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NEUTROSOPHIC WEAKLY REGULAR SEMI CONTINUOUS FUNCTIONS

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ABSTRACT. In this paper, we introduce the concept of neutrosophic weakly regular semi continuous, neutrosophic regular semi q-neighbourhood in neutrosophic topological spaces. Also, we investigate the relationship among neutrosophic weakly regular semi continuous and other existing continuous functions. Moreover, some counter examples to show that these types of mappings are not equivalent. Finally, Neutrosophic retracts, neutrosophic regular semi retracts, neutrosophic regular semi quasi Urysohn space and neutrosophic regular semi Hausdorff spaces are introduced and studied.

1. INTRODUCTION

The study of fuzzy sets was initiated by Zadeh [18] in 1965. Thereafter the paper of Chang [3] paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Currently fuzzy topology has been observed to be very beneficial in fixing many realistic problems. Several mathematicians have tried almost all the pivotal concepts of General Topology for extension to the fuzzy settings. In 1983, Atanassov [1] introduced the concept of intuitionistic fuzzy set which was generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. Later, Coker [5] introduced the concept of intuitionistic topological spaces, by using the notion of the intuitionitic fuzzy set. Smarandache [8, 9, 10] introduced the concept of neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminacy and non membership function that are independently related. In 2012, Salama and Alblowi [12, 13, 14] introduced the concept of neutrosophic topology. Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology. In 2014, Salama et. al., [13] introduced the concept of neutrosophic closed sets and neutrosophic continuous functions.

In general topology, the concept of regular semiopen set was introduced by Cameron [2] in 1978. In 2021, Praveena and Vijayalakshmi [15, 16] introduced the concept of neutrosophic regular semiopen and neutrosophic regular semiclosed sets, neutrosophic regular

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semi continuous, neutrosophic regular semi open , neutrosophic regular semi closed, neutrosophic regular semi irresolute mapping in neutrosophic topological spaces. Recently, Elavarasan et. al., [6, 7] introduced neutrosophic regular semi Baire spaces and somewhat neutrosophic regular semi continuous, neutrosophic regular semi resolvable and irresolvable spaces. In this paper, we introduce the concept of neutrosophic weakly regular semi continuous, neutrosophic regular semi q-neighbourhood in neutrosophic topological spaces. Also, we investigate the relationship among neutrosophic weakly regular semi continuous and other existing continuous functions. Moreover, some counter examples to show that these types of mappings are not equivalent. Finally, Neutrosophic retracts, neutrosophic regular semi retracts, neutrosophic regular semi quasi Urysohn space and neutrosophic regular semi Hausdorff spaces are introduced and studied.

2. PRELIMINARIES

In this section, we recollect some relevant basic preliminaries about Neutrosophic sets and its operations.

Definition 2.1. [12] Let X be a non-empty fixed set. A Neutrosophic set [for short, Ns] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminancy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark. [12] *A Ns* $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ *can be identified to an ordered triple* $A = \langle \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ *in* $]^-0, 1^+[$ *on* X.

Remark. [12] *For the sake of simplicity, we shall use the symbol* $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ *for the Ns* $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Example 2.2. [12] Every intuitionsistic neutrosophicset A is a non-empty set in X is obviously on Ns having the form $A = \{\langle x, \mu_A(x), 1 - \mu_A(x) + \gamma_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic sets 0_N and 1_N in X as follows:
$0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 2.3. [12] Let $A = \langle(\mu_A, \sigma_A, \gamma_A)\rangle$ be a Ns on X, then the complement of the set A(A^c or $C(A)$ for short) may be defined as $C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$.

Definition 2.4. [12] Let X be a non-empty set and Ns's A and B in the form $A = \{\langle x, \mu_A, \sigma_A, \gamma_A\rangle : x \in X\}$ and $B = \{\langle x, \mu_B, \sigma_B, \gamma_B\rangle : x \in X\}$. Then $(A \subseteq B)$ may defined as: $(A \subseteq B) \Leftrightarrow \mu_A(x) \subseteq \mu_B(x), \sigma_A(x) \subseteq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x) \forall x \in X$.

Definition 2.5. [12] Let X be a non-empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ are Ns's. Then $A \cap B$ and $A \cup B$ may defined as:

(I_1) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle$
(U_1) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle$

Definition 2.6. [12] A Neutrosophic topology (for short, NT or nt) is a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:

(i) $0_N, 1_N \in \tau_N$,
(ii) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
(iii) $\cup G_i \in \tau_N$ for every $\{G_i : i \in J\} \subseteq \tau_N$.

Throughout this paper, the pair of (X, τ_N) is called a neutrosophic topological space (for short, nts). The elements of τ_N or τ are called neutrosophic open set (for short, nos). A neutrosophic set F is neutrosophic closed if and only if F^c is nos.

Definition 2.7. [12] Let (X, τ_N) be nts and $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ be a Ns in X. Then the neutrosophic closure and neutrosophic interior of A are defined by $NCl(A) = \cap\{K : K$ is a NCS in X and $A \subseteq K\}$, $NInt(A) = \{G : G$ is a NOS in X and $G \subseteq A\}$. It can be also shown that $NCl(A)$ is NCS and $NInt(A)$ is a NOS in X. A is NOS if and only if $A = NInt(A)$, A is NCS if and only if $A = NCl(A)$.

