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DYNAMIC FUZZY SETS AND NEW DFS MADM TECHNIQUE

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ABSTRACT. Dynamics fuzzy sets outperform fuzzy sets for dealing with unclear situations. There are many applications for fuzzy similarity metrics, including cluster analysis, problem classification, and even medical diagnosis. Lean entropy measurements are essential to determine the weights of the criteria in a situation involving multi-criteria decision-making. In this paper, we introduce and suggest alternative similarity measures for Dynamics fuzzy collections. We developed some new entropy metrics for using recommended similarity assessments. Dynamics fuzzy collections. Finally, in a Dynamics fuzzy environment, a novel multi-attribute decision-making method is developed that solves a significant limitation of the famous decision-making methodology, namely, the technique for order preference by similarity to the ideal solution.

1. INTRODUCTION

The fuzzy set is a novel notion that Zadeh [1] invented. According to him, it is a class of objects that have a continuum of membership grades. Fuzzy set theory forms the basis of formal decision-making analysis when human decision-making involves uncertainty variables. When uncertainty variables are present in human decision-making, fuzzy set theory offers the framework for formal decision-making analysis. For risks without a suitable quantitative probability model, a fuzzy logic system can help with modeling cause-and-effect relationships, estimating risk exposure, and equally ranking the principal hazards while accounting for expert opinions and accessible data. Meanwhile, generalized fuzzy numbers were defined by Chen and Hsieh [2]. Recently, a large number of research have been conducted on the topic, including those that rank and compare generalized fuzzy numbers [3]. Fuzzy risk analysis was introduced by Chen [4]. Chen [5] also proposed a new method for estimating the similarity between two fuzzy numbers [6]. The study analyzes risk in chicken farming using the method that has been described. Chutia and Gogoi [7] offer a concept where they address risk assessments in poultry creation using a new similarity on generalized fuzzy numbers.

Delgada et al. [8] sought to address fuzzy numbers in decision-making problems by developing canonical representations presented the value and ambiguity parameters. Dubois and Prade [9] extended the conventional algebraic operations on real numbers to fuzzy numbers by employing a fuzzy logic approach. Hsieh and Chen [10] investigated triangle

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fuzzy numbers and user similarity and applied their findings to collaborative filtering recommendations. The concept of construction project analysis was introduced using fuzzy set theory, and a technique for linguistic risk assessment was presented by Kangari and Riggs [11]. In 2017, Khorshidi and Nikfalazar published an enhanced method for computing the similarity between generalized trapezoidal fuzzy numbers [12]. Lee [13] presented a novel method for integrating disparate fuzzy concepts into the greatest possible group consensus. Patra and Mondal [14] published a novel method of establishing the similarity between the geometrical distance, area, and height of generalized trapezoidal fuzzy numbers.

Schmucke [15] presents a novel method for organizing triangular fuzzy numbers using the eNagl point of a triangle. Xu and associates. Additional similarity measures are given in Chutia and Gogoi [16], Khorshidi and Nikfalazar [17, 18]. Jun et al. employed both interval-valued fuzzy sets and fuzzy sets. In 2018, the novel concept of the cubic set was presented. We don't think computing cubic numbers takes a long time because we execute the computations using a variety of tools, including MATLAB. It is a mix of two fuzzy numbers, therefore using it manually will take longer. Our results, however, are superior. We focus on representing ambiguity and uncertainty more accurately since mathematicians are always searching for ways to improve fuzzy numbers. In this range of cubic numbers, one has a greater choice. Crisp sets, also known as classical sets, are used to represent collections of items where each object is identified as a member of a particular set by a membership degree. Generalized trapezoidal cubic numbers (GTCNs) are a hybrid of generalized trapezoidal fuzzy numbers and generalized trapezoidal interval-valued fuzzy numbers. Next, an element's membership degree is represented by a binary number in the range of 0 to 1, where a membership degree of 0 denotes a non-member and a membership degree of 1 denotes a member.

2. Basic Definitions

Definition 2.1. Peng et al. [19] A function $SM : PFS(V) \times PFS(V) \to [0,1]$ is called a PF similarity measure if $\forall N_1, N_2$ and $N_3 \in PFS(V)$, we have:

(1) $SM(N_1, N_2) = SM(N_2, N_1)$;
(2) $SM(N_1, N_2) = 1$ iff $N_1 = N_2$;
(3) $SM(N_1, (N_1)^c) = 0$ iff N_1 is a crisp set;
(4) If $N_1 \subseteq N_2 \subseteq N_3$, then $SM(N_1, N_2) \geq SM(N_1, N_3)$ and $SM(N_2, N_3) \geq SM(N_1, N_3)$.

Definition 2.2. Peng et al. [19] A function $EN : PFS(V) \to [0,1]$ is referred to as a PF entropy measure if $\forall N_1$ and $N_2 \in PFS(V)$, we have:

(1) $EN(N_1) = 0$ iff N_1 is a crisp set;
(2) $EN(N_1) = 1$ iff $\mu_{N_1}(n_k) = \vartheta_{N_1}(n_k) \forall n_k \in V$;
(3) $EN(N_1) = EN((N_1)^c)$;
(4) $EN(N_1) \leq EN(N_2)$ if $\mu_{N_1}(n_k) \leq \mu_{N_2}(n_k) \leq \vartheta_{N_2}(n_k) \leq \vartheta_{N_1}(n_k)$ or $\mu_{N_1}(n_k) \geq \mu_{N_2}(n_k) \geq \vartheta_{N_2}(n_k) \geq \vartheta_{N_1}(n_k) \forall n_k \in V$.

3. Dynamic Fuzzy Sets

Particular the potential for change in the membership function of a particular fuzzy set when dealing with time variables, Wang et al. [20] provided an expanded description of fuzzy sets, which they dubbed "dynamic fuzzy sets".

Definition 3.1. Let X be a universe of elements, and $\mathcal{T} \subset R^+$ be a discrete set of time, where, $R^+ = [0, \infty)$. Then the dynamics fuzzy sets (DFS) $\mathcal{A}$ on X is defined and characterized by the following membership function $\mathcal{A} : X \times \mathcal{T} \to [0, \infty)$.

Let

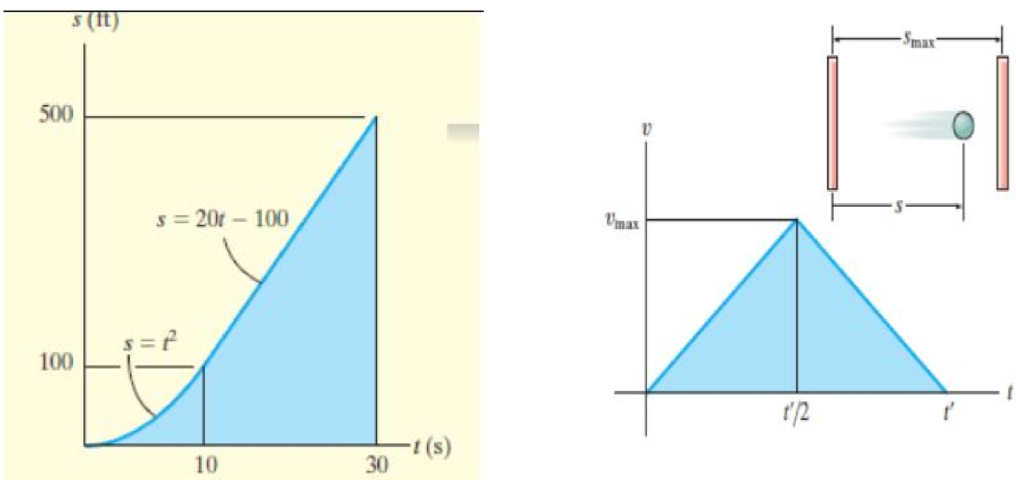
$$\mathcal{A}(t) = \sum_{j=1}^{n} \left(\sum_{i=1}^{m} \left(\mathcal{A}_{t_j}^{x_i} \right) \right),$$

$$\mathcal{B}(t) = \sum_{j=1}^{m} \left(\sum_{i=1}^{n} \left(\mathcal{B}_{t_j}^{x_i} \right) \right)$$

Are two DFS on X. The following equations provide some fundamental functions and characteristics of Union, Intersection, Complementation, and Equality.

