvable and neutrosophic regular semi irresolvabe spaces and we have given characterizations of neutrosophic regular semi-resolvable and neutrosophic regular semi-irresolvable spaces. Several examples are given to illustrate the concepts introduced in this paper.

## 2. Preliminaries

**Definition 2.1.** [**?**] Let $X$ be a non-empty fixed set. A Neutrosophic set [for short, Ns] $A$ is an object having the form $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $B_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminancy and the degree of non-membership function respectively of each element $x \in X$ to the set $A$.

**Remark.** [**?**] *A Ns $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $A = \langle B_A(x), \sigma_A(x), \gamma_A(x)\rangle$ in $]^{-}0, 1^{+}[$ on $X$.*

**Remark.** [**?**] *For the sake of simplicity, we shall use the symbol $A = \langle B_A, \sigma_A, \gamma_A\rangle$ for the Ns $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.*

**Example 2.2.** [**?**] Every intuitionsistic fuzzy set $A$ is a non-empty set in $X$ is obviously on Ns having the form $A = \{\langle x, B_A(x), 1 - B_A(x) + \gamma_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic sets $0_N$ and $1_N$ in $X$ as follows:
$0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

**Definition 2.3.** [**?**] Let $A = \langle (B_A, \sigma_A, \gamma_A)\rangle$ be a Ns on $X$, then the complement of the set $A$( $A^c$ or $C(A)$ for short) may be defined as $C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), B_A(x)\rangle : x \in X\}$.

**Definition 2.4.** [**?**] Let $X$ be a non-empty set and Ns's $A$ and $B$ in the form $A = \{\langle x, B_A, \sigma_A, \gamma_A\rangle : x \in X\}$ and $B = \{\langle x, B_B, \sigma_B, \gamma_B\rangle : x \in X\}$. Then $(A \subseteq B)$ may defined as: $(A \subseteq B) \Leftrightarrow B_A(x) \subseteq B_B(x), \sigma_A(x) \subseteq \sigma_B(x), \gamma_A(x) \supseteq \gamma_B(x) \forall x \in X$.

**Definition 2.5.** [**?**] Let $X$ be a non-empty set and $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, B_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ are Ns's. Then $A \cap B$ and $A \cup B$ may defined as:

(1) $A \cap B = \langle x, B_A(x) \wedge B_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle$
(2) $A \cup B = \langle x, B_A(x) \vee B_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle$

**Definition 2.6.** [**?**] A Neutrosophic topology (for short, $NT$ or nt) is a non-empty set $X$ is a family $\tau_N$ of neutrosophic subsets in $X$ satisfying the following axioms:

(1) $0_N, 1_N \in \tau_N$,
(2) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
(3) $\cup G_i \in \tau_N$ for every $\{G_i : i \in J\} \subseteq \tau_N$.

Throughout this paper, the pair of $(X, \tau_N)$ is called a neutrosophic topological space (for short, nts). The elements of $\tau_N$ or $\tau$ are called neutrosophic open set (for short, nos). A neutrosophic set $F$ is neutrosophic closed set (for short, ncs)if and only if $F^c$ is nos.

**Definition 2.7.** [?] Let $(X, \tau_N)$ be nts and $A = \langle x, B_A, \sigma_A, \gamma_A \rangle$ be a Ns in $X$. Then the neutrosophic closure and neutrosophic interior of $A$ are defined by $NCl(A) = \cap\{K : K$ is a ncs in $X$ and $A \subseteq K\}$, $NInt(A) = \{G : G$ is a nos in $X$ and $G \subseteq A\}$. It can be also shown that $NCl(A)$ is ncs and $NInt(A)$ is a nos in $X$. $A$ is nos if and only if $A = NInt(A)$, $A$ is ncs if and only if $A = NCl(A)$.

**Definition 2.8.** Let $A = \{\langle x, B_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ be a Ns on a nts $(X, \tau_N)$ then $A$ is called:

(1) neutrosophic regular open (for short, nro) [?] iff $A = NInt(NCl(A))$.
(2) neutrosophic regular closed (for short, nrc) [?] iff $A = NCl(NInt(A))$.
(3) neutrosophic semi-open (for short, nso) [?] iff $A \subseteq NInt(NCl(A))$
(4) neutrosophic semi-closed (for short, nsc) [?] iff $A \supseteq NCl(NInt(A))$.
(5) neutrosophic regular semiopen (for short, nrso) [?] if there exists an nro set $B$ in $X$ such that $B \subseteq A \subseteq NCl(B)$.
(6) neutrosophic regular semiclosed (for short, nrsc) [?] if there exists an nrc set $B$ in $X$ and $NInt(B) \subseteq A \subseteq B$.

