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QUOTIENT NEARNESS d-ALGEBRAS

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ABSTRACT. BCK/BCI-algebra is a class of logical algebras that was defined by K. Iseki and S. Tanaka. BCK-algebras have a lot of generalizations. One of them is d-algebras. Near set theory which is a generalization of rough set theory. This theory is based on the determination of universal sets according to the available information of the objects. Based on the image analysis, the near set theory was created. Öztürk applied the notion of near sets defined by J. F. Peters to the theory of d-algebras.

In this paper we introduce upper-nearness d-ideal, upper-near (upper-nearness) $d^{\#}$-ideal, upper-near (upper-nearness) d^{*}-ideal. We explored what conditions we should put on the ideal for quotient nearness d-algebra to become an nearness d-algebra again. Moreover, we introduce quotient nearness d-algebras with the help of upper-nearness d^{*}-ideals of nearness d-algebras. Finally, we present a theorem involving the canonical homomorphism and the structure of the kernel for nearness d-algebras. Thus, we aim to make preliminary preparations for proving isomorphism theorems for nearness d-algebras.

1. INTRODUCTION

Set theory is very important tool especially for engineers and mathematicians. They use set theory as a base in their studies. Researchers defined new approaches when ordinary set theory is insufficicent. Because the real world is uncertain, imprecise and absolute. The uncertainty that rough set theory focuses on is caused by indistinguishable elements with different values in the decision properties ([25]). In last years, rough set theory and its applications have been studied by many researchers.

Near set theory which is a generalization of rough set theory was introduced in 2002 by J. F. Peters. This theory is based on the determination of universal sets according to the available information of the objects. Based on the image analysis, the near set theory was created. Near set theory is a tool for identifying and distinguishing similarities between perceived properties of different objects. A property of physical objects is represented by a real-valued search function. In [28], an indistinguishability relation based on the properties of the objects is given to describe the proximity of objects. In more recent work, it has been adopted as a generalized approximation theory for investigating the approximation of similar non-empty sets (see [21], [22], [23], [24], [26], [27], [29], and [30]).

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We may be used near set theory to turn elements in algebraic structures into concrete elements. Algebraic structures as we know them consist of non-empty abstract points. But this is not useful for the problems we face in daily life. All researchers who study algebraic structures consider abstract elements. But using them in our study for some time is insufficient. We use perceptual objects (non-abstract points) in near set theory. Perceptual objects have some features such as color and degree of maturation for an apple. In algebraic structures built on nearness approximation spaces or weak nearness approximation spaces, the main tool is upper approximations of subsets of perceptual objects. Nearness is studied with non-abstract points in algebraic structures, and superapproximations of perceptual objects are taken into account for the nearness of binary operations. This is the important difference between classical algebraic structures and nearness algebraic structures.

In 2012, İnan and Öztürk analyzed the concept of nearness groups and investigated their basic properties ([2]). After, in [3] and [13] the nearness semigroups and nearness rings were established and their basic properties were investigated, respectively (and other algebraic approaches of near sets in see [10], [12], [14], [15], [16], [17], [19], [31], [32], and [33]).

BCK/BCI-algebra is a class of logical algebras that was defined by K. Iseki and S. Tanaka ([4]). BCK-algebras have a lot of generalizations. One of them is d-algebras. After that some further aspects were studied ([1], [5], [6], [7], [8] and [9]).

In 2015, Öztürk, Çelik Siner, and Jun introduced BCK-algebras on nearness approximation spaces [11]. Afterwards, in [20], Öztürk and Jun defined quotient $NBCK$-algebras defined via ideals and also analyzed some properties of the quotient $NBCK$-algebras.

Recently, Öztürk [18] has defined the notion of nearness d-algebras and investigate several relations between nearness d-algebras and nearness BCK-algebras. Furthermore, he has shown that the notions of nearness d-subalgebra, nearness d-ideal in nearness d-algebras, and investigate relations among them.

In this paper, essentially, our approach is to investigate which ideal type we can define quotient d-algebras. That is, we explore what conditions we should put on the ideal for nearness quotient d-algebra to become a nearness d-algebra again.

2. PRELIMINARIES

Let $\mathcal{O}$ be an approximation space defined with respect to a set of perceived objects, $\mathcal{F}$ be set of probe functions, $\sim_{B_r}$ be indiscernibility relation. A weak nearness approximation space is a tuple $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$, where the approximation space is defined with respect to a set of perceived objects $\mathcal{O}$, set of probe functions $\mathcal{F}$ representing object features, $\sim_{B_r}$indiscernibility relation, $N_r(B)$ be collection of partitions (families of neighbourhoods) $N_r(B)$. A weak nearness approximation space is denoted by a tuple $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$. For detailed information on weak nearness approximation space, refer to [26] and [14].

Definition 2.1. ([18]) Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B))$ be a weak nearness approximation space; $\emptyset \neq \mathcal{X} \subseteq \mathcal{O}$, 0 be a constant on $\mathcal{O}$ and $\boxdot : \mathcal{X} \times \mathcal{X} \to N_r(B)^- \mathcal{X}$ be a well-defined operation. A subset $\mathcal{X}$ of the set $\mathcal{O}$ is called d-algebra on weak nearness approximation space $\mathcal{O}$ or nearness d-algebra for short if the following properties are satisfied for all $a, b, c \in \mathcal{X}$

$Nd{-}\mathcal{I}$) The property $a \boxdot a = 0$ holds in $N_r(B)^- \mathcal{X}$,

$Nd{-}\mathcal{II}$) The property $0 \boxdot a = 0$ holds in $N_r(B)^- \mathcal{X}$,

$Nd{-}\mathcal{III}$) If the property $a \boxdot b = 0$, $b \boxdot a = 0$ then $a = b$ holds in $\mathcal{X}$.