Definition 2.8. Let $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ be a Ns on a nts (X, τ_N) then A is called:

(i) neutrosophic regular open (for short, nro) [17] if $A = NInt(NCl(A))$.
(ii) neutrosophic regular closed (for short, nrc) [17] if $A = NCl(NInt(A))$.
(iii) neutrosophic semi-open (for short, nso) [17] if $A \subseteq NInt(NCl(A))$.
(iv) neutrosophic regular semiopen (for short, nrso) [15] if there exists an nro set B in X such that $B \subseteq A \subseteq NCl(B)$.
(v) neutrosophic regular semiclosed (for short, nrsc) [15] if there exists an nrc set B in X and $NInt(B) \subseteq A \subseteq B$.

We shall denote the family of all nrso sets (nrsc sets) of a nts (X, τ) by $NRSOS(X)$, $NRSCS(X)$.

Definition 2.9. [15] Let $(X,\ \tau)$ be a nts. Then

(1) the neutrosophic regular closure of A, denoted by $nrcl(A)$ or $NRCl(A)$, and is defined by $nrcl(A) = \bigcap\{B|B \supseteq A, B$ is nrc $\}$.
(2) the neutrosophic regular interior of A, denoted by $nrint(A)$ or $NRInt(A)$, and is defined by $nrint(A) = \bigcup\{B|B \subseteq A, B$ is nro $\}$.
(3) the neutrosophic regular semiclosure of A defined by $NRSCl(A)$ or $nrscl(A) = \bigcap\{B \mid A \subseteq B$ and $B \in NRSCS(X, \tau)\}$ is a neutrosophic set.
(4) the neutrosophic regular semiinterior of A defined by $NRSInt(A)$ or $nrsint(A) = \bigcup\{B \mid B \subseteq A$ and $B \in NRSOS(X, \tau)\}$ is a neutrosophic set.

Definition 2.10. [4] Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$, then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point (briefly NP) in X given by

$$x_{r,t,st}(x_p) = \begin{cases} (r,t,s), & \text{if } x = x_p \\ (0,0,1) & \text{if } x \neq x_p. \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t the degree of indeterminacy and s the degree of non-membership value of $x_{r,t,s}$.

Definition 2.11. [11] Let A be a Ns over X, let $x_{\alpha,\beta,\gamma}$ be NP in X.

(i) $x_{\alpha,\beta,\gamma}$ is contained in A if $\alpha \leq T_A(x)$, $\beta \geq I_A(x)$, $\gamma \geq F_A(x)$.
(ii) $x_{\alpha,\beta,\gamma}$ is belong to A if $\alpha \leq T_A(x)$, $\beta \geq I_A(x)$, $\gamma \geq F_A(x)$.

Definition 2.12. [11] A NP $x_{\alpha,\beta,\gamma} \in N(X)$ is said to be quasi-coincident with a NS $A \in N(X)$ or $x_{\alpha,\beta,\gamma} \in N(X)$ quasi-coincides with a NS $A \in N(X)$, denoted by $x_{\alpha,\beta,\gamma} qA$, iff $\alpha > T_{A^c}(x)$ or $\beta < I_{A^c}(x)$ or $\gamma < F_{A^c}(x)$, i.e., $\alpha > F_A(x)$ or $\beta < 1 - I_A(x)$ or $\gamma < T_A(x)$. A NS A is said to be quasi-coincident with a NS B at $x \in X$ or A quasi-coincides with B, denoted by AqB at x iff $T_A(x) > T_{B^c}(x)$ or $I_A(x) < I_{B^c}(x)$ or $F_A(x) < F_{B^c}(x)$.

Definition 2.13. [11] A neutrosophic set A is called an neutrosophic quasi-neighbourhood (for short, N-q-nbd) of a neutrosophic point $x_{\alpha,\beta,\gamma}$ iff there exists a NS $B \in \tau$ such that $x_{r,t,s} q B \subseteq A$.

Theorem 2.1. *For any Ns A in an nts (X, τ), $NRInt(A) \subseteq NRSInt(A) \subseteq A \subseteq NRSCl(A) \subseteq NCl(A)$.*

Theorem 2.2. [14] *Let $f : X \to Y$ a function. Then*

(i) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$,
(ii) $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,
(iii) $A \subseteq f^{-1}(f(A))$ *and if f is injective, then* $A = f^{-1}(f(A))$,
(iv) $f^{-1}(f(B)) \subseteq B$ *and if f is surjective, then* $f^{-1}(f(B)) = B$,
(v) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$,
(vi) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$ *and if f is injective, then* $f(\cap A_i) = \cap f(A_i)$,

Definition 2.14. [14] Let (X, τ) and (Y, σ) be any two nts's. A map $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic continuous if the inverse image of every neutrosophic closed set in (Y, σ) is neutrosophic closed set in (X, τ).

Definition 2.15. [16] Let (X, τ) and (Y, σ) be any two nts's. A map $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic regular continuous (for short, NRC) if the inverse image of every neutrosophic closed set in (Y, σ) is neutrosophic regular semi closed set in (X, τ).

Definition 2.16. [16] Let (X, τ) and (Y, σ) be any two nts's. A map $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic regular open (for short, NRO) if the image of every neutrosophic open set in (X, τ) is neutrosophic regular open set in (Y, σ).

Definition 2.17. [16] Let (X, τ) and (Y, σ) be any two nts's. A map $f : (X, \tau) \to (Y, \sigma)$ is neutrosophic regular semi continuous (for short, $NRSC$) (neutrosophic regular semi irresolute (for short, $NRSI$)) if the inverse image of every neutrosophic closed (neutrosophic regular semi closed) set in (Y, σ) is neutrosophic regular semi closed (neutrosophic regular semi closed) set in (X, τ).