(1) $\mathcal{A}(t) = \mathcal{B}(t) \Leftrightarrow \mathcal{A}_{t_j}^{x_i} = \mathcal{B}_{t_j}^{x_i} \forall x_i \in X, t_j \in \mathcal{T}$,
(2) $\mathcal{A}(t) \subseteq \mathcal{B}(t) \Leftrightarrow \mathcal{A}_{t_j}^{x_i} \leq \mathcal{B}_{t_j}^{x_i} \forall x_i \in X, t_j \in \mathcal{T}$,
(3) $\overline{\mathcal{A}}(t) = \mathcal{A}_{x_i}^{t_j} \forall x_i \in X, t_j \in \mathcal{T}$,
(4) $\mathcal{A}(t) \cap \mathcal{B}(t) = \min(\mathcal{A}_{t_j}^{x_i}, \mathcal{B}_{t_j}^{x_i}) \forall x_i \in X, t_j \in \mathcal{T}$
(5) $\mathcal{A}(t) \cup \mathcal{B}(t) = \max(\mathcal{A}_{t_j}^{x_i}, \mathcal{B}_{t_j}^{x_i}) \forall x_i \in X, t_j \in \mathcal{T}$

Example 3.2. Example 1 Assuming X to be a universal in relation to the time interval $\mathcal{T}_1 = \{t | t = 4k, k = 0, 1, \ldots, 100\}, \mathcal{T}_2 = \{t | t = 3k, k = 0, 1, \ldots, 100\}$.



(a) (b)

FIGURE 1. a)Example of DFS. A train moves along a straight road such that its position is described by the graph,
b)Example of DFS. A particle moving through an electric field from one plate to another has the shape shown.

Then, DFSs A, B defined by

$$\mathcal{A}(\mathcal{T}_1)(x) = \begin{cases} \frac{1}{2} & t = 4k, k = 1, \ldots, 50 \\ \frac{1}{9} & t = 4k + 2, k = 51, \ldots, 100 \end{cases}$$

$$\mathcal{B}(\mathcal{T}_2)(x) = \begin{cases} \frac{2}{7} & t = 3k, k = 1, \ldots, 40 \\ \frac{3}{8} & t = 3k+1, k = 41, \ldots, 80 \\ \frac{1}{11} & t = 3k+2, k = 81, \ldots, 100 \end{cases}$$

Example 3.3. Example 2 Let $X = \{x_1, x_2, x_3, x_4\}$ be an elemental universe, and $\mathcal{T} = \{t_1, t_2, t_3, t_4\}$ exist a distinct period of time, where , $R^+ = [0, \infty)$. Then the dynamics fuzzy sets (DFS) A on X is identified and typified by

TABLE 1. Dynamics fuzzy set (DFS)

	x_1	x_2	x_3	x_4
t_1	0.3	0.6	0.1	0.5
t_2	0.4	0.3	0.2	0.6
t_3	0.9	0.1	0.1	0.6
t_4	0.8	0.2	0.3	0.4

$$\begin{aligned}\mathcal{A}(t) &= \sum_{j=1}^{n} (\mathcal{A}_{t_j}^{x_1} + \mathcal{A}_{t_j}^{x_2} + \mathcal{A}_{t_j}^{x_3} + \mathcal{A}_{t_j}^{x_4}) \\ &= (\mathcal{A}_{t_1}^{x_1} + \mathcal{A}_{t_2}^{x_1} + \mathcal{A}_{t_3}^{x_1} + \mathcal{A}_{t_4}^{x_1}) + (\mathcal{A}_{t_1}^{x_2} + \mathcal{A}_{t_2}^{x_2} + \mathcal{A}_{t_3}^{x_2} + \mathcal{A}_{t_4}^{x_2}) \\ &+ (\mathcal{A}_{t_1}^{x_3} + \mathcal{A}_{t_2}^{x_3} + \mathcal{A}_{t_3}^{x_3} + \mathcal{A}_{t_4}^{x_3}) + (\mathcal{A}_{t_1}^{x_4} + \mathcal{A}_{t_2}^{x_4} + \mathcal{A}_{t_3}^{x_4} + \mathcal{A}_{t_4}^{x_4}) \\ &= (0.3 + 0.4 + 0.9 + 0.8) + (0.6 + 0.3 + 0.1 + 0.2) + (0.1 + 0.2 + 0.1 + 0.3) \\ &+ (0.5 + 0.6 + 0.6 + 0.4) = 6.4\end{aligned}$$

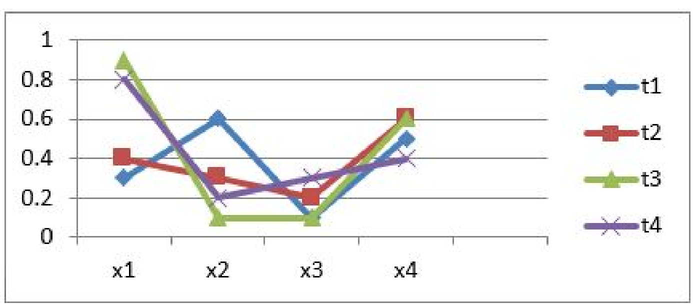


FIGURE 2. Dynamics fuzzy set (DFS)

Definition 3.4. Let $\mathcal{A}$ and $\mathcal{B}$ be two DFSs that are defined on the discourse universe $X = \{x_1, x_2, x_3, \ldots, x_n\}$ and the instants in time $\mathcal{T} = \{t_1, t_2, t_3, \ldots, t_m\}$. The following yields A and B's correlation coefficient:

$$\mathcal{K}(\mathcal{A}, \mathcal{B}) = \frac{\mathcal{C}(\mathcal{A}, \mathcal{B})}{\sqrt{\mathcal{T}(\mathcal{A})\mathcal{T}(\mathcal{B})}}$$

Where

$$\mathcal{C}(\mathcal{A},\mathcal{B}) = \left(\sum_{j=1}^{m}\left(\sum_{i=1}^{n} A_{t_j}^{x_i}\right)\right)\left(\sum_{j=1}^{m}\left(\sum_{i=1}^{n} B_{t_j}^{x_i}\right)\right)$$

Given two DFSs, $\mathcal{A}$ and $\mathcal{B}$, the correlation

$$\mathcal{T}(\mathcal{A}) = \left(\sum_{j=1}^{m}\left(\sum_{i=1}^{n} (A_{t_j}^{x_i})^2\right)\right)$$

$$\mathcal{T}(\mathcal{B}) = \left(\sum_{j=1}^{m}\left(\sum_{i=1}^{n} (B_{t_j}^{x_i})^2\right)\right)$$

The temporal fuzzy sets energies of $\mathcal{A}$ and $\mathcal{B}$, respectively, are represented by the information.