**Definition 2.9.** [?] Let $(X,\ \tau)$ be a nts. Then

(1) the neutrosophic regular semiclosure of $A$ defined by $nrscl(A) = \bigcap\{B \mid A \subseteq B$ and $B \in NRSCS(X, \tau)\}$ is a neutrosophic set.
(2) the neutrosophic regular semiinterior of $A$ defined by $nrsint(A) = \bigcup\{B \mid B \subseteq A$ and $B \in NRSOS(X, \tau)\}$ is a neutrosophic set.

**Definition 2.10.** [?] Let $(X, \tau)$ and $(Y, \sigma)$ be any two nts's. A map $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic continuous (for short, $NC$) if the inverse image of every neutrosophic closed set in $(Y, \sigma)$ is neutrosophic closed set in $(X, \tau)$.

## 3. Somewhat neutrosophic regular semi continuous functions

**Definition 3.1.** Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two nts's. A function $f : (X,\ \tau) \to (Y,\ \sigma)$ is called

(1) somewhat neutrosophic regular continuous (for short, $SNRC$) if for each nos $R$ of $Y$ and $f^{-1}(R) \neq 0_N$ there exists a nro $S \neq 0_N$ of $X$, such that $S \subseteq f^{-1}(R)$.
(2) somewhat neutrosophic regular semi continuous (for short, $SNRSC$) if for each nos $R$ of $Y$ and $f^{-1}(R) \neq 0_N$ there exists a nrso $S \neq 0_N$ of $X$, such that $S \subseteq f^{-1}(R)$.
(3) somewhat neutrosophic semi continuous (for short, $SNSC$) if for each nos $R$ of $Y$ and $f^{-1}(R) \neq 0_N$ there exists a nso $S \neq 0_N$ of $X$, such that $S \subseteq f^{-1}(R)$.

**Example 3.2.** Let $X = \{a, b\}$, $\tau = \{0_N, 1_N, A, B\}$, $Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, D\}$, where $A$ and $B$ are Ns of $X$ and $C$ is Ns of $Y$, defined as follows:
$A = \left\langle (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.5}) \right\rangle$,
$B = \left\langle (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.4}, \frac{\gamma_b}{0.5}) \right\rangle$,
$C = \left\langle (\frac{\mu_p}{0.5}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}) \right\rangle$.
$D = \left\langle (\frac{\mu_p}{0.5}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.6}) \right\rangle$.
Clearly $\tau$ and $\sigma$ are NT on $X$ and $Y$. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then $f$ is $SNRSC$. Since a nos $D$ of $Y$, $f^{-1}(D) \neq 0_N$, there exist a nrso set $C \neq 0_N$ such that $C \subseteq f^{-1}(D)$.

**Remark.** *(1) Every $SNRC$ function is $SNRSC$ but not conversely.*
*(2) Every $SNRSC$ function is $SNSC$ but not conversely.*

**Example 3.3.** In Example **??**, $f$ is $SNRSC$ but not $SNRC$, because $C$ is not nrc set in $X$ such that $C \subseteq f^{-1}(D)$.

**Example 3.4.** Let $X = \{a, b\}$ and $\tau = \{0_N, 1_N, A, B\}, Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, D\}$, where $A$ and $B$ are Ns of $X$ and $C$ is Ns of $Y$, defined as follows:
$A = \left\langle (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.5}) \right\rangle$,
$B = \left\langle (\frac{\mu_a}{0.6}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.5}, \frac{\gamma_b}{0.5}) \right\rangle$,
$C = \left\langle (\frac{\mu_p}{0.4}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}) \right\rangle$.
$D = \left\langle (\frac{\mu_p}{0.4}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.6}) \right\rangle$.
Clearly $\tau$ and $\sigma$ are NT on $X$ and $Y$. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then $f$ is $SNSC$ but not $SNRSC$, Since for each nos $D$, $f^{-1}(D) \neq 0_N$, there exist a nso set $C \neq 0_N$ (since $\exists$ a nos $B$ such that $B \subseteq C \subseteq NCl(B)$) such that $C \subseteq f^{-1}(D)$. but $C$ is not nrso.

**Definition 3.5.** A Ns $R$ in a nts $(X,\ \tau)$ is called $NRS$-dense if there exists no nrsc set $S$ such that $R \subset S \subset 1_N$.

**Proposition 3.1.** *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two nts's and $f : (X,\ \tau) \to (Y,\ \sigma)$ be a function. Then the following are equivalent:*

*(1) $f$ is $SNRSC$,*
*(2) If $R$ is a ncs of $Y$ such that $f^{-1}(R) \neq 1_N$, then there exists a nrsc set $S \neq 1_N$ of $X$ such that $S \supseteq f^{-1}(R)$,*
*(3) If $R$ is a $NRS$-dense set in $X$, then $f(R)$ is a $NRS$-dense set in $Y$.*

*Proof.* $(1) \Rightarrow (2)$. Suppose $f$ is $SNRSC$-continuous and $R$ is a ncs in $Y$ such that $f^{-1}(R) \neq 1_N$. Clearly $R^c$ is nos and $f^{-1}(R^c) = (f^{-1}(R))^c \neq 0_N$ (since $f^{-1}(R) \neq 1_N$). By (1), there exists a nrso set $T$ in $X$ such that $T \subseteq f^{-1}(R^c) = (f^{-1}(R))^c$. That is, $f^{-1}(R) \subseteq T^c$. Clearly $T^c$ is nrsc and taking $T^c = S$, we find that $(1) \Rightarrow (2)$ is proved.