Equivalently, A mapping $f : X \to Y$ is $NRSC$ iff for any neutrosophic point $x_{r,t,s}$ in X and any nos B in Y with $f(x_{r,t,s}) \in B$, there exists $A \in NRSO(X)$ such that $x_{r,t,s} \in A$ and $f(A) \subseteq B$.

Theorem 2.3. [16] *If $f : X \to Y$ is $NRSC$ and NAO, then f is $NRSI$.*

3. Neutrosophic weakly regular semi continuous functions

Definition 3.1. A mapping $f : X \to Y$ is called the :

(i) neutrosophic weakly regular continuous (for short, $NWRC$) iff for any NP $x_{r,t,s}$ in X and any nos B in Y containing $f(x_{r,t,s})$, there exists an nro set A containing $x_{r,t,s}$ such that $f(A) \subseteq NRCl(B)$.
(ii) neutrosophic weakly regular semi continuous (for short, $NWRSC$) iff for any NP $x_{r,t,s}$ in X and any nos B in Y containing $f(x_{r,t,s})$, there exists an nrso set A containing $x_{r,t,s}$ such that $f(A) \subseteq NRSCl(B)$.
(iii) neutrosophic weakly semi continuous (for short, $NWSC$) iff for any NP $x_{r,t,s}$ in X and any nos B in Y containing $f(x_{r,t,s})$, there exists an nso set A containing $x_{r,t,s}$ such that $f(A) \subseteq NSCl(B)$.

Example 3.2. Let $X = \{a, b\}$, $\tau = \{0_N, 1_N, A, B\}$, $Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, C\}$, where A and B are Ns's of X and C is Ns of Y, defined as follows:
$A = \left\langle (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.5}) \right\rangle,$
$B = \left\langle (\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.4}, \frac{\gamma_b}{0.5}) \right\rangle,$
$C = \left\langle (\frac{\mu_p}{0.5}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}) \right\rangle.$
Clearly τ and σ are NT on X and Y. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then f is $NWRSC$ but not $NWRC$, for any NP $x_{0.5,0.6,0.6}$ in X and a nos C of Y containing $f(x_{0.5,0.6,0.6})$, there exists a nrso $D = \left\langle (\frac{\mu_p}{0.5}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}) \right\rangle$ [D is nrso set of X, since $\exists$ a nro set B such that $B \subseteq C \subseteq NCl(B)$] containing $x_{0.5,0.6,0.6}$ such that $f(D) \subseteq NRSCl(C)$. But D is not nro set.

Example 3.3. Let $X = \{a, b\}$ and $\tau = \{0_N, 1_N, X, A, B\}$, $Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, C\}$, where A and B are Ns of X and C is Ns of Y, defined as follows:
$A = \left\langle (\frac{\mu_a}{0.3}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.5}) \right\rangle,$
$B = \left\langle (\frac{\mu_a}{0.6}, \frac{\mu_b}{0.5}), (\frac{\sigma_a}{0.5}, \frac{\sigma_a}{0.5}), (\frac{\gamma_a}{0.5}, \frac{\gamma_b}{0.5}) \right\rangle,$
$C = \left\langle (\frac{\mu_p}{0.4}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}) \right\rangle.$
Clearly τ and σ are NT on X and Y. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then f is $NWSC$ but not $NWRSC$, for any NP $x_{0.4,0.5,0.6}$ in X and a nos C of Y containing $f(x_{0.5,0.6,0.6})$, there exists a nrso $D = \left\langle (\frac{\mu_p}{0.4}, \frac{\mu_q}{0.5}), (\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}), (\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}) \right\rangle$ [D is nso set of X, since $\exists$ a nos B such that $B \subseteq C \subseteq NCl(B)$] containing $x_{0.4,0.5,0.6}$ such that $f(D) \subseteq NSCl(C)$. But D is not nrso.

Remark. *The above Definition and Examples 3.2 and 3.3, it clear that*

(i) Every $NWRC$ functions is $NWRSC$ but not conversely.
(ii) Every $NWRSC$ functions is $NWSC$ but not conversely.

Definition 3.4. An Ns A is called an neutrosophic q-nbd (neutrosophic regular-q-nbd (for short, NR-q-nbd), neutrosophic regular semi-q-nbd (for short, NRS-q-nbd)) of an NP $x_{r,t,s}$ in an nts (X, τ) iff there exists an nos (nro, nrso) set B of X such that $x_{r,t,s} qB \subseteq A$.

Definition 3.5. An nts (X, τ) is NR-regular iff for each NP $x_{r,t,s}$ in X and each N-open-q-nbd A of $x_{r,t,s}$, there exists NR-open-q-nbd B of $x_{r,t,s}$ such that $NRCl(B) \subseteq A$.

Theorem 3.1. *If Y is an NR-regular space, then a mapping $f : X \to Y$ is $NWRSC$ iff f is $NRSC$.*

Proof. The necessary part follows from Remark 3. We prove only the sufficient part. Let f be $NWRSC$ and Y be an NR-regular space. Let $x_{r,t,s}$ be any NP of X and B be any nro (it is nos) set in Y containing $f(x_{r,t,s})$. Since Y is NR-regular, there exists an NR-q-nbd C of $f(x_{r,t,s}) = Y_{r,t,s}$ (where $y = f(x)$) such that $NRSCl(C) \subseteq B$. Since f is $NWRSC$ and C is an NR-q-nbd of $f(x_{r,t,s})$, there exists $A \in NRSOS(X)$ with $x_{r,t,s} \in A$ such that $f(A) \subseteq NRSCl(C)$. By Theorem 2.3, $NRSCl(C) \subseteq NRSCl(C)$ and so $f(A) \subseteq NRSCl(C) \subseteq NCl(C) \subseteq B$. Thus f is $NRSC$ by Definition 3.1 and this completes the proof. □

In the following theorems we give some characterization of $NWRSC$ functions.