Example 3.5. Example 3 Assume that $X = \{x_1, x_2, x_3\}$ in relation to the designated time $\mathcal{T} = \{t_1, t_2, t_3\}$. Specifics of a DFS $\mathcal{A}(\mathcal{T})$ and DFS $\mathcal{B}(\mathcal{T})$ explained in Table 2

TABLE 2. DFS $\mathcal{A}(\mathcal{T})$ and DFS $\mathcal{B}(\mathcal{T})$

DFS $\mathcal{A}(\mathcal{T})$				DFS $\mathcal{B}(\mathcal{T})$			
	t_1	t_2	t_3		t_1	t_2	t_3
x_1	0.2	0.1	0.3	x_1	0.1	0.4	0.3
x_2	0.6	0.1	0.6	x_1	0.4	0.6	0.3
x_3	0.7	0.8	0.5	x_1	0.8	0.7	0.5
x_4	0.6	0.2	0.8	x_1	0.9	0.2	0.5
x_5	0.4	0.1	0.3	x_1	0.4	0.1	0.1
x_6	1	0.4	0.6	x_1	0.3	0.1	0.9

Then $\mathcal{K}(\mathcal{A},\mathcal{B}) = 13.2967$ the correlation coefficient $between\ DFS\ \ \mathcal{A}(\mathcal{T})$ and $DFS\ \ \mathcal{B}(\mathcal{T})$.

Definition 3.6. Over a DFS $\mathcal{A}$, we define the following two operators: f and g.

$$f(\mathcal{A}(\mathcal{T})) = \left\{(x, \left\{\left(x, \frac{\max}{t \in \mathcal{T}}\ A_{t_j}^{x_i}\right)\right\} : x \in X\right\}$$

$$g(\mathcal{A}(\mathcal{T})) = \left\{(x, \left\{\left(x, \frac{\min}{t \in \mathcal{T}}\ A_{t_j}^{x_i}\right)\right\} : x \in X\right\}$$

$$h(\mathcal{A}(\mathcal{T})) = \left\{(x, \left\{\left(x, \frac{\max}{t \in X}\ A_{t_j}^{x_i}\right)\right\} : x \in X\right\}$$

$$g(\mathcal{A}(\mathcal{T})) = \left\{(x, \left\{\left(x, \frac{\min}{t \in \mathcal{T}}\ A_{t_j}^{x_i}\right)\right\} : x \in X\right\}$$

$$l(\mathcal{A}(T)) = \left\{(x, \left\{\left(x, \frac{\min}{t \in X}\ A_{t_j}^{x_i}\right)\right\} : x \in X\right\}$$

Theorem 3.1. *$f(\mathcal{A}(\mathcal{T}))$ and $g(\mathcal{A}(\mathcal{T}))$ are $DFSs$*

Proof

Assume that $\frac{\max}{t\in\mathcal{T}}\ A_{t_j}^{x_i} = A_{t_1}^{x_i} \le 1$, for some $t_1 \in \mathcal{T}$, then $f(\mathcal{A}(\mathcal{T}))$ is DFSs, Additionally, in the same manner $g(\mathcal{A}(\mathcal{T}))$ are DFS.

Theorem 3.2. *For every* $DFS \ \mathcal{A}(\mathcal{T})$,

(1) $f(f(\mathcal{A}(\mathcal{T}))) = f(\mathcal{A}(\mathcal{T}))$;
(2) $g(g(\mathcal{A}(\mathcal{T}))) = g(\mathcal{A}(\mathcal{T}))$;
(3) $f(g(\mathcal{A}(\mathcal{T}))) = g(\mathcal{A}(\mathcal{T}))$;
(4) $g(f(\mathcal{A}(\mathcal{T}))) = f(\mathcal{A}(\mathcal{T}))$.

Proof Clear.

Theorem 3.3. *For each and every* $DFS \ \mathcal{A}(\mathcal{T})$,

(1) $f(f(\mathcal{A}(\mathcal{T}))) = f(f(\mathcal{A}(\mathcal{T})))$;
(2) $g(g(\mathcal{A}(\mathcal{T}))) = g(g(\mathcal{A}(\mathcal{T})))$.

Proof

(1)

$$\begin{aligned} h(f(\mathcal{A}(\mathcal{T}))) &= \left\{(x, \left\{\left(x, \frac{\max}{t \in X} \frac{\max}{t \in \mathcal{T}} A_{t_j}^{x_i}\right)\right\} : (x_i, t_j) \in X \times \mathcal{T}\right\} \\ &= \left\{(x, \left\{\left(x, \frac{\max}{t \in \mathcal{T}} \frac{\max}{t \in X} A_{t_j}^{x_i}\right)\right\} : (x_i, t_j) \in X \times \mathcal{T}\right\} \\ &= f(f(\mathcal{A}(\mathcal{T}))); \end{aligned}$$

(2) By the same fashion.

Proposition 3.4. *Let* $\mathcal{A}(\mathcal{T}_1)$ *and* $\mathcal{B}(\mathcal{T}_2)$ *be two* $DFSs$. *Then:*

(1) $\mathcal{T}(\mathcal{A}) = \mathcal{T}(\overline{\mathcal{A}})$;
(2) $\mathcal{C}(\mathcal{A}, \mathcal{A}) = \mathcal{T}(\mathcal{A})$;
(3) $\mathcal{C}(\mathcal{A}, \mathcal{B}) = \mathcal{C}(\mathcal{B}, \mathcal{A})$.

Proof

(1)

$$\mathcal{T}(\mathcal{A}) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)^2\right)\right) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{x_i}^{t_j}\right)^2\right)\right) = \mathcal{T}(\overline{\mathcal{A}});$$

(2)

$$\mathcal{C}(\mathcal{A}, \mathcal{A}) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right)\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)^2\right)\right) = \mathcal{T}(\mathcal{A});$$

(3)

$$\begin{aligned} \mathcal{C}(\mathcal{A}, \mathcal{B}) &= \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right)\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right) \\ &= \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right)\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) = \mathcal{C}(\mathcal{B}, \mathcal{A}). \end{aligned}$$

Theorem 3.5. *Let* $\mathcal{A}$ *and* $\mathcal{B}$ *be two DFSs. Then,*

(1) $\mathcal{K}(\mathcal{A}, \mathcal{B}) = 1$ *if and only if* $\mathcal{A}$ *and* $\mathcal{B}$ *are identical*
(2) $\mathcal{K}(\mathcal{A}, \mathcal{B}) = k(\mathcal{B}, \mathcal{A})$
(3) $0 \leq \mathcal{K}(\mathcal{A}, \mathcal{B}) \leq 1$.

Proof

(1) Assume that $\mathcal{A}$ and $\mathcal{B}$ are two DFSs based on the discourse universe $X = \{x_1, x_2, x_3, \ldots\ldots., x_n\}$ and the momentous times $\mathcal{T} = \{t_1, t_2, t_3, \ldots\ldots., t_m\}$.
The following yields $\mathcal{A}$ and $\mathcal{B}$ correlation coefficient:

$$\mathcal{K}(\mathcal{A}, \mathcal{B}) = \frac{\mathcal{C}(\mathcal{A}, \mathcal{B})}{\sqrt{\mathcal{T}(\mathcal{A})\ \mathcal{T}(\mathcal{B})}}$$

if $\mathcal{A} = \mathcal{B}$, then $\sqrt{\mathcal{T}(\mathcal{A})\ \mathcal{T}(\mathcal{B})} = \sqrt{\mathcal{T}(\mathcal{A})^2} = \mathcal{T}(\mathcal{A})$. Then from Proposition 3.4

(2) $\mathcal{C}(\mathcal{A}, \mathcal{A}) = \mathcal{T}(\mathcal{A})$, then

$$\mathcal{K}(\mathcal{A}, \mathcal{B}) = \frac{\mathcal{T}(\mathcal{A})}{\mathcal{T}(\mathcal{A})} = 1$$