$(2) \Rightarrow (3)$. Let $R$ be a $NRS$-dense set in $X$ and suppose $f(R)$ is not $NRS$-dense in $Y$. Then there exists a nrsc set $T$ (say) in $Y$ such that

$$f(R) \subset T \subset 1_N \tag{3.1}$$

Since $T \subset 1_N$, $f^{-1}(T) \neq 1_N$ and so by (2) there exists a nrsc set $U$ ($U \neq 1_N$) such that $U \supseteq f^{-1}(T) \supseteq f^{-1}f(R) \supseteq R$ [from **??**]. That is, there exists a nrsc set $U$ such that $U \supset R$ which is a contradiction to the assumption on $R$. Therefore $(2) \Rightarrow (3)$ is proved.

$(3) \Rightarrow (1)$. Suppose $R \neq 0_N$ be a nro set and obviously nos in $Y$ and $f^{-1}(R) \neq 0_N$. Suppose there exists no nrso set $S$ in $X$ such that $S \subseteq f^{-1}(R)$. Then $(f^{-1}(R))^c$ is a Ns in $X$ such that there is no nrsc set $U$ in $X$ with $(f^{-1}(R))^c \subset U \subset 1_N$. [Otherwise $(f^{-1}(R))^c \subset U$ implies $U^c \subset f^{-1}(R)$ and $U^c$ is nrso, a contradiction]. That is, $(f^{-1}(R))^c$ is a $NRS$-dense set in $X$. Then by (3) $f((f^{-1}(R))^c)$ is $NRS$-dense in $(Y, \sigma)$. But $f((f^{-1}(R))^c) = f((f^{-1}(R^c))) \subseteq R^c \subset 1_N$ and $f((f^{-1}(R))^c) \subseteq R^c \subset 1_N$ implies $NRSCl(f((f^{-1}(R))^c)) \subseteq NRSCl(R^c)$. Then since $f([f^{-1}(R)]^c)$ is $NRS$-dense, we have $1_N \subseteq NRSCl(R^c) = R^c$ [$R^c$ is nrc $\Rightarrow$ $R^c$ is nrsc]. This implies $R \subseteq 0_N$. That is, $R = 0_N$. But this a contradiction to the fact that $R \neq 0_N$. Therefore there exists a nrso set $S$ in $X$ such that $S \subseteq f^{-1}(R)$. This proves that $f$ is $SNRSC$. □

**Remark.** *3. Product of nrso sets is nrso sets.*

**Proposition 3.2.** *Let $(X_1,\ \tau_1)$, $(X_2,\ \tau_2)$, $(Y_1,\ \sigma_1)$ and $(Y_2,\ \sigma_2)$ be nts's such that $X_1$ is product related to $X_2$ and $Y_1$ is product related to $Y_2$. Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ be $SNRSC$. Then $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is a $SNRSC$.*

*Proof.* Let $R = \bigcup_{i,\, j}(R_i \times S_j)$ be a nos in $Y_1 \times Y_2$ where $R_i$ and $S_j$ are nrso sets in $Y_1$ and $Y_2$ respectively. We can assume that $R_i \neq 0_N$ and $S_j \neq 0_N$. If any one is zero, that factor can be omitted. Now $(f_1 \times f_2)^{-1}(R) = (f_1 \times f_2)^{-1}\big(\bigcup_{i,j}(R_i \times S_j)\big) = \bigcup_{i,j}\big(f_1 \times f_2\big)^{-1}(R_i \times S_j) = \bigcup_{i,j}\big(f_1^{-1}(R_i) \times f_2^{-1}(S_j)\big)$. Since $f_1 : X_1 \to Y_1$ is $SNRSC$ and $R_i$ is nos in $Y_1$ and $f_1^{-1}(R_1) \neq 0_N$, there exists a nrso set $U_i$ in $X_1$ such that $U_i \subseteq f_1^{-1}(R_i)$. Since $f_2 : X_2 \to Y_2$ is $SNRSC$ and $S_j$ is nos in $Y_2$ and $f_2^{-1}(S_j) \neq 0_N$, there exists a nrso set $T_j$ in $X_2$ such that $T_j \subseteq f_2^{-1}(S_j)$. Therefore $U_i \times T_j \subseteq f_1^{-1}(R_i) \times f_2^{-1}(S_j)$ and $U_i \times T_j$ is a nrso set [Remark **??**]. Also $\bigcup_{i,j} U_i \times T_j \subseteq \bigcup_{i,j} f_1^{-1}(R_i) \times f_2^{-1}(S_j)$ and $\bigcup_{i,j} U_i \times T_j$ is nrso in $X_1 \times X_2$ [Remark **??**]. That is $\bigcup_{i,j} U_i \times T_j \subseteq \bigcup_{i,j}(f_1 \times f_1)^{-1}(R_i \times S_j)$. Thus $f_1 \times f_2$ is a $SNRSC$. □