Theorem 3.2. *A mapping $f : X \to Y$ is $NWRSC$ iff for each nos B in Y, $f^{-1}(B) \subseteq NRSInt(f^{-1}(NRSCl(B)))$.*

Proof. Let f be $NWRSC$ and B be any nos set in Y. Let $x_{r,t,s}$ be an NP in $f^{-1}(B)$. Thus $f(x_{r,t,s}) \in B$, f is $NWRSC$ implies that there exists an $A \in NRSOS(X)$ such that $x_{r,t,s} \in A$ and $f(A) \subseteq NRSCl(B)$. By Theorem 2.2(2) and (3) we have $A \subseteq$

$f^{-1}(NRSCl(B))$. Hence $NRSInt(A) \subseteq NRSInt(f^{-1}(NRSCl(B)))$ and since A is nrso, $A \subseteq NRSInt(f^{-1}(NRSCl(B)))$. So $f^{-1}(B) \subseteq A \subseteq NRSInt(f^{-1}(NRSCl(B)))$.

Conversely let $x_{r,t,s}$ be an NP in X and B be any nos set in Y such that $f(x_{r,t,s}) \in B$. By hypothesis, $f^{-1}(B) \subseteq NRSInt(f^{-1}(NRSCl(B)) = A$(say). Hence $x_{r,t,s} \in f^{-1}(B) \subseteq A$, which implies that A is an nrso set in X containing $x_{r,t,s}$. So $A = NRSInt(f^{-1}(NRSCl(B))) \subseteq f^{-1}(NRSCl(B)$, i.e., $f(A) \subseteq NRSCl(B)$ (by Theorem 2.2(4)). Hence f is $NWRSC$ and this proves the result. □

Theorem 3.3. *A mapping $f : X \to Y$ is $NWRSC$ if for each nos B in Y, $f^{-1}(NRSCl(B)) \in NRSOS(X)$.*

Proof. Straightforward. □

Theorem 3.4. *If $f : X \to Y$ is $NRSI$ and $g : Y \to Z$ is $NWRSC$, then $g \circ f : X \to Z$ is $NWRSC$.*

Proof. Let $x_{r,t,s}$ be an NP in X and C be any nos in Z containing $((g \circ f)(x_{r,t,s})) = g(f(x_{r,t,s}))$. Since g is $NWRSC$ there exists an nro B in Y containing $f(x_{r,t,s})$ such that $g(B) \subseteq NRSCl(C)$. Also since f is $NRSI$ and B [every nro set is nos and every nro set is nrso] is nos in Y, it follows that $f^{-1}(B)$ is nrso in X. Let $A = f^{-1}(B)$. Now $(g \circ f)(A) = g(f(A)) \subseteq g(B) \subseteq NRSCl(C)$. So $g \circ f$ is $NWRSC$ and this completes the proof. □

Corollary 3.5. *If $f : X \to Y$ is $NRSC$ and NRO and $g : Y \to Z$ is $NWRSC$, then $g \circ f$ is $NWRSC$.*

Proof. The proof follows from Theorems 2.3 and 3.4. □

4. Neutrosophic weakly regular semi continuous in terms of q-coincidence, q-neighborhoods and θ-cluster points

Definition 4.1. A Ns A in an nts (X, τ) is said to be an neutrosophic-θ-nbd (neutrosophic regular-θ-nbd (for short, NR-θ-nbd), neutrosophic regular semi-θ-nbd (for short, NRS-θ-nbd)) of an NP $x_{r,t,s}$ iff there exists an N-closed-q-nbd (NR-closed-q-nbd, NRS-closed-q-nbd) B of $x_{r,t,s}$ such that $B\overline{q}A^c$, i.e., $B \subseteq A$.

Definition 4.2. A NP $x_{r,t,s}$ in a nts (X, τ) is called an neutrosophic regular semi cluster point (for short, NRS-cluster point) of an Ns A iff every NRS-q-nbd of $x_{r,t,s}$ is q-coincident with A. The set of all NRS-cluster points of an Ns A is called neutrosophic regular semi closure of A.

Definition 4.3. A NP $x_{r,t,s}$ in a nts (X, τ) is called an neutrosophic regular semi θ-cluster point (for short, $NRS\theta$-cluster point) of an Ns A iff every NRS-open-q-nbd B of $x_{r,t,s}$, $NRSCl(B)$ is q-coincident with A. The set of all $NRS\theta$-cluster points of an Ns A is called neutrosophic regular semi θ closure of A and it is denoted by $NRS\theta Cl(A)$.

Theorem 4.1. *A mapping $f : X \to Y$ is $NWRSC$ iff corresponding to each NR-open-q-nbd B of $y_{r,t,s}$ in Y, there exists an NRS-open-q-nbd A of $x_{r,t,s}$ in X such that $f(A) \subseteq NRSCl(B)$, where $f(x_{r,t,s}) = (f(x))_{r,t,s} = y_{r,t,s}$.*

Proof. Let f be $NWRSC$ and B be an NR-open-q-nbd of $y_{r,t,s}$, where $f(x) = y$ in Y. So, $B(y) + (r, t, s) > 1_N$. We can choose a positive real number (u, v, w) such that $B(y) > (u, v, w) > 1 - (r, t, s)$. Hence B is an NR-open-nbd of $y_{u,v,w}$ in Y. Since f is $NWRSC$, there exists an nrso set A containing $x_{u,v,w}$ such that $f(A) \subseteq NRSCl(B)$.