Example 2.2. ([18]) Let $\mathcal{O} = \{0, x, y, z, t, w\}$ be a set of perceptual objects, $r = 1$, $B = \{\psi_1, \psi_2, \psi_3\} \subseteq \mathcal{F}$ be a set of probe functions, and $\mathcal{X} = \{x, y, z\} \subset \mathcal{O}$. Let us take

$$\psi_1 : \mathcal{O} \to V_1 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\},$$
$$\psi_2 : \mathcal{O} \to V_2 = \{\sigma_2, \sigma_3, \sigma_4\},$$
$$\psi_3 : \mathcal{O} \to V_3 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}$$

and

TABLE 1. Values of the probe functions ψ_1, ψ_2, ψ_3.

	0	x	y	z	t	w
ψ_1	σ_1	σ_1	σ_2	σ_3	σ_1	σ_4
ψ_2	σ_2	σ_3	σ_2	σ_3	σ_3	σ_4
ψ_3	σ_2	σ_3	σ_2	σ_4	σ_1	σ_5

We can write

$$N_1 (B)^- \mathcal{X} = \bigcup_{[a]_{\psi_\mathcal{I}} \cap \mathcal{X} \neq \emptyset} [a]_{\psi_\mathcal{I}} = \{0, x, y, z, t\}.$$

Then $\mathcal{X} := \{x, y, z\}$ is a nearness d-algebra by Definition 2.1 where

TABLE 2. Cayley table of the operation $\boxdot$.

$\boxdot$	0	x	y	z	t
0	0	0	0	0	0
x	x	0	0	0	0
y	y	y	0	t	t
z	z	x	x	0	0
t	t	t	t	t	0

Definition 2.3. ([18]) Let $\mathcal{X}$ be a nearness d-algebra, and S be a non-empty subset of $\mathcal{X}$. Then, S is called a nearness d-subalgebra of $\mathcal{X}$ if $a \boxdot b \in N_r (B)^- S$ for all $a, b \in S$. S is called an upper-near d-subalgebra of $\mathcal{X}$ if $0 \in N_r (B)^- S$ and $a \boxdot b \in N_r (B)^- S$ for all $a, b \in N_r (B)^- S$.

3. NEARNESS d-IDEALS IN d-ALGEBRAS

Definition 3.1. ([18]) Let $(\mathcal{X}, \boxdot, 0)$ be a nearness d-algebra, and $\emptyset \neq \mathcal{I} \subseteq \mathcal{X}$.
i) If the following assertions are provided in $\mathcal{I}$:
$N\mathcal{I}1)$ $a \boxdot b \in N_r (B)^- \mathcal{I}$, $b \in \mathcal{I}$ *implies* $a \in \mathcal{I}$ for all $a, b \in \mathcal{X}$,
$N\mathcal{I}2)$ $a \in \mathcal{I}$, $b \in X$ implies $a \boxdot b \in N_r (B)^- \mathcal{I}$,
then $\mathcal{I}$ is called a nearness d-ideal of $\mathcal{X}$
ii) If the following assertions are provided in $\mathcal{I}$:
$UN\mathcal{I}1)$ $0 \in N_r (B)^- \mathcal{I}$,
$UN\mathcal{I}2)$ $a \boxdot b \in N_r (B)^- \mathcal{I}$, $b \in \mathcal{I}$ *implies* $a \in \mathcal{I}$ for all $a, b \in \mathcal{X}$,

$UN\mathcal{I}3)$ $a \in N_r(B)^- \mathcal{I}, b \in N_r(B)^- X$ implies $a \boxdot b \in N_r(B)^- \mathcal{I}$,
then $\mathcal{I}$ is called an upper-near d-ideal of $\mathcal{X}$.

Definition 3.2. Let $(\mathcal{X}, \boxdot, 0)$ be a nearness d-algebra and $\emptyset \neq \mathcal{I} \subseteq \mathcal{X}$. If the following assertions are provided,

$UN'\mathcal{I}1)$ $0 \in N_r(B)^- \mathcal{I}$,

$UN'\mathcal{I}2)$ $a \boxdot b \in N_r(B)^- \mathcal{I}, b \in N_r(B)^- \mathcal{I}$ *implies* $a \in N_r(B)^- \mathcal{I}$ for all $a, b \in \mathcal{X}$,

$UN'\mathcal{I}3)$ $a \in N_r(B)^- \mathcal{I}, b \in N_r(B)^- X$ implies $a \boxdot b \in N_r(B)^- \mathcal{I}$,

then $\mathcal{I}$ is called an upper-nearness d-ideal of $\mathcal{X}$.

Example 3.3. Let $\mathcal{O} = \{0, x, y, z, t, w, u\}$ be a set of perceptual objects, $r = 1$, $B = \{\psi_1, \psi_2, \psi_3\} \subseteq \mathcal{F}$ be a set of probe functions, and $\mathcal{X} = \{x, z, t, w\} \subset \mathcal{O}, \mathcal{I} = \{x, z\} \subset \mathcal{X}$. Let us take

$$\psi_1 : \mathcal{O} \to V_1 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\},$$
$$\psi_2 : \mathcal{O} \to V_2 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\},$$
$$\psi_3 : \mathcal{O} \to V_3 = \{\sigma_1, \sigma_2, \sigma_3\}$$

and

TABLE 3. Values of the probe functions ψ_1, ψ_2, ψ_3.

	0	x	y	z	t	w	u
ψ_1	σ_1	σ_1	σ_2	σ_1	σ_3	σ_3	σ_4
ψ_2	σ_1	σ_4	σ_2	σ_4	σ_3	σ_3	σ_2
ψ_3	σ_2	σ_1	σ_3	σ_1	σ_2	σ_2	σ_3

Then, we get $N_1(B)^- \mathcal{X} = \{0, x, z, t, w\}$ and $N_1(B)^- \mathcal{I} = \{0, x, z\}$. Therefore $(\mathcal{X}, \boxdot, 0)$ is a d-algebra on $\mathcal{O}$ by Definition 2.1. Then $\mathcal{I}$ is an upper-near d-ideal of $\mathcal{X}$ where

TABLE 4. Cayley table of the operation $\boxdot$.