**Lemma 3.3.** *Let $g : X \to X \times Y$ be the graph of a function $f : X \to Y$. If $R$ is a Ns of $X$ and $S$ is a Ns of $Y$, then $g^{-1}(R \times S) = R \cap f^{-1}(S)$.*

**Proposition 3.4.** *Let $f : (X,\, \tau) \to (Y,\, \sigma)$ be a function. Then, if the graph $g : X \to X \times Y$ of $f$ is $SNRSC$, then $f$ is also $SNRSC$.*

*Proof.* Let $R \neq 0_N$ be a nos in $Y$. Then, by Lemma **??**, we have $f^{-1}(R) = 1_N \cap f^{-1}(R) = g^{-1}(1_N \times R)$. Since $g$ is $SNRSC$ and $1_N \times R(\neq 0_N)$ is a nos in $X \times Y$, there exists a nrso set $S \neq 0_N$ (say) of $X$ such that $S \subseteq g^{-1}(1_N \times R) = f^{-1}(R)$. This proves that $f$ is a $SNRSC$ function. □

**Proposition 3.5.** *Let $X$, $X_1$ and $X_2$ be nts's and $p_i : X_1 \times X_2 \to X_i$ $(i = 1,\, 2)$ be any $NC$ function. If $f : X \to X_1 \times X_2$ is $SNRSC$, then $p_i \circ f$ is also a $SNRSC$ function for $i = 1,\, 2$.*

*Proof.* For any nos $R \neq 0_N$ of $X_i$, we have $(p_i \circ f)^{-1}(R) = f^{-1}\big(p_i^{-1}(R)\big)$. Now $p_i^{-1}(R) \neq 0_N$ (since $R \neq 0_N$). Since $p_i$ is neutrosophic continuous, $p_i^{-1}(R)$ is nos and since $f$ is a $SNRSC$ function, then there exists a nrso set $S$ of $X$ such that $S \subseteq f^{-1}\big(p_i^{-1}(R)\big) = (p_i \circ f)^{-1}(R)$. Therefore $p_i \circ f$ is a $SNRSC$ function. □

**Proposition 3.6.** *Let $(X,\, \tau)$ and $(Y,\, \sigma)$ be any two nts's. If the function $f : (X,\, \tau) \to (Y,\, \sigma)$ is $SNRSC$ and onto and if $NRSInt(R) = 0_N$ for any Ns $R \neq 0_N$ in $(X,\, \tau)$, then $NRSInt\big(f(R)\big) = 0_N$ in $(Y,\, \sigma)$.*

*Proof.* Let $R \neq 0_N$ be a Ns in $(X,\, \tau)$ such that $NRSInt(R) = 0_N$. Then $(NRSInt(R))^c = 1_N$ implies that $NRSCl(R^c) = 1_N$. Since $f$ is $SNRSC$ and $R^C$ is $NRS$-dense in $(X,\, \tau)$, $f(R^c)$ is $NRS$-dense in $(Y,\, \sigma)$ [by Proposition **??**]. That is, $NRSCl(f(R^c)) = 1_N$. Then $NRSCl((f(R))^c) = 1_N$. [since $f$ is onto]. Therefore we have $(NRSInr(f(R))^c = 1_N$ which implies that $NRSInt(f(R)) = 0_N$. Hence the proposition. □

**Definition 3.6.** A nts $(X,\, \tau)$ is called a $NRS$-$D$-space if every nos $R \neq 0_N$ of $X$ is $NRS$-dense in $X$.

**Proposition 3.7.** *If $f : (X,\, \tau) \to (Y,\, \sigma)$ is a $SNRSC$ function from $X$ onto $Y$ and $X$ is a $NRS$-$D$-space, then $Y$ is a $NRS$-$D$-space.*

*Proof.* Let $R \neq 0_N$ be a nos in $Y$. We want to show that $R$ is $NRS$-dense in $Y$. Suppose not. Then there exists a nrsc set $S$ in $Y$ such that $R \subset S \subset 1_N$ and $f^{-1}(R) \subset f^{-1}(S) \subset f^{-1}(1_N) = 1_N$. Since $R \neq 0_N$, $f^{-1}(R) \neq 0_N$ and since $f$ is $SNRSC$, there exists a nrso set $T \neq 0_N$ in $X$ such that $T \subset f^{-1}(R)$ and $T \subset f^{-1}(R) \subset f^{-1}(S) \subset NRSCl(f^{-1}(S)) \subset 1_N$. That is, $T \subset NRSCl(f^{-1}(S)) \subset 1_N$. This contradicts the fact that $(X,\ \tau)$ is a $NRS$-$D$-space and it proves that $Y$ is a $NRS$-$D$-space. □

**Definition 3.7.** Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two nts's. A function $f : (X,\ \tau) \to (Y,\ \sigma)$ is called somewhat neutrosophic regular semi open function (for short, $SNRS$-$O$) if for each nos $R$ and $R \neq 0_N$, there exists a nrso set $S \neq 0_N$ of $(Y,\ \sigma)$ such that $S \subseteq f(R)$.