Now $A(x) \geq (u, v, w)$ implies $A(x) > 1 - (r, t, s)$, i.e., $A(x) + (r, t, s) > 1_N$. Thus $x_{r,t,s}qA$. So A is an NRS-open-q-nbd of $x_{r,t,s}$.

Conversely, let the condition of the theorem hold, i.e., let $x_{r,t,s}$ be an NP in X and B be an nos set in Y containing $y_{r,t,s} = (f(x))_{r,t,s}$. So $x_{r,t,s} \in f^{-1}(B) = C$ (say). Hence $C(x) \geq (r, t, s)$. We can choose a (u, v, w) such that $C(x) \geq 1/(u, v, w)$. Put $(r, t, s)_n = 1 + (1/n) - C(x)$, for any positive integer $n \geq (u, v, w)$. Clearly $0 < (r, t, s)_n \subseteq 1$ for all $n \geq (u, v, w)$. Now $B(y) + (r, t, s)_n = B(y) + 1 + (1/n) - C(x) = 1 + (1/n) > 1$ (Since $C(x) = f^{-1}(B)(x) = B(f(x)) = B(y)$). Hence $y_{r,t,s_n}qB$, i.e., B is an NR-open-q-nbd of y_{r,t,s_n} for all $n \geq (u, v, w)$. So by hypothesis there exists an NRS-open-q-nbd A_n of x_{r,t,s_n} such that $f(A_n) \subseteq NRSCl(B)$, for all $n \geq (u, v, w)$. Now $A = \cup_{n \geq (u,v,w)} A_n$ is nrso in X. It remains to show that $x_{r,t,s} \in A$. We have $A_n(X) > 1 - (r, t, s)_n = C(x) - (1/n)$ for all $n \geq (u, v, w)$. Thus $A(x) > C(x) - (1/n)$ for all $n \geq (u, v, w)$. Since $x_{r,t,s} \in C$, $A(x) \geq C(x) \geq (r, t, s)$. So A is an nrso set in X such that $f(A) = f(\cup_{n \geq (u,v,w)} A_n) = \cup_{n \geq (u,v,w)} f(A_n) \subseteq NRSCl(B)$. Hence f is $NWRSC$ and this completes the proof. □

Lemma 4.2. *For any two Ns's A and B in X, $A \subseteq B$ iff for each NP $x_{r,t,s}$ in X, $x_{r,t,s} \in A$ then $x_{r,t,s} \in B$.*

Lemma 4.3. *Let $f : X \to Y$ be any neutrosophic function and $x_{r,t,s}$ be any NP in X, then*

(i) for $A \subseteq X$ and $x_{r,t,s}qA$, we have $f(x_{r,t,s})qf(A)$.
(ii) for $B \subseteq Y$ and $f(x_{r,t,s})qB$, we have $x_{r,t,s}qf^{-1}(B)$.

Theorem 4.4. *If $f : X \to Y$ is an $NWRSC$, then for each nos B in Y, $NRSCl(f^{-1}(B) \subseteq f^{-1}(NRSCl(B)$.*

Proof. A Ns is the union of all of its NP. Suppose that there is an NP $x_{r,t,s} \in NRSCl(f^{-1}(B))$ but $x_{r,t,s} \notin f^{-1}(NRSCl(B))$. Since $f(x_{r,t,s}) \notin NRCl(B)$ there exists an nro set C in Y with $f(x_{r,t,s}) \in C$ such that $C\overline{q}NRCl(B)$. Thus $C\overline{q}B$ and $NRCl(C)\overline{q}B$. Since f is $NWRSC$, there exists an $A \in FRSOS(X)$ with $x_{r,t,s} \in A$ such that $f(A) \subseteq NRSCl(C)$. Hence $f(A)\overline{q}B$. Since $f(A) \subseteq NRSCl(C) \subseteq NRCl(C)$. But on the other hand, since each nrso set is NRS-q-nbd of each of its NP $x_{r,t,s} \in NRSCl(f^{-1}(B))$ and A is an NRS-q-nbd of $x_{r,t,s}$. By Definition 4.2, $Aqf^{-1}(B)$. By Lemma 4.3(1), $f(A)qf(f^{-1}(B))$, and hence by theorem 2.2(iv), $f(A)qB$ which is a contradiction. This completes the proof of the theorem. □

Theorem 4.5. Let $f : X \to Y$ be an NRO and $NWRSC$ mapping. Then $f(NRSCl(A) \subseteq NRCl(f(A))$, for each nro set A in X.