$\boxdot$	0	x	y	z	t	w	u
0	0	0	0	0	0	0	0
x	x	0	0	0	0	0	0
y	y	y	0	0	w	w	y
z	z	x	x	0	0	0	w
t	t	t	t	t	0	t	w
w	w	t	t	t	t	0	w
u	u	u	u	u	u	u	0

Example 3.4. Let $\mathcal{O} = \{0, x, y, z, t, w, u\}$ be a set of perceptual objects, $r = 1$, $B = \{\psi_1, \psi_2, \psi_3\} \subseteq \mathcal{F}$ be a set of probe functions, and $\mathcal{X} = \{0, x, y, z, t, w\} \subset \mathcal{O}, \mathcal{I} = \{0, x, t, w\} \subset \mathcal{X}$. Let us take

$$\psi_1 : \mathcal{O} \to V_1 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\},$$
$$\psi_2 : \mathcal{O} \to V_2 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\},$$
$$\psi_3 : \mathcal{O} \to V_3 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}$$

and

TABLE 5. Values of the probe functions ψ_1, ψ_2, ψ_3.

	0	x	y	z	t	w	u
ψ_1	σ_1	σ_1	σ_2	σ_2	σ_3	σ_5	σ_4
ψ_2	σ_1	σ_4	σ_2	σ_2	σ_3	σ_3	σ_2
ψ_3	σ_2	σ_1	σ_4	σ_5	σ_1	σ_3	σ_4

In this case, we get $N_1(B)^- \mathcal{X} = \{0, x, y, z, t, w, u\}$ and $N_1(B)^- \mathcal{I} = \{0, x, t, w\}$. Then, $(\mathcal{X}, \boxdot, 0)$ is a d-algebra on $\mathcal{O}$ by Definition 2.1 and $\mathcal{I}$ is a nearness d-ideal of $\mathcal{X}$. Moreover, it is an upper-nearness d-ideal of $\mathcal{X}$ where

TABLE 6. Cayley table of the operation $\boxdot$.

$\boxdot$	0	x	y	z	t	w	u
0	0	0	0	0	0	0	0
x	x	0	0	0	0	0	0
y	y	y	0	0	y	y	0
z	z	z	z	0	z	z	0
t	t	t	t	t	0	t	0
w	w	w	w	w	w	0	0
u	u	u	u	u	u	u	0

Let $\mathcal{X}$ be a nearness d-algebra. Let $A \boxdot B = \{a \boxdot b \mid a \in A, b \in B\}$, where A and B are subsets of $\mathcal{X}$.

Lemma 3.1. *Let $\mathcal{X}$ be a nearness d-algebra and $\mathcal{I}$ be a nearness d-ideal of $\mathcal{X}$. Then $0 \in N_r(B)^- \mathcal{I}$.*

Proof. Since $\mathcal{I} \neq \emptyset$, there exists $a \in \mathcal{I}$ and hence $0 = a \boxdot a \in N_r(B)^- \mathcal{I}$ from $(N\mathcal{I}2)$. □

Proposition 3.2. *Let $\mathcal{I}$ be a nearness d-ideal of nearness d-algebra $\mathcal{X}$ and $a \in \mathcal{I}$. If $b \boxdot a = 0$, then $b \in \mathcal{I}$.*

Proof. Let $b \boxdot a = 0$ for $a \in \mathcal{I}$. $b \boxdot a = 0 \in N_r(B)^- \mathcal{I}$, and so $b \in \mathcal{I}$ from $(N\mathcal{I}1)$. □

Definition 3.5. Let $\mathcal{X}$ is a nearness d-algebra. An upper-near d-ideal of $\mathcal{X}$ is called an upper-near $d^{\#}$-ideal of $\mathcal{X}$ if, for all $a, b, c \in \mathcal{X}$,

$UN\mathcal{I}4)$ $a \boxdot b \in N_r(B)^- \mathcal{I}$ and $b \boxdot c \in N_r(B)^- \mathcal{I}$ imply $a \boxdot c \in N_r(B)^- \mathcal{I}$.

Example 3.6. In Example 3.4, $\mathcal{I} = \{0, x, t, w\}$ is an upper near $d^{\#}$-ideal.

Every upper-near $d^{\#}$-ideal is an upper-near d-ideal, but its converse can not be true.

Definition 3.7. Let $\mathcal{X}$ be a nearness d-algebra. An upper-nearness d-ideal of $\mathcal{X}$ is called an upper-nearness $d^{\#}$-ideal of $\mathcal{X}$ if, for all $a, b, c \in \mathcal{X}$,

$UN'\mathcal{I}4)$ $a \boxdot b \in N_r(B)^- \mathcal{I}$ and $b \boxdot c \in N_r(B)^- \mathcal{I}$ imply $a \boxdot c \in N_r(B)^- \mathcal{I}$.

Every upper-nearness $d^{\#}$-ideal is an upper-nearness d-ideal, but its converse can not be true.

Example 3.8. Let $\mathcal{O} = \{0, x, y, z, t, w, u\}$ be a set of perceptual objects, $r = 1, B = \{\varphi_1, \varphi_2, \varphi_3\} \subseteq \mathcal{F}$ be a set of probe functions, and $\mathcal{X} = \{x, y, z\} \subset \mathcal{O}$, $I = \{x\} \subset \mathcal{X}$. Values of the probe functions

$$\begin{aligned}
\varphi_1 &: \mathcal{O} \to V_1 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\},\\
\varphi_2 &: \mathcal{O} \to V_2 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\},\\
\varphi_3 &: \mathcal{O} \to V_3 = \{\sigma_1, \sigma_3, \sigma_4\}
\end{aligned}$$

are given in the following table:

TABLE 7. Values of the probe functions $\varphi_1, \varphi_2, \varphi_3$.