**Example 3.8.** In Example **??**, $f : Y \to X$ as $f(p) = a$ and $f(q) = b$, then $f$ is $SNRS$-$O$. Since a nos $D \neq 0_N$ of $Y$, there exist a nrso set $C \neq 0_N$ of $X$ such that $C \subseteq f(D)$.

**Proposition 3.8.** *Let $(X,\ \tau)$, $(Y,\ \sigma)$ and $(Z,\ \gamma)$ be nts's. Suppose that $f : (X,\ \tau) \to (Y,\ \sigma)$ is neutrosophic open and $g : (Y,\ \sigma) \to (Z,\ \gamma)$ is $SNRS$-$O$, then $g \circ f : (X,\ \tau) \to (Z,\ \gamma)$ is $SNRS$-$O$.*

*Proof.* Straightforward. □

**Proposition 3.9.** *Suppose that $(X,\ \tau)$ and $(Y,\ \sigma)$ be nts's. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an onto function. Then the following conditions are equivalent:*

*(1) $f$ is $SNRS$-$O$.*
*(2) If $R$ is a $NRS$-dense set in $Y$, then $f^{-1}(R)$ is a $NRS$-dense set in $X$.*

*Proof.* $(1) \Rightarrow (2)$. Assume (1). Suppose $R$ is a $NRS$-dense set in $Y$. We want to show that $f^{-1}(R)$ is a $NRS$-dense set in $X$. Suppose not. Then there exists a nrsc set $S$ in $X$ such that $f^{-1}(R) \subset S \subset 1_N$. Since $f$ is $SNRS$-$O$ and $S^c$ is nrso, there exists a nrso set $T \neq 0_N$ in $Y$ such that $T \subset f(S^c)$. Therefore $T \subset f(S^c) \subset f(f^{-1}(R^c)) \subseteq R^c$. That is, $R \subset T^c \subset 1_N$ (since $T \neq 0_N$). Now $T^c \neq 0_N$ is a nrsc set and $R \subset T^c \subset 1_N \Rightarrow R$ is not a $NRS$-dense set in $Y$, which is a contraction. Therefore $f^{-1}(R)$ must be a $NRS$-dense set in $X$. This prove $(1) \Rightarrow (2)$.

$(2) \Rightarrow (1)$. Assume (2). Suppose for a nro set (obviously nos) $R$ and $R \neq 0_N$. We want to show that $NRSInt(f(R)) \neq 0_N$. Suppose $NRSInt(f(R)) = 0_N$. Now $NRSC((f(R))^c) = (NRSInt(f(R)))^c = 1_c$. That is $(f(R))^c$ is $NRS$-dense in $Y$. Therefore by assumption (2), $f^{-1}((f(R))^c)$ is $NRS$-dense in $X$. But $f^{-1}((f(R))^c) = (f^{-1}(f(R)))^c \subseteq R^c$. Then we have $NRSCl(f^{-1}((f(R))^c)) \subseteq NRSCl(R^c) = R^c$ [since $R^c$ is nrc set $\Rightarrow R^c$ is nrsc set]. That is, $1_N \subseteq R^c$. Then $R \subseteq 0_N$. That is, $R = 0_N$ which is a contradiction to $R \neq 0_N$. Therefore $NRSInt(f(R)) \neq 0_N$. This proves that $f$ is $SNRS$-$O$. □

**Proposition 3.10.** *Suppose $(X,\ \tau)$ and $(Y,\ \sigma)$ be nts's. Let the function $f : (X,\ \tau) \to (Y,\ \sigma)$ be a bijection function. Then the following are equivalent:*

*(1) $f$ is $SNRS$-$O$.*
*(2) If $R$ is a ncs in $X$ such that $f(R) \neq 1_N$, then there exists a nrsc set $S$ in $Y$ such that $S \neq 1_N$ and $S \supset f(R)$.*

*Proof.* $(1) \Rightarrow (2)$: Let $S$ be a ncs on $X$ such that $f(S) \neq 1_N$. Since $f$ is bijective and $S^c$ is a nos on $X$, $f(S^c) = (f(S))^c \neq 0_N$. From the definition, there exists a nrso set $U \neq 0_N$ on $Y$ such that $U \subset f(S^c) = (f(S))^c$. Consequently, $f(S) \subset U^c = V \neq 1_N$ and $V$ is a nrsc set on $Y$.