Proof. Let A be an nro (it is nos) set in X and let $f(A) = B$. Since f is NRO, we see that B is an nro (it is also nos) set in Y. Hence by Theorem 2.2(iii), $A \subseteq f^{-1}(f(A)) = f^{-1}(B)$. Since f is $NWRSC$, we have from Theorem 4.4, $NRSCl(f^{-1}(B)) \subseteq f^{-1}(NRCl(B)$. Thus $NRSCl(A) \subseteq f^{-1}(NRCl(B)$, i.e., $f(NRSCl(A)) \subseteq NRCl(B) = NRCl(f(A))$ and this proves the result. □

Theorem 4.6. *A function $f : X \to Y$ is $NWRSC$ iff for each nos B in Y, $(x_{r,t,s})qf^{-1}(B)$ implies $(x_{r,t,s})qf^{-1}(NRSInt(NRSCl(B))$ for each NP $x_{r,t,s}$ in X.*

Proof. Let f is $NWRSC$. Let $x_{r,t,s}$ be NP in X and B be any nro (nos) set in Y such that $(x_{r,t,s})qf^{-1}(B)$. Then $f(x_{r,t,s})qB$. Since f is $NWRSC$ by Theorem 4.1, there exists an nrso set A in X such that $(x_{r,t,s})qA$ and $f(A) \subseteq NRSCl(B)$. By

Theorem 2.2(ii) and (iii) we have $A \subseteq f^{-1}(NRSCl(B))$. Since $A \in NRSOS(X)$, $A \subseteq f^{-1}(NRSInt(NRSC(B))$. So $(x_{r,t,s})qf^{-1}(NRSInt(NRSCl(B))$.

Converesely, let the condition given in the statement hold. Let $x_{r,t,s}$ be an neutrosophic point in X and B be an NR-open-q-nbd of $f(x_{r,t,s})$ such that $(x_{r,t,s})qf^{-1}(B)$. By hypothesis, $(x_{r,t,s})qf^{-1}(NRSInt(NRSCl(B))$. Put $A = f^{-1}(NRSInt(NRSCl(B))$. Hence $A \in NRSOS(X)$. $(x_{r,t,s})qA$ implies that A is an NRS-open-q-nbd of $x_{r,t,s}$. Also $f(f^{-1}(NRSInt(NRSCl(B)))) \subseteq f(f^{-1}(NRSCl(B)))$, i.e., $f(A) \subseteq NRSCl(B)$. Thus f is $NWRSC$ and this completes the proof. □

Theorem 4.7. *If $f : X \to Y$ is $NWRSC$ then for each NP $x_{r,t,s}$ in X and each NR-θ-nbd B of $f(x_{r,t,s})$, $f^{-1}(B)$ is an NRS-q-nbd of $x_{r,t,s}$.*

Proof. Let $f : X \to Y$ is $NWRSC$ function and $x_{r,t,s}$ be an NP in X. Let $NRSCl(B)$ be an NR-θ-nbd of $f(x_{r,t,s})$. So there is an NR-open-q-nbd C of $f(x_{r,t,s})$ such that $NRCl(C)\overline{q}B^c$, i.e., $NRCl(C) \subseteq B$. Since C is an NR-open-q-nbd of $f(x_{r,t,s})$, by Theorem 4.1, there is an NRS-open-q-nbd A of $x_{r,t,s}$ such that $f(A) \subseteq NRSCl(C)$ and thus $f(A) \subseteq NRSCl(C) \subseteq NRCl(C) \subseteq B$. So $A \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is an NRS-q-nbd of $x_{r,t,s}$ and this completes the proof. □

Theorem 4.8. *If $f : X \to Y$ is a function such that for each NP $x_{r,t,s}$ in X and each NRS-θ-nbd B of $f(x_{r,t,s})$ in Y, $f^{-1}(B)$ is NRS-q-nbd of $x_{r,t,s}$ in X, then f is $NWRSC$.*

Proof. Let $x_{r,t,s}$ be an NP in X and B be any NR-open-q-nbd of $f(x_{r,t,s})$. We note that B is an NRS-open-q-nbd of $f(x_{r,t,s})$. So $NRSCl(B)$ is an NRS-θ-nbd of $f(x_{r,t,s})$. By hypothesis, $f^{-1}(NRSCl(B))$ is an NRS-q-nbd of $x_{r,t,s}$. So there exists an nrso set A in X such that $x_{r,t,s}qA \subseteq f^{-1}(NRSCl(B))$, i.e., $f(A) \subseteq NRSCl(B)$. Hence f is $NWRSC$ and this completes the proof. □

Theorem 4.9. *If $f : X \to Y$ is $NWRSC$, then*

(i) $f(NRSC(A) \subseteq NRS\theta Cl(f(A))$ for each Ns A in X,
(ii) $f(NRSCl(f^{-1}(NRSCl(NRSInt(B))))) \subseteq NRS\theta Cl(B)$ for each Ns B in Y.

Proof. (i) Let $x_{r,t,s} \in NRSCl(A)$ and S be an NR-closed-q-nbd of $f(x_{r,t,s})$. Then there exists an NR-open-q-nbd V of $f(x_{r,t,s})$ such that $V \subseteq S$. Since f is $NWRSC$, by Theorem 4.1 there exists an NRS-open-q-nbd U of $x_{r,t,s}$ such that $f(U) \subseteq NRSCl(V)$. Since $x_{r,t,s} \in NRSCl(A)$, by Definition 4.2, $x_{r,t,s}$ is an NRS-cluster point of A. Hence UqA and also $f(U)qf(A)$. Since $f(U) \subseteq NRSCl(V)$, $NRSCl(V)qf(A)$. Again since, S is nrc (it is ncs), we have $NRSCl(V) \subseteq NRCl(V) \subseteq S$. Therefore $Sqf(A)$. By Definition 4.3, $f(x_{r,t,s}) \in NRS\theta Cl(f(A))$, i.e., $x_{r,t,s} \in f^{-1}(NRS\theta Cl(f(A))$. Therefore, $NRSCl(A) \subseteq f^{-1}(NRS\theta Cl(f(A)))$; which implies $f(NRSCl(A)) \subseteq NRS\theta Cl(f(A))$, proving(i).