2 From Proposition 3.4 (3) $\mathcal{C}(\mathcal{A}, \mathcal{B}) = \mathcal{C}(\mathcal{B}, \mathcal{A})$. Then

$$\mathcal{K}(\mathcal{A}, \mathcal{B}) = \frac{\mathcal{C}(\mathcal{B}, \mathcal{A})}{\sqrt{\mathcal{T}(\mathcal{A})\ \mathcal{T}(\mathcal{B})}} = \mathcal{K}(\mathcal{B},\mathcal{A})$$

(3) Well demonstrate that. $\mathcal{K}(\mathcal{A}, \mathcal{B}) < 1$ so that it is clear $0 < \mathcal{K}(\mathcal{A}, \mathcal{B})$, so assume that

$$\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)^2\right)\right) = \rho_1$$

$$\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(\left(B_{t_j}^{x_i}\right)^2\right)\right)\right) = \rho_2$$

Then

$$\mathcal{T}(\mathcal{A})\,\mathcal{T}(\mathcal{B}) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)^2\right)\right) \times \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(\left(B_{t_j}^{x_i}\right)^2\right)\right)\right)$$

$$\sqrt{\mathcal{T}(\mathcal{A})\,\mathcal{T}(\mathcal{B})} = \left(\left(\sum_{j=1}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)^2\right) + \sum_{j=1}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)^2\right)\right)\left(\sum_{j=1}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)^2\right) + \sum_{j=1}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)^2\right)\right)\right)^{\frac{1}{2}}$$

Then
$\sqrt{\mathcal{T}(\mathcal{A})\,\mathcal{T}(\mathcal{B})} = [(\rho_1 + \rho_2) \times (\rho_1 + \rho_2)]^{\frac{1}{2}} = (\rho_1 + \rho_2)^{\frac{1}{2}} \times (\rho_1 + \rho_2)^{\frac{1}{2}}$,

$$\mathcal{C}(\mathcal{A}, \mathcal{B}) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) \times \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right),$$

Then

$$\mathcal{C}(\mathcal{A}, \mathcal{B}) = (\rho_1 \times \rho_2)^{\frac{1}{2}}$$

Then

$$\mathcal{K}^2(\mathcal{A},\mathcal{B}) \leq \frac{\rho_1 \times \rho_2}{(\rho_1+\rho_2)\times(\rho_1+\rho_2)}$$

But

$$\mathcal{K}^2(\mathcal{A},\mathcal{B}) - 1 \leq \frac{\rho_1 \times \rho_2}{(\rho_1+\rho_2)\times(\rho_1+\rho_2)} - 1$$
$$= \frac{[\rho_1 \times \rho_2] - [(\rho_1+\rho_2)\times(\rho_1+\rho_2)]}{(\rho_1+\rho_2)\times(\rho_1+\rho_2)} \leq 0$$

Hence $\mathcal{K}^2(\mathcal{A},\mathcal{B}) - 1 \leq 0$, then $\mathcal{K}(\mathcal{A},\mathcal{B}) \leq 1$.

Definition 3.7. Allow $S : DFSs\ (X,\mathcal{T}) \times DFSs\ (X,\mathcal{T}) \rightarrow [0,1]$ be a function, and let $\mathcal{A}(\mathcal{T}),\ \mathcal{B}(\mathcal{T})\ and\ \mathcal{C}(\mathcal{T})\ be$ DFSs in the universal $X = \{x_1, x_2, x_3, \ldots, x_n\}$ Regarding the time set $\mathcal{T} = \{t_1, t_2, t_3, \ldots\ldots\ldots, t_m\}$. Then $S(\mathcal{A},\mathcal{B})$ is defined as the degree of similarity between $DFSs$ $\mathcal{A}$ and $\mathcal{B}$. Satisfies the ensuing claims

(1) $0 \leq S(\mathcal{A},\mathcal{B}) \leq 1$;
(2) $S(\mathcal{A},\mathcal{B}) = 1$ if $\mathcal{A}= \mathcal{B}$;
(3) $S(\mathcal{A},\mathcal{B}) = S(\mathcal{B},\mathcal{A})$
(4) If $\mathcal{A} \subseteq \mathcal{B} \subseteq C$. Then $S(\mathcal{A},C) \leq S(\mathcal{A},\mathcal{B}),\ S(\mathcal{A},C) \leq S(\mathcal{B},C)$.

The degrees of similarity between $\mathcal{A}$ and $\mathcal{B}$ that now satisfy requirements 1 through 4 are as follows,

$$\mathcal{C}(\mathcal{A},\mathcal{A}) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) \times \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right)$$

$S_{\mathcal{A}}(i,j) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right)$,
$S_{\mathcal{B}}(i,j) = \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right)$,

$$\psi_{\mathcal{A}}(i,j) = \frac{\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) + 1 - \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right)}{2},$$

Where $i = 1, 2, 3, \ldots.., n$, $j = 1, 2, 3, \ldots\ldots, m$. Then

$$S_1(\mathcal{A},\mathcal{B}) = 1 - \frac{1}{2mn}\left|\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) - \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right)\right|$$

$$S_2(\mathcal{A},\mathcal{B}) = 1 - \frac{1}{2mn}\left|\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(\left(A_{t_j}^{x_i}\right)^2\right)\right)\right) - \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(\left(B_{t_j}^{x_i}\right)^2\right)\right)\right)\right|$$

$$S_0(\mathcal{A},\mathcal{B}) = 1 - \frac{1}{\sqrt{2mn}}\left|\sqrt{\left(\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) - \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right)\right)}\right|$$

$$S_3(\mathcal{A},\mathcal{B}) = 1 - \frac{1}{\sqrt[y]{2mn}} \sqrt{\left| \frac{\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) + \left(1 - \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right)\right)}{2} - \frac{\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right) + \left(1 - \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right)\right)}{2} \right|}, \quad 1 \leq$$

$y < +\infty$,

A comparison of the similarity metrics $S_0(\mathcal{A},\mathcal{B})$, $S_1(\mathcal{A},\mathcal{B})$, $S_2(\mathcal{A},\mathcal{B})$, $S_3(\mathcal{A},\mathcal{B})$, we give the following example.

Example 3.8. Example 4 Assume that $\mathcal{A}(\mathcal{T})$ and $\mathcal{B}(\mathcal{T})$ is DFSs specified on $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ in relation to the designated time $\mathcal{T} = \{t_1, t_2, t_3, t_4, t_5, t_6\}$. The specifics of a DFS $\mathcal{A}(\mathcal{T})$ explained in Table (4), Table (5) explained DFS $\mathcal{B}(T)$ and Table (6) provided an explanation of a comparison of similarity metrics $S_0(\mathcal{A},\mathcal{B})$, $S_1(\mathcal{A},\mathcal{B})$, $S_2(\mathcal{A},\mathcal{B})$, $S_3(\mathcal{A},\mathcal{B})$, DFS $\mathcal{A}(\mathcal{T})$ and DFS $\mathcal{B}(\mathcal{T})$.