	0	x	y	z	t	w	u
φ_1	σ_1	σ_1	σ_2	σ_2	σ_3	σ_5	σ_4
φ_2	σ_1	σ_4	σ_2	σ_2	σ_3	σ_2	σ_5
φ_3	σ_1	σ_1	σ_4	σ_1	σ_3	σ_4	σ_3

Let us now determine the near equivalence classes according to the indiscernibility relation of $\sim_{B_r}$ of elements of $\mathcal{O}$:

$$\begin{aligned}
[0]_{\varphi_1} &= \{a \in \mathcal{O} \mid \varphi_1(a) = \varphi_1(0) = \sigma_1\} = \{0, x\}\\
&= [x]_{\varphi_1}\\
[y]_{\varphi_1} &= \{a \in \mathcal{O} \mid \varphi_1(a) = \varphi_1(y) = \sigma_2\} = \{y, z\}\\
&= [z]_{\varphi_1},\\
[t]_{\varphi_1} &= \{a \in \mathcal{O} \mid \varphi_1(a) = \varphi_1(t) = \sigma_3\} = \{t\},\\
[w]_{\varphi_1} &= \{a \in \mathcal{O} \mid \varphi_1(a) = \varphi_1(w) = \sigma_5\} = \{w\},\\
[u]_{\varphi_1} &= \{a \in \mathcal{O} \mid \varphi_1(a) = \varphi_1(u) = \sigma_4\} = \{u\}.
\end{aligned}$$

Then we get that $\xi_{\varphi_1} = \left\{[0]_{\varphi_1}, [y]_{\varphi_1}, [t]_{\varphi_1}, [w]_{\varphi_1}, [u]_{\varphi_1}\right\}$.

$$\begin{aligned}
[0]_{\varphi_2} &= \{a \in \mathcal{O} \mid \varphi_2(a) = \varphi_2(0) = \sigma_1\} = \{0\},\\
[x]_{\varphi_2} &= \{a \in \mathcal{O} \mid \varphi_2(a) = \varphi_2(x) = \sigma_4\} = \{x\},\\
[y]_{\varphi_2} &= \{a \in \mathcal{O} \mid \varphi_2(a) = \varphi_2(y) = \sigma_2\} = \{y, z, w\}\\
&= [z]_{\varphi_2} = [w]_{\varphi_2},\\
[t]_{\varphi_2} &= \{a \in \mathcal{O} \mid \varphi_2(a) = \varphi_2(t) = \sigma_3\} = \{t\},\\
[u]_{\varphi_2} &= \{a \in \mathcal{O} \mid \varphi_2(a) = \varphi_2(u) = \sigma_5\} = \{u\}.
\end{aligned}$$

Thus, we have that $\xi_{\varphi_2} = \left\{ [0]_{\varphi_2}, [x]_{\varphi_2}, [y]_{\varphi_2}, [t]_{\varphi_2}, [u]_{\varphi_2} \right\}$.

$$\begin{aligned} [0]_{\varphi_3} &= \{a \in \mathcal{O} \mid \varphi_3(a) = \varphi_3(0) = \sigma_1\} = \{0, x, z\} \\ &= [x]_{\varphi_3} = [z]_{\varphi_3}, \\ [y]_{\varphi_3} &= \{a \in \mathcal{O} \mid \varphi_3(a) = \varphi_3(y) = \sigma_4\} = \{y, w\} \\ &= [w]_{\varphi_3} \\ [t]_{\varphi_3} &= \{a \in \mathcal{O} \mid \varphi_3(a) = \varphi_3(t) = \sigma_3\} = \{t, u \\ &= [u]_{\varphi_3}. \end{aligned}$$

From hence, we obtain that $\xi_{\varphi_3} = \left\{ [0]_{\varphi_3}, [y]_{\varphi_3}, [t]_{\varphi_3} \right\}$. Therefore, for $r = 1$, a set of partitions of $\mathcal{O}$ is $N_1(B) = \{\xi_{\varphi_1}, \xi_{\varphi_2}, \xi_{\varphi_3}\}$. Then, we can write

$$\begin{aligned} N_1(B)^- \mathcal{X} &= \bigcup_{[x]_{\varphi_i} \cap \mathcal{X} \neq \emptyset} [x]_{\varphi_i} \\ &= \{0, x, y, z, w\}. \end{aligned}$$

Furthermore,

$$\begin{aligned} N_1(B)^- I &= \bigcup_{[x]_{\varphi_i} \cap I \neq \emptyset} [x]_{\varphi_i} \\ &= \{0, x, z\}. \end{aligned}$$

Considering the following table of operation:

TABLE 8. Cayley table of the operation $\circledast$.

$\circledast$	0	x	y	z	t	w	u
0	0	0	0	0	0	0	0
x	x	0	0	0	0	0	0
y	y	y	0	0	w	w	y
z	z	x	x	0	0	0	w
t	t	t	t	t	0	t	w
w	w	t	t	t	t	0	w
u	u	u	u	u	u	u	0

In that case; by Definition 2.1, $\mathcal{X}$ is a nearness d-algebra. Then, I is a upper-nearness d-ideal of $\mathcal{X}$, but is not upper-near $d^{\#}$-ideal, since $y, z, x \in \mathcal{X}$, $y \circledast z = 0 \in N_r(B)^- I$ and $z \circledast x = x \in N_r(B)^- I$ *implies* $y \circledast x = y \notin N_r(B)^- I$.

Example 3.9. In Example 3.4, $\mathcal{I} = \{0, x, t, w\}$ is an upper-near $d^{\#}$-ideal. Moreover, it is an upper-nearness $d^{\#}$-ideal of $\mathcal{X}$.

Definition 3.10. A nearness d-algebra $\mathcal{X}$ is called a nearness d^*-algebra if it satisfies the identity in $N_r(B)^- \mathcal{X}$, $(a \boxdot b) \boxdot a = 0$ for all $a, b \in X$.

Example 3.11. In Example 3.3 , $\mathcal{X}$ is not nearness d^*-algebra, since $(w \boxdot t) \boxdot w = t \neq 0$. By the way in Example 3.4, $\mathcal{X}$ is a nearness d^*-algebra.