(ii) Let B be an Ns in Y and $x_{r,t,s}$ be an NP in X such that $x_{r,t,s} \in NRSCl(f^{-1}(NRSCl(NRSInt(B))))$. Let V be any NR-open-q-nbd of $f(x_{r,t,s})$. By Theorem 4.1, there exists NRS-open-q-nbd U of $x_{r,t,s}$ such that $f(U) \subseteq NRSCl(V)$. Since $NRSCl(f^{-1}(NRSInt(B))) \subseteq NRSCl(f^{-1}(B))$, we have $x_{r,t,s} \in NRSCl(f^{-1}(NRSCl(NRSInt(B))))$. By Definition 4.3, $Uqf^{-1}(B)$, i.e.,$f(U)qB$. Thus $NRSCl(V)qB$, which implies $f(x_{r,t,s}) \in NRS\theta Cl(B)$. So $f(NRSCl(f^{-1}(NRSCl(NRSInt(B))))) \subseteq NRS\theta Cl(B)$, proving(ii). □

5. NEUTROSOPHIC WEAKLY REGULAR SEMI CONTINUOUS FUNCTIONS AND NEUTROSOPHIC RETRACTS

Definition 5.1. Let X be a nts and $A \subseteq X$. Then the subspace (crisp) A of X is called a neutrosophic retract (for short, N-retract) of X if there exists a neutrosophic continuous function $r : X \to A$ such that $r(a) = a$ for all $a \in A$. In this case r is called a neutrosophic retraction. A is called an NO-retract (NRO-retract, $NRSC$-retract) of X if r is N-open function (NRO-function, $NRSC$-function). Similarly A is called $NWRSC$-retract of X if r is $NWRSC$.

Theorem 5.1. *If A is an NRO-retract, $NRSC$-retract of the nts X then for every nts Y, any $NWRSC$ function $g : A \to Y$ can be extended to an $NWRSC$ function of X into Y.*

Proof. Let Y be an arbitrary nts and $g : A \to Y$ be an $NWRSC$ function. By Corollary 3.5, $g \circ r : X \to Y$ is $NWRSC$ and $g \circ r(a) = g(r(a)) = g(a)$ for all $a \in A$, where $r : X \to A$ is a NRO-retract, $NRSC$-retraction. Hence $g \circ r$ is an $NWRSC$ extension of g to X and this completes the proof. □

Theorem 5.2. *If A is an NRO-retract, $NRSC$-retract of X and B is an $NWRSC$-retract of A then B is an $NWRSC$-retract of X.*

Proof. Let $r : X \to A$ be an NRO and $NRSC$ mapping such that $r(a) = a$ for all $a \in A$, Let $s : A \to B$ be an $NWRSC$ retraction of A such that $s(b) = b$ for all $b \in B$. By corollary 3.5, $s \circ r : X \to B$ is $NWRSC$ and $s \circ r(b) = b$ for all $b \in B$. Hence B is $NWRSC$ retract of X and this proves the result. □

Definition 5.2. An nts X is said to be NR-quasi Urysohn space if for any two distinct NP's $x_{r,t,s}$ and $y_{r,t,s}$, there exist nro sets U_1 and U_2 in X such that $x_{r,t,s}qU_1$, $y_{r,t,s}qU_2$ and $NCl(U_1) \cap NCl(U_2) = 0_N$.

Definition 5.3. An nts X is said to be NRS quasi Hausdorff if distinct neutrosophic points in X have disjoint NRS-q-nbds, i.e., if $x_{r,t,s}$ and $y_{r,t,s}$ are distinct NP's in X, then there exist NRS-q-nbds V_1 and V_2 such that $x_{r,t,s}qV_1$, $y_{r,t,s}qV_2$ and $V_1 \cap V_2 = 0_N$.

Theorem 5.3. *If Y is an NR-quasi Urysohn space and $f : X \to Y$ is an $NWRSC$ injection, then X is a NRS quasi Hausdorff space.*

Proof. Let $x_{r,t,s}$ and $y_{r,t,s}$ be two distinct NP's in X. f being injective, $f(x_{r,t,s})$ and $f(y_{r,t,s})$ are distinct NP's in Y. Since Y is NR quasi Urysohn, there exists nro sets V_1 and V_2 in Y such that $f(x_{r,t,s})qV_1$, $f(y_{r,t,s}qV_2)$ and $NCl(V_1) \cap NCl(V_2) = 0_N$, i.e., $f^{-1}(NRSInt(NCl(V_1)) \cap f^{-1}(NRSInt(NCl(V_2)) = 0_N$, By Theorem 3.2, $x_{r,t,s}qf^{-1}(V_1) \subseteq f^{-1}(NRSInt(NRSCl(V_1))) \subseteq f^{-1}(NRSInt(NCl(V_1))$. Similarly $y_{r,t,s}qf^{-1}(V_2) \subseteq f^{-1}(NRSInt(NRSCl(V_2))) \subseteq f^{-1}(NRSInt\,(NCl(V_2))$. So,$f^{-1}(NRSInt(NCl(V_1))$ and $f^{-1}(NInt(NCl(V_2))$ are disjoint NRS-q-nbds of $x_{r,t,s}$ and $y_{r,t,s}$, respectively. So X is NRS quasi Hausdorff and this proves the result. □

6. NEUTROSOPHIC WEAKLY REGULAR SEMI OPEN FUNCTIONS

Definition 6.1. A Neut. function $f : (X, \tau) \to (Y, \sigma)$ is said to be:

(i) neutrosophic weakly regular open (for short, $NWRO$) if $f(R) \subseteq NRInt(f(NCl(R)))$ for each nos R of X.