TABLE 3. DFS $\mathcal{A}(\mathcal{T})$

	t_1	t_2	t_3	t_4	t_5	t_6
x_1	0.1	0.2	0.3	0.1	0.3	0.4
x_2	0.3	0.3	0.6	0.2	0.2	0.1
x_3	0.2	0.4	0.2	0.1	0.3	0.4
x_4	0.4	0.5	0.3	0.1	0.2	0.5
x_5	0.5	0.1	0.1	0.1	0.3	0.1
x_6	0.1	0.2	0.2	0.5	0.2	0.1

TABLE 4. DFS $\mathcal{B}(\mathcal{T})$

	t_1	t_2	t_3	t_4	t_5	t_6
x_1	0.3	0.1	0.1	0.4	0.1	0.3
x_2	0.2	0.2	0.2	0.4	0.3	0.5
x_3	0.1	0.3	0.3	0.3	0.2	0.5
x_4	0.2	0.4	0.2	0.5	0.4	0.6
x_5	0.5	0.5	0.1	0.2	0.1	0.3
x_6	0.1	03	0.4	0.2	0.1	0.2

TABLE 5. A comparison of the similarity metrics $S_0(\mathcal{A},\mathcal{B})$, $S_1(\mathcal{A},\mathcal{B})$, $S_2(\mathcal{A},\mathcal{B})$, $S_3(\mathcal{A},\mathcal{B})$

	t_1	t_2	t_3	t_4	t_5	t_6
x_1	$S_0(\mathcal{A},\mathcal{B})=0.95$	$S_0(\mathcal{A},\mathcal{B})=0.46$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=1.$	$S_0(\mathcal{A},\mathcal{B})=0.95$	$S_0(\mathcal{A},\mathcal{B})=1.$
	$S_1(\mathcal{A},\mathcal{B})=0.998$	$S_1(\mathcal{A},\mathcal{B})=0.98$	$S_1(\mathcal{A},\mathcal{B})=0.72$	$S_1(\mathcal{A},\mathcal{B})=1.$	$S_1(\mathcal{A},\mathcal{B})=0.8$	$S_1(\mathcal{A},\mathcal{B})=1.$
	$S_2(\mathcal{A},\mathcal{B})=1.00$	$S_2(\mathcal{A},\mathcal{B})=1.0$	$S_2(\mathcal{A},\mathcal{B})=0.92$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=0.80$	$S_2(\mathcal{A},\mathcal{B})=1.$
	$S_3(\mathcal{A},\mathcal{B})=0.91$	$S_3(\mathcal{A},\mathcal{B})=0.93$	$S_3(\mathcal{A},\mathcal{B})=0.82$	$S_3(\mathcal{A},\mathcal{B})=1.$	$S_3(\mathcal{A},\mathcal{B})=0.16$	$S_3(\mathcal{A},\mathcal{B})=1.$
x_2	$S_0(\mathcal{A},\mathcal{B})=1.$	$S_0(\mathcal{A},\mathcal{B})=0.75$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=0.82$	$S_0(\mathcal{A},\mathcal{B})=0.92$	$S_0(\mathcal{A},\mathcal{B})=1$
	$S_1(\mathcal{A},\mathcal{B})=0.94$	$S_1(\mathcal{A},\mathcal{B})=0.19$	$S_1(\mathcal{A},\mathcal{B})=0.30$	$S_1(\mathcal{A},\mathcal{B})=0.80$	$S_1(\mathcal{A},\mathcal{B})=0.91$	$S_1(\mathcal{A},\mathcal{B})=0.96$
	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=0.86$	$S_2(\mathcal{A},\mathcal{B})=0.80$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=1.$
	$S_3(\mathcal{A},\mathcal{B})=0.94$	$S_3(\mathcal{A},\mathcal{B})=0.33$	$S_3(\mathcal{A},\mathcal{B})=0.95$	$S_3(\mathcal{A},\mathcal{B})=0.96$	$S_3(\mathcal{A},\mathcal{B})=0.958$	$S_3(\mathcal{A},\mathcal{B})=0.91$
x_3	$S_0(\mathcal{A},\mathcal{B})=1.$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=0.64$	$S_0(\mathcal{A},\mathcal{B})=0.92$	$S_0(\mathcal{A},\mathcal{B})=1$
	$S_1(\mathcal{A},\mathcal{B})=1.$	$S_1(\mathcal{A},\mathcal{B})=0.75$	$S_1(\mathcal{A},\mathcal{B})=0.58$	$S_1(\mathcal{A},\mathcal{B})=0.68$	$S_1(\mathcal{A},\mathcal{B})=0.96$	$S_1(\mathcal{A},\mathcal{B})=0.90$
	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=0.30$	$S_2(\mathcal{A},\mathcal{B})=0.98$	$S_2(\mathcal{A},\mathcal{B})=0.68$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=0.90$
	$S_3(\mathcal{A},\mathcal{B})=1.$	$S_3(\mathcal{A},\mathcal{B})=0.69$	$S_3(\mathcal{A},\mathcal{B})=0.9823$	$S_3(\mathcal{A},\mathcal{B})=0.68$	$S_3(\mathcal{A},\mathcal{B})=0.9941$	$S_3(\mathcal{A},\mathcal{B})=0.95$
x_4	$S_0(\mathcal{A},\mathcal{B})=0.6$	$S_0(\mathcal{A},\mathcal{B})=0.29$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=0.92$	$S_0(\mathcal{A},\mathcal{B})=0.3$	$S_0(\mathcal{A},\mathcal{B})=1$
	$S_1(\mathcal{A},\mathcal{B})=0.91$	$S_1(\mathcal{A},\mathcal{B})=0.95$	$S_1(\mathcal{A},\mathcal{B})=0.96$	$S_1(\mathcal{A},\mathcal{B})=0.91$	$S_1(\mathcal{A},\mathcal{B})=0.980$	$S_1(\mathcal{A},\mathcal{B})=0.96$
	$S_2(\mathcal{A},\mathcal{B})=0.91$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=0.95$	$S_2(\mathcal{A},\mathcal{B})=0.991$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=0.9886$
	$S_3(\mathcal{A},\mathcal{B})=0.99$	$S_3(\mathcal{A},\mathcal{B})=0.75$	$S_3(\mathcal{A},\mathcal{B})=0.91$	$S_3(\mathcal{A},\mathcal{B})=0.99$	$S_3(\mathcal{A},\mathcal{B})=0.9175$	$S_3(\mathcal{A},\mathcal{B})=0.9841$
x_5	$S_0(\mathcal{A},\mathcal{B})=0.98$	$S_0(\mathcal{A},\mathcal{B})=0.45$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=1.$	$S_0(\mathcal{A},\mathcal{B})=1$
	$S_1(\mathcal{A},\mathcal{B})=0.29$	$S_1(\mathcal{A},\mathcal{B})=0.68$	$S_1(\mathcal{A},\mathcal{B})=0.44$	$S_1(\mathcal{A},\mathcal{B})=0.94$	$S_1(\mathcal{A},\mathcal{B})=1.$	$S_1(\mathcal{A},\mathcal{B})=0.96$
	$S_2(\mathcal{A},\mathcal{B})=0.31$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=0.92$	$S_2(\mathcal{A},\mathcal{B})=0.92$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=1.$
	$S_3(\mathcal{A},\mathcal{B})=0.899$	$S_3(\mathcal{A},\mathcal{B})=0.44$	$S_3(\mathcal{A},\mathcal{B})=0.94$	$S_3(\mathcal{A},\mathcal{B})=0.82$	$S_3(\mathcal{A},\mathcal{B})=1.$	$S_3(\mathcal{A},\mathcal{B})=0.91$
x_6	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=0.45$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=1$	$S_0(\mathcal{A},\mathcal{B})=0.93$	$S_0(\mathcal{A},\mathcal{B})=0.93$
	$S_1(\mathcal{A},\mathcal{B})=0.94$	$S_1(\mathcal{A},\mathcal{B})=0.68$	$S_1(\mathcal{A},\mathcal{B})=0.94$	$S_1(\mathcal{A},\mathcal{B})=0.98$	$S_1(\mathcal{A},\mathcal{B})=0.98$	$S_1(\mathcal{A},\mathcal{B})=0.98$
	$S_2(\mathcal{A},\mathcal{B})=0.94$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=0.92$	$S_2(\mathcal{A},\mathcal{B})=0.96$	$S_2(\mathcal{A},\mathcal{B})=1.$	$S_2(\mathcal{A},\mathcal{B})=1.$
	$S_3(\mathcal{A},\mathcal{B})=0.46$	$S_3(\mathcal{A},\mathcal{B})=0.44$	$S_3(\mathcal{A},\mathcal{B})=0.98$	$S_3(\mathcal{A},\mathcal{B})=0.93$	$S_3(\mathcal{A},\mathcal{B})=0.96$	$S_3(\mathcal{A},\mathcal{B})=0.96$

4. NOVEL MEASUREMENTS OF SIMILARITY FOR DYNAMICS FUZZY SETS (DFS)

The fuzzy set (DFS) for measures of dynamics is extended in the following formulation.