Definition 3.12. An upper-near (upper-nearness) $d^{\#}$-ideal $\mathcal{I}$ of a nearness d-algebra $\mathcal{X}$ is called an upper-near (upper-nearness) d^*-ideal if it satisfies the property

$UN\mathcal{I}5)$ $a \boxdot b \in N_r(B)^- \mathcal{I}$ and $b \boxdot a \in N_r(B)^- \mathcal{I}$ imply $(a \boxdot c) \boxdot (b \boxdot c) \in N_r(B)^- \mathcal{I}$ and $(c \boxdot a) \boxdot (c \boxdot b) \in N_r(B)^- \mathcal{I}$ for all $x, y, z \in \mathcal{X}$.

Example 3.13. In Example 3.4, $\mathcal{I} = \{0, x, t, w\}$ is an upper-near-d^*-ideal and it is an upper-nearness d^*-ideal of $\mathcal{X}$.

4. Quotient Nearness d-Algebras

Let $(\mathcal{X}, \boxdot, 0)$ be a nearness d-algebra, and $\mathcal{I}$ be an upper-nearness $d^{\#}$-ideal of $\mathcal{X}$. The relation $\approx_{\mathcal{I}}$ is defined on $\mathcal{X}$ as follows:

$$a \approx_{\mathcal{I}} b \Leftrightarrow a \boxdot b, b \boxdot a \in N_r(B)^- \mathcal{I}, \text{ for all } a, b \in \mathcal{X}.$$

We notice that $\approx_{\mathcal{I}}$ is an equivalence relation on $\mathcal{X}$. Since for all $a \in \mathcal{X}$, $a \boxdot a = 0 \in N_r(B)^- \mathcal{I}$, we get $a \approx_{\mathcal{I}} a$. That is, $\approx_{\mathcal{I}}$ is reflexive. For $a, b \in \mathcal{X}$, if $a \approx_{\mathcal{I}} b$, it is clear that $b \approx_{\mathcal{I}} a$ from definition of $\approx_{\mathcal{I}}$. Thus $\approx_{\mathcal{I}}$ is symmetric. Let us take $a \approx_{\mathcal{I}} b$ and $b \approx_{\mathcal{I}} c$ for $a, b, c \in \mathcal{X}$. Since $a \approx_{\mathcal{I}} b$, we have $a \boxdot b$, $b \boxdot a \in N_r(B)^- \mathcal{I}$ and since $b \approx_{\mathcal{I}} c$, we have $b \boxdot c$, $c \boxdot b \in N_r(B)^- \mathcal{I}$. Since $\mathcal{I}$ is an upper-nearness $d^{\#}$-ideal and $a \boxdot b$, $b \boxdot c \in N_r(B)^- \mathcal{I}$, we get $a \boxdot c \in N_r(B)^- \mathcal{I}$. Similarly, we get $c \boxdot a \in N_r(B)^- \mathcal{I}$. Therefore, $\approx_{\mathcal{I}}$ is transitive.

A class defined by relation $\approx_{\mathcal{I}}$ is called coset. The coset that contains the element a in $\mathcal{X}$ is denoted by $[a]_{\mathcal{I}}$, i.e.,

$$\begin{aligned} [a]_{\mathcal{I}} &= \left\{ b \in N_r(B)^- \mathcal{X} \mid b \approx_{\mathcal{I}} a \right\} \\ &= \left\{ b \in N_r(B)^- \mathcal{X} \mid a \boxdot b, b \boxdot a \in N_r(B)^- \mathcal{I} \right\}. \end{aligned}$$

We denote the set of all equivalence classes of $\mathcal{X}$, $\mathcal{X}/\mathcal{I} = \{[a]_{\mathcal{I}} \mid a \in \mathcal{X}\}$.

Lemma 4.1. *Let $\mathcal{X}$ be a nearness d-algebra and $\mathcal{I}$, $\mathcal{J}$ be upper-nearness $d^{\#}$-ideals of $\mathcal{X}$. If $\mathcal{I} \subseteq \mathcal{J}$, then for all $a \in \mathcal{X}$, $[a]_{\mathcal{I}} \subseteq [a]_{\mathcal{J}}$.*

Proof. Let $b \in [a]_{\mathcal{I}}$. Thus, $b \approx_{\mathcal{I}} a \Rightarrow b \boxdot a$, $a \boxdot b \in N_r(B)^- \mathcal{I}$. Since $\mathcal{I} \subseteq \mathcal{J}$, we have $N_r(B)^- (\mathcal{I}) \subseteq N_r(B)^- \mathcal{J}$. From here, $b \boxdot a$, $a \boxdot b \in N_r(B)^- \mathcal{J}$. By the definition of $\approx_{\mathcal{J}}$, we get $b \approx_{\mathcal{J}} a$. That is, $b \in [a]_{\mathcal{J}}$. □

Lemma 4.2. *Let $\mathcal{X}$ be a nearness d-algebra and $\mathcal{I}$ be an upper-nearness $d^{\#}$-ideal of $\mathcal{X}$, and $\approx_{\mathcal{I}}$ be an equivalence relation on $\mathcal{X}$. Then $[0]_{\mathcal{I}} = N_r(B)^- \mathcal{I}$.*

Proof. Let $a \in N_r(B)^- \mathcal{I}$. Since $\mathcal{I}$ is an upper-nearness $d^{\#}$-ideal of $\mathcal{X}$ and $0 \in N_r(B)^- \mathcal{X}$, then $a \boxdot 0 \in N_r(B)^- \mathcal{I}$. Since $0 \boxdot a = 0 \in N_r(B)^- \mathcal{I}$, we get $a \approx_{\mathcal{I}} 0$. Thus, from $a \in [0]_{\mathcal{I}}$, we get $N_r(B)^- \mathcal{I} \subseteq [0]_{\mathcal{I}}$.