(ii) neutrosophic weakly regular semi open (for short, $NWRSO$) if $f(R) \subseteq NRSInt(f(NCl(R)))$ for each nos R of X.

(iii) neutrosophic weakly semi open (for short, $NWSO$) if $f(R) \subseteq NSInt(f(NCl(R)))$ for each nos R of X.

Remark. *Clearly, every $NWRO$ function is $NWRSO$ and every $NWRSO$ function is also $NWSO$.*

Theorem 6.1. *Let $f : (X, \tau) \to (Y, \sigma)$ be a Neut. function. Then the following statements are equivalent:*

(i) f is $NWRSO$.

(ii) For each NP $x_{r,t,s}$ in X and each nos S of X containing $x_{r,t,s}$, there exists a nrso set T containing $f(x_{r,t,s})$ such that $T \subseteq f(NCl(S))$.

Proof. (i)→ (ii): Let $x_{r,t,s} \in X$ and S be a nos in X containing $x_{r,t,s}$. Since f is $NWRSO$. $f(S) \subseteq NRSInt(f(NCl(S)))$.Let $T = NRSInt(f(NCl(S)))$. Hence $T \subseteq f(NCl(S))$, with T containing $f(x_{r,t,s})$.

(ii)→ (i): Let S be a nos in X and let $y_{r,t,s} \in f(S)$. It following from (ii) $T \subseteq f(NCl(S))$ for some nrso T in Y containing $y_{r,t,s}$. Hence we have, $y_{r,t,s} \in T \subseteq NRSInt(f(NCl(S)))$. This shows that $f(S) \subseteq NRSInt(f(NCl(S)))$, i.e., f is a $NWRSO$ functions. □

Theorem 6.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then the following statements are equivalent:*

(i) f is $NWRSO$.

(ii) $NRSCl(f(R)) \subseteq f(NCl(R))$ for each nos R in X.

(iii) $NRSCl(f(NInt(S))) \subseteq f(S)$ for each ncs S in X.

Proof. (i)→ (iii): Let S be a ncs in X. Then we have $f(1-S) = 1-f(S) \subseteq NRSInt(f(NCl(1-S)))$ and so $1 - f(S) \subseteq 1 - NRSCl(f(NInt(S)))$. Hence $NRSCl(f(NInt(S))) \leq f(S)$.

(iii)→ (ii): Let R be a nos in X. Since $NCl(R)$ is a ncs and $R \subseteq NInt(NCl(R))$ by (iii) we have $NRSCl(f(R)) \subseteq NRSCl(f(NInt(NCl(R)))) \subseteq f(NCl(R))$.

(ii)→ (iii): Similar to (iii)→ (ii).

(iii)→ (i): Clear. □

Definition 6.2. Two non-null Ns's R and S in a nts X are said to be NRS-separated if $R\overline{q}NRSCl(S)$ and $S\overline{q}NRSCl(R)$ or equivalently if there exist two nrso sets T and U such that $R \subseteq T$, $S \subseteq U$, $R\overline{q}U$ and $S\overline{q}T$.

Definition 6.3. A nts X which can not be expressed as the union of two NRS-separated sets is said to be a NRS-connected space.

Theorem 6.3. *If $f : (X, \tau) \to (Y, \sigma)$ is a $NWRSO$ of a space X onto a NRS-connected space Y, then X is NRS-connected.*

Proof. If possible, let X be not NRS-connected. Then there exist NRS-separated sets T and U in X such that $X = T \cup U$. Since T and U are NRS-separated, there exist two nro sets R and S such that $T \subseteq R$, $U \subseteq S$, $T\overline{q}S$ and $U\overline{q}R$. Hence we have $f(T) \subseteq f(R)$, $f(U) \subseteq f(S)$, $f(T)\overline{q}f(S)$ and $f(U)\overline{q}f(R)$. Since f is $NWRSO$, we have $f(R) \subseteq NRSInt(f(NCl(R)))$ and $f(S) \subseteq NRSInt(f(NCl(S)))$ and since R and S are nrso and also nrsc by Theorem 3.2, we have $f(NRSCl(R)) = f(R)$, $f(NRSCl(S)) = f(S)$. Hence $f(R)$ and $f(S)$ are nrso in Y. Therefore, $f(T)$ and $f(U)$ are NRS-separated sets in Y and $Y = f(X) = f(T \cup U) = f(T) \cup f(U)$. Hence this contrary to the fact that Y is NRS-connected. Thus X is NRS-connected. □

7. Conclusions

The basic aim of this paper we introduced the concept of neutrosophic weakly regular semi continuous, neutrosophic regular semi q-neighbourhood in neutrosophic topological spaces. Moreover, we investigate the relationship among neutrosophic weakly regular semi continuous and other existing continuous functions and some counter examples to show that these types of mappings are not equivalent. Finally, we introduced Neutrosophic retracts, neutrosophic regular semi retracts, neutrosophic regular semi quasi Urysohn space and neutrosophic regular semi Hausdorff spaces. In future, we promote this thought into neutrosophic soft regular semi continuous mappings, neutrosophic contra regular semi continuous and neutrosophic contra regular semi irresolute mappings in neutrosophic topological spaces. Further, we work may include the extension of this work for Nano Topology and AntiTopology which got some attention from researchers.

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