Definition 4.1. If $A(t)$ is DFS, $t \in \mathcal{T}$. Define

$$G(A(t)) = \frac{1 - \left(\sum_{j=1,}^{n}\left(\sum_{i=1}^{m}\left(A_{t_j}^{x_i}\right)\right)\right)}{1 + \alpha\left(\sum_{j=1,}^{n}\left(\sum_{i=1}^{m}\left(A_{t_j}^{x_i}\right)\right)\right)}, \quad \alpha > 0$$

Then a generator DFS (GDFS) $\mathcal{A}$ is given by

$$\mathcal{A}^{\alpha}(T) = \left\{\left(\left(\sum_{j=1,}^{n}\left(\sum_{i=1}^{m}\left(A_{t_j}^{x_i}\right)\right)\right), \frac{1 - \left(\sum_{j=1,}^{n}\left(\sum_{i=1}^{m}\left(A_{t_j}^{x_i}\right)\right)\right)}{1 + \alpha\left(\sum_{j=1,}^{n}\left(\sum_{i=1}^{m}\left(A_{t_j}^{x_i}\right)\right)\right)}\right)\right\}$$

The hesitation degree of a GDFS $\mathcal{A}$ is

$$\pi_{\mathcal{A}^{\alpha}}(x,t) = 1 - \left(\sum_{j=1,}^{n}\left(\sum_{i=1}^{m}\left(A_{t_j}^{x_i}\right)\right)\right) - \frac{1 - \left(\sum_{j=1,}^{n}\left(\sum_{i=1}^{m}\left(A_{t_j}^{x_i}\right)\right)\right)}{1 + \alpha\left(\sum_{j=1,}^{n}\left(\sum_{i=1}^{m}\left(A_{t_j}^{x_i}\right)\right)\right)}$$

Example 4.2. Example 5 Assume that $\mathcal{A}(\mathcal{T})$ is DFS defined on $X = \{x_1, x_2, x_3\}$ in relation to the designated time $\mathcal{T} = \{t_1, t_2, t_3\}$. The specifics of a $GDFS\ \mathcal{A}(\mathcal{T})$ outlined in Table (6), when If $\alpha = 1$, and Table (7) explained $the\ hesitation\ degree\ of\ a\ DFS\ \mathcal{A}$.

If $\rho = 1$, then

TABLE 6. GDFS $\mathcal{A}^1$

	t_1	t_2	t_3
x_1	(0.4,0.1)	(0.2,0.6)	(0.3,0.5)
x_2	(0.6,0.5)	(0.1,0.6)	(0.6,0.4)
x_3	(0.7,0.2)	(0.1,0.5)	(0.8,0.5)

TABLE 7. *The hesitation degree of a GFS* $\mathcal{A}$

	x_1	x_2	x_3
t_1	0.133	0.34	0.16
t_2	0.65	0.76	0.652
t_3	0.43	0.65	0.76

Definition 4.3. Assume that $\mathcal{A}(\mathcal{T})$ and $\mathcal{B}(\mathcal{T})$ is DFSs inside the global $X = \{x_1, x_2, x_3, \ldots\ldots\ldots, x_n\}$ in relation to the designated time $\mathcal{T} = \{t_1, t_2, t_3, \ldots\ldots\ldots, t_m\}$.
Then, a comparison of the cosine similarity between $\mathcal{A}(\mathcal{T})$ and $\mathcal{B}(\mathcal{T})$ is suggested to be done as follows:

$$C_T(\mathcal{A}(x_i, t_j), \mathcal{B}(x_i, t_j)) = \frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(A_{t_j}^{x_i}\right)\left(B_{t_j}^{x_i}\right)}{\sqrt{\left(A_{t_j}^{x_i}\right)^2} + \sqrt{\left(B_{t_j}^{x_i}\right)^2}}\right)$$

Where i = 1,2,3,. . . ..,n, j = 1,2,3,. . . . . . ,m

Theorem 4.1. *Assume that* $\mathcal{A}(T\mathcal{T})$ and $\mathcal{B}(\mathcal{T})$ *is DFSs in* X *with respect to* $\mathcal{T}$*.. Then,*

(1) $C_{\mathcal{T}}(\mathcal{A}, \mathcal{B}) = 1$ *if* $\mathcal{A} = \mathcal{B}$*;*
(2) $C_{\mathcal{T}}(\mathcal{A}, \mathcal{B}) = C_{\mathcal{T}}(\mathcal{B}, \mathcal{A})$ *,;*
(3) $-1 \leq C_{\mathcal{T}}(\mathcal{A}, \mathcal{B}) \leq 1$*,;*
(4) If $n = m = 1,$ *then* $C_{\mathcal{T}}(\mathcal{A}, \mathcal{B}) = (\mathcal{A}, \mathcal{B})$ *.*

Proof

(1) Allow $\mathcal{A}$ and $\mathcal{B}$ be two DFSs which are specified in the discourse universe. $X = \{x_1, x_2, x_3, \ldots\ldots., x_n\}$ and the instants in time $\mathcal{T} = \{t_1, t_2, t_3, \ldots\ldots., t_m\}$. The cosine similarity measure between the following $\mathcal{A}(\mathcal{T})$ and $\mathcal{B}(\mathcal{T})$:

$$C_{\mathcal{T}}(\mathcal{A}(x_i, t_j), \mathcal{B}(x_i, t_j)) = \frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(A_{t_j}^{x_i}\right)\left(B_{t_j}^{x_i}\right)}{\sqrt{\left(A_{t_j}^{x_i}\right)^2}.\sqrt{\left(B_{t_j}^{x_i}\right)^2}}\right)$$

if $\mathcal{A} = \mathcal{B}$, then

$$C_{\mathcal{T}}(\mathcal{A}, \mathcal{B}) = \frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(A_{t_j}^{x_i}\right)^2}{\left(A_{t_j}^{x_i}\right)^2}\right) = 1$$

(2)

$$C_{\mathcal{T}}\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)=\frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(A_{t_j}^{x_i}\right)\left(B_{t_j}^{x_i}\right)}{\sqrt{\left(A_{t_j}^{x_i}\right)^2}.\sqrt{\left(B_{t_j}^{x_i}\right)^2}}\right)$$

$$C_{\mathcal{T}}\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)=\frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(B_{t_j}^{x_i}\right)\left(A_{t_j}^{x_i}\right)}{\sqrt{\left(B_{t_j}^{x_i}\right)^2}.\sqrt{\left(A_{t_j}^{x_i}\right)^2}}\right)$$

$= C_{\mathcal{T}}(\mathcal{B},\mathcal{A})$

(3) By the same manner in (3) Theorem 3.4.