$(2) \Rightarrow (1)$: Let $S$ be a nos on $X$ such that $f(S) \neq 0_N$. Then $S^c$ is a ncs on $X$ and $f(S^c) \neq 1_N$. Hence there exists a nrsc set $V \neq 1_N$ on $Y$ such that $f(S^c) \subset V$. Since $f$ is bijective, $f(S^c) = (f(S))^c \subset V$. Thus $V^c \subset f(S)$ and $V^c \neq 0_c$ is a nrso set on $Y$. Therefore, $f$ is $SNRS$-$O$. □

**Proposition 3.11.** *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two nts's. If the function $f:(X,\ \tau)\rightarrow(Y,\ \sigma)$ is $SNRS$-open and if $NRSInt(R)=0_N$ for any Ns $R\neq 0_N$ in $(Y,\ \sigma)$, then $NRSInt(f^{-1}(R))=0_N$ in $(X,\ \tau)$.*

*Proof.* Let $R\neq 0_N$ be a Ns in $(Y,\ \sigma)$ such that $NRSInt(R)=0_N$. Then $(NRSInt(R))^c=1_N$ implies that $NRSCl(R^c)=1_N$. Since the function $f$ is $SNRS$-$O$ and $R^c$ is $NRS$-dense in $(Y,\sigma)$, $f^{-1}(R^c)$ is $NRS$-dense in $(X,\tau)$ [by Proposition **??**]. That is, $NRSCl(f^{-1}(R^c))=1_N$. Then $NRSCl([f^{-1}(R))^c)=1_N$. Therefore $(NRSInt(f^{-1}(R)))^c=1_N$ implies that $NRSInt(f^{-1}(R))=0_N$. Hence the proposition. □

## 4. Neutrosophic regular semi resolvable and irresolvable spaces

**Definition 4.1.** A nts $(X,\ \tau)$ is said to be $NRS$-resolvable if there exists a $NRS$-dense set $R\neq 0_N$ in $(X,\ \tau)$ such that $NRSCl(R^c)=1_N$. Otherwise $(X,\ \tau)$ is called a $NRS$-irresolvable space.

**Proposition 4.1.** *A nts $(X,\ \tau)$ is a $NRS$-resolvable space if and only if $(X,\ \tau)$ has a pair of $NRS$-dense sets $R_1$ and $R_2$ such that $R_1\subseteq {R_2}^c$.*

*Proof.* Let $(X,\ \tau)$ be a $NRS$-resolvable space. Suppose that for all $NRS$-dense sets $R_i$ and $R_j$, we have $R_i\nsubseteq {R_j}^c$. Then we have $R_i\supset {R_j}^c$ for some $i$ and $j$. Then, we have $NRSCl(R_i)\supset NRSCl({R_j}^c)$ which implies that $1_N\supset NRSCl({R_j}^c)$. Then $NRSCl({R_j}^c)\neq 1_N$. Also $R_j\supset {R_i}^c$. Then $NRSCl(R_j)\supset NRSCl({R_i}^c)$ which implies that $1_N\supset NRSCl({R_i}^c)$. Then $NRSCl({R_i}^c)\neq 1_N$. Hence $NRSCl(R_i)=1_N$, but $NRSCl({R_i}^c)\neq 1_N$ for all $NRS$-dense sets $R_i$ in $(X,\ \tau)$, which is a contradiction to $(X,\ \tau)$ being a $NRS$-resolvable space. Therefore $(X,\ \tau)$ has a pair of $NRS$-dense sets $R_1$ and $R_2$ such that $R_1\subseteq {R_2}^c$.

Conversely, suppose that the nts $(X,\ \tau)$ has a pair of $NRS$-dense sets $R_1$ and $R_2$, such that $R_1\subseteq {R_2}^c$. We want to show that $(X,\ \tau)$ is $NRS$-resolvable. Suppose that $(X,\ \tau)$ is a $NRS$-irresolvable space. Then for all $NRS$-dense sets $R_i$ in $(X,\ \tau)$, we have $NRSCl({R_i}^c)\neq 1_N$. In particular $NRSCl({R_2}^c)\neq 1_N$ implies that there exist a nrsc set $S$ in $(X,\ \tau)$ such that $({R_2}^c)\subset S\subset 1_N$. Then $R_1\subseteq {R_2}^c\subset S\subset 1_N\Rightarrow R_1\subset S\subset 1_N$, which is a contradiction to $NRSCl(R_1)=1_N$. Hence our assumption that $(X,\ \tau)$ is a $NRS$-irresolvable space, is wrong. Hence $(X,\ \tau)$ is a $NRS$-resolvable space. □

**Proposition 4.2.** *A nts $(X,\ \tau)$ is a $NRS$-resolvable space if $\bigcup\limits_{i=1}^{i=n} R_i=1_N$ where $NRSInt(R_i)=0_N$.*