If $a \in [0]_{\mathcal{I}}$, then $a \approx_{\mathcal{I}} 0$. By the definition of $\approx_{\mathcal{I}}$, we have $a \boxdot 0$, $0 \boxdot a \in N_r(B)^- \mathcal{I}$. Since $\mathcal{I}$ is an upper-nearness $d^{\#}$-ideal, we get $a \in N_r(B)^- \mathcal{I}$, and so we obtain that $[0]_{\mathcal{I}} = N_r(B)^- \mathcal{I}$. □

Lemma 4.3. *Let $\mathcal{X}$ be a nearness d-algebra, $\mathcal{I}$ be an upper-nearness d^*-ideal of $\mathcal{X}$. If for all $a, b, c, d \in \mathcal{X}$, $a \approx_{\mathcal{I}} b$ and $c \approx_{\mathcal{I}} d$ imply $a \boxdot c \approx_{\mathcal{I}} b \boxdot d$.*

Proof. Let $a \approx_{\mathcal{I}} b$ and $c \approx_{\mathcal{I}} d$, for all $a, b, c, d \in \mathcal{X}$. Since $a \approx_{\mathcal{I}} b$, $a \boxdot b$, $b \boxdot a \in N_r(B)^- \mathcal{I}$ and since $c \approx_{\mathcal{I}} d$, we have $c \boxdot d$, $d \boxdot c \in N_r(B)^- \mathcal{I}$. Since $\mathcal{I}$ is an upper-nearness d^*-ideal of $\mathcal{X}$, we get

$$(a \boxdot c) \boxdot (a \boxdot d) \in N_r(B)^- \mathcal{I} \text{ and } (a \boxdot d) \boxdot (a \boxdot c) \in N_r(B)^- \mathcal{I}$$

and so $(a \boxdot c) \approx_{\mathcal{I}} (a \boxdot d)$. Since $\mathcal{I}$ is an upper-nearness d^*-ideal of $\mathcal{X}$, we get

$$(a \boxdot d) \boxdot (b \boxdot d) \in N_r(B)^- \mathcal{I} \text{ and } (b \boxdot d) \boxdot (a \boxdot d) \in N_r(B)^- \mathcal{I}.$$

Then, we have $(a \boxdot d) \approx_{\mathcal{I}} (b \boxdot d)$. From transitivity of $\approx_{\mathcal{I}}$, we obtain that $(a \boxdot c) \approx_{\mathcal{I}} (b \boxdot d)$. □

Theorem 4.4. *Let $(\mathcal{X}, *, 0)$ be a d-algebra and $\mathcal{I}$ be an upper-near d^*-ideal of $\mathcal{X}$. If we define $[a]_{\mathcal{I}} \boxtimes [b]_{\mathcal{I}} = [a \boxdot b]_{\mathcal{I}}$, then $(\mathcal{X}/\mathcal{I}, \boxtimes, [0]_{\mathcal{I}})$ is a nearness d-algebra, called quotient nearness d-algebra.*

Proof. Since $\approx_{\mathcal{I}}$ is congruence relation on $\mathcal{X}$, $(a \boxdot c) \approx_{\mathcal{I}} (b \boxdot d)$ for any $a \approx_{\mathcal{I}} b$ and $c \approx_{\mathcal{I}} d$. From here, $[a]_{\mathcal{I}} \boxtimes [b]_{\mathcal{I}} = [a \boxdot b]_{\mathcal{I}}$ is well-defined. Now we will prove that $(\mathcal{X}/\mathcal{I}, \boxtimes, [0]_{\mathcal{I}})$ is nearness d-algebra.
$Nd{-}\mathcal{I}$) For all $[a]_{\mathcal{I}} \in \mathcal{X}/\mathcal{I}$, $[a]_{\mathcal{I}} \boxtimes [a]_{\mathcal{I}} = [a \boxdot a]_{\mathcal{I}} = [0]_{\mathcal{I}}$ holds in $N_r(B)^- (\mathcal{X}/\mathcal{I})$,
$Nd{-}\mathcal{II}$) For all $[a]_{\mathcal{I}} \in \mathcal{X}/\mathcal{I}$, $[0]_{\mathcal{I}} \boxtimes [a]_{\mathcal{I}} = [0 \boxdot a]_{\mathcal{I}} = [0]_{\mathcal{I}}$ holds in $N_r(B)^- (\mathcal{X}/\mathcal{I})$,
$Nd{-}\mathcal{III}$) Let $[a]_{\mathcal{I}} \boxtimes [b]_{\mathcal{I}} = [0]_{\mathcal{I}}$ and $[b]_{\mathcal{I}} \boxtimes [a]_{\mathcal{I}} = [0]_{\mathcal{I}}$ for $[a]_{\mathcal{I}}, [b]_{\mathcal{I}} \in \mathcal{X}/\mathcal{I}$. Thus, we have $[a \boxdot b]_{\mathcal{I}} = [0]_{\mathcal{I}}$ and $[b \boxdot a]_{\mathcal{I}} = [0]_{\mathcal{I}}$. Since $a \boxdot b \approx_{\mathcal{I}} 0$ and $b \boxdot a \approx_{\mathcal{I}} 0$, we get $a \boxdot b \in N_r(B)^- \mathcal{I}$ and $b \boxdot a \in N_r(B)^- \mathcal{I}$. Therefore we get $a \approx_{\mathcal{I}} b$, and so $[a]_{\mathcal{I}} = [b]_{\mathcal{I}}$. □

Example 4.1. We will obtain all cosets of $\mathcal{X}$ by $\mathcal{I}$ in Example 3.4. By using the definition of coset,

$$\begin{aligned}
[0]_{\mathcal{I}} &= \left\{a \in N_r(B)^- \mathcal{X} \mid 0 \boxdot a, a \boxdot 0 \in N_r(B)^- \mathcal{I}\right\} \\
&= \left\{a \in N_r(B)^- \mathcal{X} \mid a \boxdot 0 \in N_r(B)^- \mathcal{I}\right\} \\
&= \{0, x, t, w\} \\
&= [x]_{\mathcal{I}} = [t]_{\mathcal{I}} = [w]_{\mathcal{I}}, \\
[y]_{\mathcal{I}} &= \left\{a \in N_r(B)^- \mathcal{X} \mid y \boxdot a, a \boxdot y \in N_r(B)^- \mathcal{I}\right\} \\
&= \{y\}, \\
[z]_{\mathcal{I}} &= \left\{a \in N_r(B)^- \mathcal{X} \mid z \boxdot a, a \boxdot z \in N_r(B)^- \mathcal{I}\right\} \\
&= \{z\}.
\end{aligned}$$

Thus we have that $\mathcal{X}/\mathcal{I} = \{[0]_{\mathcal{I}}, [y]_{\mathcal{I}}, [z]_{\mathcal{I}}\}$.