(4) If $n = m = 1$, then

$$C_T\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)=\frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(A_{t_j}^{x_i}\right)\left(B_{t_j}^{x_i}\right)}{\sqrt{\left(A_{t_j}^{x_i}\right)^2}.\sqrt{\left(B_{t_j}^{x_i}\right)^2}}\right)$$

$$C_T\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)=\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\frac{\left(A_{t_j}^{x_i}\right)\left(B_{t_j}^{x_i}\right)}{\sqrt{\left(A_{t_j}^{x_i}\right)^2}.\sqrt{\left(B_{t_j}^{x_i}\right)^2}}\right)\right)$$

$$= k(\mathcal{A},\mathcal{B})$$

Definition 4.4. Assume that $\mathcal{A}(T)$ and $\mathcal{B}(T)$ is DFSs in $X = \{x_1, x_2, x_3, \ldots\ldots\ldots, x_n\}$ regarding the designated time $T = \{t_1, t_2, t_3, \ldots\ldots\ldots, t_m\}$. Then, the distance measure is suggested to be as follows:

$$d\left(\mathcal{A},\mathcal{B}\right)=\cos^{-1}\left(C_T\left(\mathcal{A},\mathcal{B}\right)\right)$$

Theorem 4.2. *Assume that $\mathcal{A}(T)$ and $\mathcal{B}(T)$ is DFSs in X regarding the designated time T. Then,*

(1) $C_T\left(\mathcal{A},\mathcal{B}\right)=1$, then $d\left(\mathcal{A},\mathcal{B}\right)=0$;
(2) $C_T\left(\mathcal{A},\mathcal{B}\right)=C_T(\mathcal{B},\mathcal{A})$, then $d\left(\mathcal{A},\mathcal{B}\right)=d\left(\mathcal{B},\mathcal{A}\right)$;
(3) If $-1\le C_T\left(\mathcal{A},\mathcal{B}\right)\le 1$, then $d\left(\mathcal{A},\mathcal{B}\right)\ge 0$;
(4) $\mathcal{A}\subseteq\mathcal{B}\subseteq C$, then $d\left(\mathcal{A},C\right)\le d\left(\mathcal{A},\mathcal{B}\right)+d\left(\mathcal{B},C\right)$.

Proof

(1), (2), and (3) are simple proof.

(4) Let $\mathcal{A}(T)$, $\mathcal{B}(T)$ *and* $C(T)$ *be* DFSs in the universal $X = \{x_1, x_2, x_3, \ldots, x_n\}$ with respect to the time set $T = \{t_1, t_2, t_3, \ldots\ldots\ldots, t_m\}$. Then, the distance measure of the angle is proposed as follows:

$$d_{(i,j)}\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)=\cos^{-1}\left(C_T\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)\right)$$

$$d_{(i,j)}\left(\mathcal{A}\left(x_i,t_j\right),C\left(x_i,t_j\right)\right)=\cos^{-1}\left(C_T\left(\mathcal{A}\left(x_i,t_j\right),C\left(x_i,t_j\right)\right)\right)$$

$$d_{(i,j)}\left(\mathcal{B}\left(x_i,t_j\right),C\left(x_i,t_j\right)\right)=\cos^{-1}\left(C_T\left(\mathcal{B}\left(x_i,t_j\right),C\left(x_i,t_j\right)\right)\right)$$

Where $i=1,2,3,\ldots..,n,\ j=1,2,3,\ldots\ldots,m$ and

$$C_T\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)=\frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(A_{t_j}^{x_i}\right)\left(B_{t_j}^{x_i}\right)}{\sqrt{\left(A_{t_j}^{x_i}\right)^2}.\sqrt{\left(B_{t_j}^{x_i}\right)^2}}\right)$$

$$C_T\left(\mathcal{B}\left(x_i,t_j\right),C\left(x_i,t_j\right)\right)=\frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(B_{t_j}^{x_i}\right)\left(C_{t_j}^{x_i}\right)}{\sqrt{\left(B_{t_j}^{x_i}\right)^2}.\sqrt{\left(C_{t_j}^{x_i}\right)^2}}\right)$$

$$C_T\left(\mathcal{A}\left(x_i,t_j\right),C\left(x_i,t_j\right)\right)=\frac{1}{mn}\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(A_{t_j}^{x_i}\right)\left(C_{t_j}^{x_i}\right)}{\sqrt{\left(A_{t_j}^{x_i}\right)^2}.\sqrt{\left(C_{t_j}^{x_i}\right)^2}}\right)$$

If $\mathcal{A}\left(x_i,t_j\right)\subseteq\mathcal{B}\left(x_i,t_j\right)\subseteq C\left(x_i,t_j\right)$, for each $i=1,2,3,\ldots..,n,\ j=1,2,3,\ldots\ldots,m$ then

$$d_{(i,j)}\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)+d_{(i,j)}\left(\mathcal{B}\left(x_i,t_j\right),C\left(x_i,t_j\right)\right)\geq d_{(i,j)}\left(\mathcal{A}\left(x_i,t_j\right),C\left(x_i,t_j\right)\right).$$

Definition 4.5. Allow $\mathcal{A}$, $\mathcal{B}$ be two DFSs defined on $X=\{x_1,x_2,x_3,\ldots\ldots.,x_n$ and $T=\{t_1,t_2,t_3,\ldots\ldots.,t_m\}$. Assume that $k(\mathcal{A},\mathcal{B})$ is correlation coefficient of $\mathcal{A}$ and $\mathcal{B}$. Then, a comparison of weight similarity between DFSs $\mathcal{A}(T)\ and\ \mathcal{B}(T)$ is suggested to be done as follows:

$$\rho_T\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)=\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\mathcal{K}(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right))\left(\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\left(A_{t_j}^{x_i}\right)\left(C_{t_j}^{x_i}\right)}{\sqrt{\left(A_{t_j}^{x_i}\right)^2}.\sqrt{\left(C_{t_j}^{x_i}\right)^2}}\right)\right)$$

and possess the ensuing qualities

(1) $\rho_T\left(\mathcal{A},\mathcal{B}\right)=1$ then $\mathcal{A}=\mathcal{B}$;
(2) $\rho_T\left(\mathcal{A},\mathcal{B}\right)=\rho_T(\mathcal{B},\mathcal{A})$;
(3) $-1\leq\rho_T\left(\mathcal{A},\mathcal{B}\right)\leq1$.

Remark. $\rho_T\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)=C_T\left(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right)\right)$ *if,*

$$\sum_{i=1}^{n}\sum_{j=1}^{m}k(\mathcal{A}\left(x_i,t_j\right),\mathcal{B}\left(x_i,t_j\right))=\frac{1}{mn}$$

A comparative analysis of similarity metrics $C_T\left(\mathcal{A},\mathcal{B}\right)$, $\rho_T\left(\mathcal{A},\mathcal{B}\right)$. *We offer the subsequent illustration.*

Example 4.6. Example 6 Assume that $\mathcal{A}(T)\ and\ \mathcal{B}(T)$ is TCIFSs defined on $X=\{x_1,x_2,x_3,x_4,x_5,x_6\}$ with time set $T=\{t_1,t_2,t_3,t_4,t_5,t_6\}$, The specifics of an $DFS\ \mathcal{A}(T)$ explained in Table (8) Table (9), explained $DFS\ \ \mathcal{B}(T)$, and Table (10,11) described a comparison of the measurements of similarity between $C_T\left(\mathcal{A},\mathcal{B}\right)$, $\rho_T\left(\mathcal{A},\mathcal{B}\right)$.