*Proof.* $\bigcup\limits_{i=1}^{n} R_i=1_N$ where $NRSInt(R_i)=0_N$, implies that $(\bigcup\limits_{i=1}^{n} R_i)^c=0_N$. Then we have $\bigcap\limits_{i=1}^{n}({R_i}^c)=0_N$. Then there must be at least two non-zero disjoint Ns's $(R_i)^c$, $(R_j)^c$ in $(X,\tau)$. Hence $((R_i)^c)+((R_j)^c)\subseteq 1_N$. Therefore $((R_i)^c)\subseteq R_j$ which implies that $NRSCl((R_i)^c)\subseteq NRSCl(R_j)$. But $NRSInt(R_i)=0_N$ implies that $NRSCl((R_i)^c)=1_N$. Hence $1_N\subseteq NRSCl(R_j)$ which implies that $NRSCl(R_j)=1_N$. Also $NRSInt(R_j)=0_N$ implies that $NRSCl((R_j)^c)=1_N$. Therefore $(X,\ \tau)$ has a $NRS$-dense set $R_j$ such that $NRSCl((R_j)^c)=1_N$. Hence $(X,\ \tau)$ is a $NRS$-resolvable space. □

**Proposition 4.3.** *If $(X,\ \tau)$ is $NRS$-irresolvable if and only if $NRSInt(R)\neq 0_N$ for all $NRS$-dense sets $R$ in $(X,\ \tau)$.*

*Proof.* Since $(X,\ \tau)$ is $NRS$-irresolvable, for all $NRS$-dense sets $R$ in $(X,\ \tau)$, we have $NRSCl(R^c) \neq 1_N$. Then $(NRSInt(R))^c \neq 1_N$ implies that $NRSInt(R) \neq 0_N$.

Conversely, let $NRSInt(R) \neq 0_N$ for each $NRS$-dense set $R$ in $(X,\ \tau)$. Suppose that $(X, \tau)$ is $NRS$-resolvable. Then there exists a $NRS$-dense set $R \neq 0_N$ in $(X,\ \tau)$ such that $NRSCl(R^c) = 1_N$. Then we have $NRSInt(R) = 1_N$ and therefore $NRSInt(R) = 0_N$ which is a contradiction. Hence $(X,\ \tau)$ is a $NRS$-irresolvable space. □

## 5. FUNCTIONS AND NEUTROSOPHIC REGULAR SEMI IRRESOLVABLE SPACES

**Definition 5.1.** A function $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ is said to be weakly somewhat neutrosophic regular semi open function (for short, $WSNRSO$)if for each $NRS$-dense set $R$ in $(Y,\ \sigma)$ with $NRSInt(R) \neq 0_N$, we have that $f^{-1}(R)$ is also a $NRS$-dense set in $(X,\ \tau)$.

The above definition leads to a characterization of $NRS$-irresolvable space as follows:

**Proposition 5.1.** *The following statements are equivalent for a nts $(Y,\ \sigma)$.*

*(1) $(Y,\ \sigma)$ is $NRS$-irresolvable.*
*(2) For every nts $(X,\ \tau)$, every $WSNRSO$ function $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ is $SNRS$-$O$.*

*Proof.* (1) $\Rightarrow$ (2) Let $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ be a $WSNRSO$ function from a nts $(X,\ \tau)$ to a $NRS$-irresolvable space $(Y,\ \sigma)$. Since $(Y,\ \sigma)$ is $NRS$-irresolvable space, $(Y,\ \sigma)$ has a pair of $NRS$-dense sets $R_1$ and $R_2$ such that $R_1 \not\subseteq (R_2)^c$. Now $NRSInt(R_1) \neq 0_N$ and $NRSInt(R_2) \neq 0_N$. For, if $NRSInt(R_1) = 0_N$ then, $(NRSCl((R_1)^c))^c = 0_N$. Now $R_1 \supset (R_2)^c \Rightarrow R_2 \supset (R_1)^c$. Therefore $NRSCl(R_2) \supset NRSCl((R_1)^c)$. In other words $(NRSCl(R_2))^c \subset (NRSCl((R_1)^c))^c = 0_N$. Then $1_N \subset NRSCl(R_2)$ implies $1_N \subset 1_N$, which is a contradiction. Therefore $NRSInt(R_1) \neq 0_N$. Similarly we can have $NRSInt(R_2) \neq 0_N$. Since $f$ is $WSNRSO$, $f^{-1}(R_1)$ and $f^{-1}(R_2)$ are $NRS$-dense sets in $(X, \tau)$. Therefore by Proposition **??** $f$ is $WSNRSO$.