Now, we will obtain the upper approximation of $\mathcal{X}/\mathcal{I}$.

$$\begin{aligned}
Q(\mathcal{X}/\mathcal{I}) &= \{\Phi(A) \mid A \in \mathcal{X}/\mathcal{I}\} \\
&= \{\Phi([0]_{\mathcal{I}}), \Phi([y]_{\mathcal{I}}), \Phi([z]_{\mathcal{I}})\} \\
&= \{\{\Phi(0), \Phi(x), \Phi(t), \Phi(w)\}, \{\Phi(y)\}, \{\Phi(z)\}\} \\
&= \{\{(\sigma_1, \sigma_1, \sigma_2), (\sigma_1, \sigma_4, \sigma_1), (\sigma_3, \sigma_3, \sigma_1), (\sigma_5, \sigma_3, \sigma_3)\}, \\
&\quad \{(\sigma_2, \sigma_2, \sigma_4)\}, \{(\sigma_2, \sigma_2, \sigma_5)\}\}.
\end{aligned}$$

For $[0]_{\mathcal{I}} \in \mathcal{X}/\mathcal{I}$,

$$\begin{aligned} Q\left([0]_{\mathcal{I}}\right) &= \{\Phi(0), \Phi(x), \Phi(t), \Phi(w)\} \\ &= \{(\sigma_1, \sigma_1, \sigma_2), (\sigma_1, \sigma_4, \sigma_1), (\sigma_3, \sigma_3, \sigma_1), (\sigma_5, \sigma_3, \sigma_3)\} \end{aligned}$$

and so $\xi_\Phi\left([0]_{\mathcal{I}}\right) = \{[0]_{\mathcal{I}}\}$.

For $[y]_{\mathcal{I}} \in \mathcal{X}/\mathcal{I}$,

$$Q\left([y]_{\mathcal{I}}\right) = \{\Phi(y)\} = \{(\sigma_2, \sigma_2, \sigma_4)\}$$

and so $\xi_\Phi\left([y]_{\mathcal{I}}\right) = \{[y]_{\mathcal{I}}\}$.

For $[z]_{\mathcal{I}} \in \mathcal{X}/\mathcal{I}$,

$$Q\left([z]_{\mathcal{I}}\right) = \{\Phi(z)\} = \{(\sigma_2, \sigma_2, \sigma_5)\}$$

and so $\xi_\Phi\left([z]_{\mathcal{I}}\right) = \{[z]_{\mathcal{I}}\}$. Thus we obtain that

$$N_r(B)^-(\mathcal{X}/\mathcal{I}) = \{[0]_{\mathcal{I}}, [y]_{\mathcal{I}}, [z]_{\mathcal{I}}\}.$$

In this case, $(\mathcal{X}/\mathcal{I}, \boxtimes, [0]_{\mathcal{I}})$ is a nearness quotient d-algebra where

TABLE 9. Cayley table of the operation $\boxtimes$.

$\boxtimes$	$[0]_{\mathcal{I}}$	$[y]_{\mathcal{I}}$	$[z]_{\mathcal{I}}$
$[0]_{\mathcal{I}}$	$[0]_{\mathcal{I}}$	$[0]_{\mathcal{I}}$	$[0]_{\mathcal{I}}$
$[y]_{\mathcal{I}}$	$[y]_{\mathcal{I}}$	$[0]_{\mathcal{I}}$	$[0]_{\mathcal{I}}$
$[z]_{\mathcal{I}}$	$[z]_{\mathcal{I}}$	$[z]_{\mathcal{I}}$	$[0]_{\mathcal{I}}$

Theorem 4.5. *Let $\mathcal{X}$ be a nearness d-algebra, $\mathcal{I}$ be an upper nearness d*-ideal of $\mathcal{X}$ and $\mathcal{X}/\mathcal{I}$ be a set of all cosets of $\mathcal{X}$ by $\mathcal{I}$.Then $N_r(B)^-(\mathcal{X}/\mathcal{I}) \subseteq \left(N_r(B)^- \mathcal{X}\right)/\mathcal{I}$.*

Proof. Let take $[x]_{\mathcal{I}} \in N_r(B)^- \mathcal{I}$. Since $\xi_\Phi\left([x]_{\mathcal{I}}\right) \cap (\mathcal{X}/\mathcal{I}) \neq \emptyset$, we have $[y]_{\mathcal{I}} \in \xi_\Phi\left([x]_{\mathcal{I}}\right) \cap (\mathcal{X}/\mathcal{I})$. From here, $[y]_{\mathcal{I}} \in \xi_\Phi\left([x]_{\mathcal{I}}\right)$, $[y]_{\mathcal{I}} \in \mathcal{X}/\mathcal{I}$, and so we get $Q\left([y]_{\mathcal{I}}\right) \cap Q\left([x]_{\mathcal{I}}\right) \neq \emptyset$. That is, for $i_0 \in L$, $\psi_{i_0}(x) = \psi_{i_0}(y)$ (see [26]). Thus, $x \sim_{B_r} y$. Therefore, $y \in [x]_{\psi_{i_0}} \cap \mathcal{X}$, then $[x]_{\psi_{i_0}} \cap \mathcal{X} \neq \emptyset$. Consequently, since $x \in N_r(B)^- \mathcal{X}$, we obtain $[x]_{\mathcal{I}} \in N_r(B)^- \mathcal{X}/\mathcal{I}$. □