(2) $\Rightarrow$ (1) Suppose that nts $(Y,\ \sigma)$ is $NRS$-resolvable. This means that there exists a pair of $NRS$-dense sets $R_1$ and $R_2$ such that $R_1 \subseteq [R_2]^c$. Define $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ to be the identity function. Then $f$ is not $SNRS$-$O$, since $f^{-1}(R_2)$ is not a $NRS$-dense set in $(Y,\ \tau)$. For, $f^{-1}(R_2) = R_2$ and $R_2 \subseteq (R_1)^c \neq 1_N$. Then $R_2 \subseteq (R_1)^c \Rightarrow NRSCl(R_2) \subseteq NRSCl((R_1)^c)$. Since $(R_1)^c$ is nrc set and hence nrsc in $(Y,\ \tau)$, $NRSCl(R_2) \neq 1_N$. That is, $R_2$ is not a $NRS$-dense set. We shall now show that $f$ is $WSNRSO$. Let $R$ be any $NRS$-dense set in $(Y,\ \sigma)$ such that $NRSInt(R) \neq 0_N$. Then $f^{-1}(R) = R$. We have to show that $NRSCl[f^{-1}(R)] = NRSCl(R) = 1_N$ in $(Y,\ \tau)$. Now $NRSInt(R) \neq 0_N$ and $R_1$ is $NRS$-dense implies that $R \not\subseteq (R_1)^c$. Therefore $NRSCl(R) = 1_N$. That is, $R$ is $NRS$-dense in $(Y,\ \tau)$. This proves that $f$ is $WSNRSO$. Hence (2) $\Rightarrow$ (1) is proved. □

**Proposition 5.2.** *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two nts's. Let $f : (X,\ \tau) \rightarrow (Y,\ \sigma)$ be a $SNRS$-$O$ function. If $(X,\ \tau)$ is a $NRS$-irresolvable space, then $(Y,\ \sigma)$ is a $NRS$-irresolvable space.*

*Proof.* Let $R \neq 0_N$ be an arbitrary neutrosophic set in $(Y,\ \sigma)$ such that $NRSCl(R) = 1_N$. We claim that $NRSInt(R) \neq 0_N$. Assume the contrary. That is, $NRSInt(R) = 0_N$. Then by Proposition **??**, we have $NRSInt(f^{-1}(R)) = 0_N$ in $(X,\ \tau)$. Now $R$ is $NRS$-dense in $(Y,\ \sigma)$, then by Proposition **??**, we have $f^{-1}(R)$ is $NRS$-dense in $(X,\ \tau)$. Therefore for the $NRS$-dense that $f^{-1}(R)$, we have $NRSInt(f^{-1}(R)) = 0_N$ in $(X,\ \tau)$, which is a contradiction [since $(X,\ \tau)$ is $NRS$-irresolvable, by Proposition **??** $NRSInt(R) \neq 0_N$ for all $NRS$-dense sets $R$ in $(X,\ \tau)$]. Hence we must have $NRSInt(R) \neq 0_N$ for all $NRS$-dense sets $R$ in $(Y,\ \sigma)$. Hence by Proposition **??**, $(Y,\ \sigma)$ is a $NRS$-irresolvable space. □

**Proposition 5.3.** *Let $(X,\ \tau)$ and $(Y,\ \sigma)$ be any two nts's and $f : (X,\ \tau) \to (Y,\ \sigma)$ be a $SNRSC$ and onto function. If $(Y,\ \sigma)$ is a $NRS$-irresolvable space, then $(X,\ \tau)$ is a $NRS$-irresolvable space.*

*Proof.* Let $R \neq 0_N$ be an arbitrary neutrosophic set in $(X,\ \tau)$ such that $NRSCl(R) = 1_N$. We claim that $NRSInt(R) \neq 0_N$. Assume the contrary. That is, $NRSInt(R) = 0_N$. Then by Proposition **??**, we have $NRSInt(f(R)) = 0_N$. Now $R$ is $NRS$-dense in $(X,\ \tau)$, then by Proposition **??**, we have $f(R)$ is $NRS$-dense in $(Y,\ \sigma)$. Therefore for the $NRS$-dense set $f(R)$ in $(Y,\ \sigma)$, we have $NRSInt(f(R)) = 0_N$, which is a contradiction [since $(Y,\ \sigma)$ is $NRS$-irresolvable, $NRSInt(S) \neq 0_N$ for all $NRS$-dense sets $S$ in $(X,\ \tau)$]. Therefore we must have $NRSInt(R) \neq 0_N$ for all $NRS$-dense sets $R$ in $(Y,\ \tau)$. Hence by Proposition **??**, the nts $(X,\ \tau)$ is a $NRS$-irresolvable space. □

## 6. Conclusions

In this paper, the concept of somewhat neutrosophic regular semi continuous, somewhat neutrosophic regular semi-open functions are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are also given. More examples are given to illustrate the concepts introduced in this paper. In future, we can be extended to somewhat neutrosophic regular semi irresolute, somewhat neutrosophic regular semi irresolute homeomorphisms, somewhat neutrosophic regular semi contra continuous, neutrosophic regular semi compact and connected in nts.

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