Theorem 4.6. *Let $\mathcal{X}$ be a nearness d-algebra, $\mathcal{I}$ and $\mathcal{J}$ be upper nearness d*-ideals of $\mathcal{X}$. If $\mathcal{I} \subseteq \mathcal{J}$ and $N_r(B)^-(\mathcal{J}/\mathcal{I}) = \left(N_r(B)^- \mathcal{J}\right)/\mathcal{I}$, then $\mathcal{J}/\mathcal{I}$ is an upper-nearness d*-ideal of $\mathcal{X}/\mathcal{I}$.*

Proof. Let $\mathcal{I} \subseteq \mathcal{J}$. Since $\mathcal{I}$ is an upper nearness d^*-ideal of $\mathcal{X}$, $\mathcal{J}/\mathcal{I}$ is a nearness d-algebra and $\mathcal{J}/\mathcal{I} \subseteq \mathcal{X}/\mathcal{I}$. Since $0 \in N_r(B)^- \mathcal{J}$, we have $[0]_{\mathcal{I}} \in \left(N_r(B)^- \mathcal{J}\right)/\mathcal{I}$, and so $[0]_{\mathcal{I}} \in N_r(B)^-(\mathcal{J}/\mathcal{I})$. Let $[a]_{\mathcal{I}} \boxtimes [b]_{\mathcal{I}} \in N_r(B)^-(\mathcal{J}/\mathcal{I})$ and $[b]_{\mathcal{I}} \in N_r(B)^-(\mathcal{J}/\mathcal{I})$. Since $[a \boxdot b]_{\mathcal{I}} \in \left(N_r(B)^- \mathcal{J}\right)/\mathcal{I}$ and $[b]_{\mathcal{I}} \in \left(N_r(B)^- \mathcal{J}\right)/\mathcal{I}$, we get $a \boxdot b$, $b \in N_r(B)^- \mathcal{J}$. From here, $a \in N_r(B)^- \mathcal{J}$. Then we get $[a]_{\mathcal{I}} \in \left(N_r(B)^- \mathcal{J}\right)/\mathcal{I} = N_r(B)^-(\mathcal{J}/\mathcal{I})$.

Let $[a]_{\mathcal{I}} \in N_r(B)^-(\mathcal{J}/\mathcal{I})$ and $[b]_{\mathcal{I}} \in N_r(B)^-(\mathcal{X}/\mathcal{I})$. Then, $a \in N_r(B)^- \mathcal{J}$ and $b \in N_r(B)^- \mathcal{X}$. $a \boxdot b \in N_r(B)^- \mathcal{J}$, and so $[a \boxdot b]_{\mathcal{I}} \in \left(N_r(B)^- \mathcal{J}\right)/\mathcal{I}$. Hence we get $[a]_{\mathcal{I}} \boxtimes [b]_{\mathcal{I}} \in N_r(B)^-(\mathcal{J}/\mathcal{I})$. Thus $\mathcal{J}/\mathcal{I}$ is an upper nearness d^*-ideal of $\mathcal{X}/\mathcal{I}$. □

Definition 4.2. Let $(\mathcal{X}, \boxdot, 0)$ and $(\mathcal{Y}, \circledcirc, 0)$ be two nearness d-algebras and $f : N_r(B)^-(\mathcal{X}) \rightarrow N_r(B)^-(\mathcal{Y})$ be a function. If the property $f(a \boxdot b) = f(a) \circledcirc f(b)$ for all $a, b \in \mathcal{X}$, then f is called a homomorphism on $\mathcal{O}$.

Theorem 4.7. *Let $(\mathcal{X}, \boxdot, 0)$ be a nearness d-algebra and $\mathcal{I}$ be an upper nearness d^*-ideal of $\mathcal{X}$. If we define $\psi : N_r(B)^-(\mathcal{X}, \boxdot, 0) \rightarrow N_r(B)^-(\mathcal{X}/\mathcal{I}, \boxtimes, [0]_{\mathcal{I}})$ by $\psi(a) = [a]_{\mathcal{I}}$ for all $a \in \mathcal{X}$, then ψ is homomorphism, called canonical homomorphism. Moreover, $Ker\psi = \mathcal{X} \cap N_r(B)^- \mathcal{I}$.*

Proof. Let $a, b \in \mathcal{X}$. Then, we get $\psi(a \boxdot b) = [a \boxdot b]_{\mathcal{I}} = [a]_{\mathcal{I}} \boxtimes [b]_{\mathcal{I}} = \psi(a) \boxtimes \psi(b)$. Thus ψ is a homomorphism.

$$\begin{aligned} Ker\psi &= \{a \in \mathcal{X} \mid \psi(a) = [0]_{\mathcal{I}}\} \\ &= \{a \in \mathcal{X} \mid [a]_{\mathcal{I}} = [0]_{\mathcal{I}}\} \\ &= \{a \in \mathcal{X} \mid a \approx_{\mathcal{I}} 0\} \\ &= \mathcal{X} \cap N_r(B)^- \mathcal{I}. \end{aligned}$$

□

5. CONCLUSIONS AND DISCUSSIONS

Two important classes of abstract algebras are BCK-algebras and BCI-algebras. These algebras are linked to fields such as lattice ordered groups, MV-algebras, Wajsberg algebras, and implicative abelian semigroups. There are many generalizations of BCK-algebras, and one of them is d-algebras. Near set theory is a generalization of rough set theory. This theory begins with the selection of probe functions in order to distinguish and define affinities between perceptual objects. Based on some motivational requirements mentioned above, in this study, we applied the concept of nearness, which has a different approach for algebraic structures, to the quotient structure of d-algebras and we obtained some results. We study quotient nearness d-algebras by defining the nearness d-ideal types of d-algebras. This article provides very useful results for the nearness theory of d-algebras. In future studies, the nearness quotient structure examined here can also be examined for other algebra types such as nearness BCC-algebra, nearness BCH-algebra, nearness subtraction algebra etc. Also, as a continuation of this work, isomorphism theorems for nearness d-algebras can be studied.

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