TABLE 8. $DFS\ \mathcal{A}(\mathcal{T})$

	t_1	t_2	t_3	t_4	t_5	t_6
x_1	0.33	0.32	0.12	0.24	0.54	0.23
x_2	0.11	0.22	0.32	0.32	0.31	0.43
x_3	0.31	0.36	0.37	0.75	0.34	0.49
x_4	0.32	0.43	0.11	0.43	0.44	0.53
x_5	0.43	0.33	0.43	0.41	0.49	0.58
x_6	0.43	0.42	0.32	0.97	0.32	0.41

TABLE 9. $DFS\ \mathcal{B}(\mathcal{T})$

	t_1	t_2	t_3	t_4	t_5	t_6
x_1	0.71	0.76	0.54	0.53	0.43	0.42
x_2	0.41	0.42	0.43	0.54	0.22	0.54
x_3	0.56	0.42	0.76	0.51	0.63	0.54
x_4	0.22	0.23	0.76	0.65	0.43	0.62
x_5	0.32	0.72	0.63	0.63	0.65	0.87
x_6	0.63	0.63	0.63	0.87	0.76	0.63

5. A NEW DFS MADM TECHNIQUE

We first discuss the shortcomings of the traditional DFS TOPSIS approach in this section. Next, we provide a new MADM technique in the DFS circumstances that is akin to TOPSIS.

5.1. The DFS TOPSIS method's flaws.

5.1. The DFS TOPSIS method's flaws. The order preference by similarity to ideal solution (TOPSIS) approach, first published by Hwang and Yoon (1981) and later extended to the fuzzy environment by Chen (2000), is one of the most well-liked and effective methods for handling MADM issues.

TABLE 10. Then the similarity measures $C_T(\mathcal{A},\mathcal{B})$ and $\rho_T(\mathcal{A},\mathcal{B})$, $t_1 \to t_3$

	t_1	t_2	t_3
x_1	$C_T(\mathcal{A},\mathcal{B}) = .7652$ $\rho_T(\mathcal{A},\mathcal{B}) = .6543$	$C_T(\mathcal{A},\mathcal{B}) = .5487$ $\rho_T(\mathcal{A},\mathcal{B}) = .4321$	$C_T(\mathcal{A},\mathcal{B}) = .0013$ $\rho_T(\mathcal{A},\mathcal{B}) = .2700$
x_2	$C_T(\mathcal{A},\mathcal{B}) = .0243$ $\rho_T(\mathcal{A},\mathcal{B}) = .5431$	$C_T(\mathcal{A},\mathcal{B}) = .9871$ $\rho_T(\mathcal{A},\mathcal{B}) = .4321$	$C_T(\mathcal{A},\mathcal{B}) = .1044$ $\rho_T(\mathcal{A},\mathcal{B}) = .9321$
x_3	$C_T(\mathcal{A},\mathcal{B}) = .02707$ $\rho_T(\mathcal{A},\mathcal{B}) = .3541$	$C_T(\mathcal{A},\mathcal{B}) = .0021$ $\rho_T(\mathcal{A},\mathcal{B}) = .096$	$C_T(\mathcal{A},\mathcal{B}) = .0004$ $\rho_T(\mathcal{A},\mathcal{B}) = .6478$
x_4	$C_T(\mathcal{A},\mathcal{B}) = .5432$ $\rho_T(\mathcal{A},\mathcal{B}) = .4321$	$C_T(\mathcal{A},\mathcal{B}) = .8721$ $\rho_T(\mathcal{A},\mathcal{B}) = .4231$	$C_T(\mathcal{A},\mathcal{B}) = .0108$ $\rho_T(\mathcal{A},\mathcal{B}) = .1363$
x_5	$C_T(\mathcal{A},\mathcal{B}) = .4325$ $\rho_T(\mathcal{A},\mathcal{B}) = .4113$	$C_T(\mathcal{A},\mathcal{B}) = .0075$ $\rho_T(\mathcal{A},\mathcal{B}) = .542$	$C_T(\mathcal{A},\mathcal{B}) = .0024$ $\rho_T(\mathcal{A},\mathcal{B}) = .6148$
x_6	$C_T(\mathcal{A},\mathcal{B}) = .4327$ $\rho_T(\mathcal{A},\mathcal{B}) = .4321$	$C_T(\mathcal{A},\mathcal{B}) = .8741$ $\rho_T(\mathcal{A},\mathcal{B}) = .0033$	$C_T(\mathcal{A},\mathcal{B}) = .0044$ $\rho_T(\mathcal{A},\mathcal{B}) = .1124$

TABLE 11. Then the similarity measures $C_T(\mathcal{A},\mathcal{B})$ and $\rho_T(\mathcal{A},\mathcal{B})$,$t_4 \to t_6$

	t_4	t_5	t_6
x_1	$C_T(\mathcal{A},\mathcal{B}) = .0116$	$C_T(\mathcal{A},\mathcal{B}) = -.016$	$C_T(\mathcal{A},\mathcal{B}) = -1.006$
	$\rho_T(\mathcal{A},\mathcal{B}) = .1321$	$\rho_T(\mathcal{A},\mathcal{B}) = .007$	$\rho_T(\mathcal{A},\mathcal{B}) = -1.000$
x_2	$C_T(\mathcal{A},\mathcal{B}) = .6521$	$C_T(\mathcal{A},\mathcal{B}) = -.003$	$C_T(\mathcal{A},\mathcal{B}) = .0042$
	$\rho_T(\mathcal{A},\mathcal{B}) = .4681$	$\rho_T(\mathcal{A},\mathcal{B}) = -.0500$	$\rho_T(\mathcal{A},\mathcal{B}) = -.1356$
x_3	$C_T(\mathcal{A},\mathcal{B}) = .81$	$C_T(\mathcal{A},\mathcal{B}) = -.014$	$C_T(\mathcal{A},\mathcal{B}) = -.021$
	$\rho_T(\mathcal{A},\mathcal{B}) = .231$	$\rho_T(\mathcal{A},\mathcal{B}) = -.7200$	$\rho_T(\mathcal{A},\mathcal{B}) = -.9821$
x_4	$C_T(\mathcal{A},\mathcal{B}) = .0051$	$C_T(\mathcal{A},\mathcal{B}) = -.011$	$C_T(\mathcal{A},\mathcal{B}) = -.0102$
	$\rho_T(\mathcal{A},\mathcal{B}) = .9997$	$\rho_T(\mathcal{A},\mathcal{B}) = -.729$	$\rho_T(\mathcal{A},\mathcal{B}) = -.971$
x_5	$C_T(\mathcal{A},\mathcal{B}) = .0008$	$C_T(\mathcal{A},\mathcal{B}) = -.342$	$C_T(\mathcal{A},\mathcal{B}) = -.062$
	$\rho_T(\mathcal{A},\mathcal{B}) = -.4198$	$\rho_T(\mathcal{A},\mathcal{B}) = -1.00$	$\rho_T(\mathcal{A},\mathcal{B}) = -.6153$
x_6	$C_T(\mathcal{A},\mathcal{B}) = -.0008$	$C_T(\mathcal{A},\mathcal{B}) = -.003$	$C_T(\mathcal{A},\mathcal{B}) = -.033$
	$\rho_T(\mathcal{A},\mathcal{B}) = -.6519$	$\rho_T(\mathcal{A},\mathcal{B}) = -.528$	$\rho_T(\mathcal{A},\mathcal{B}) = .0316$

The foundation of the TOPSIS technique is the idea that the option that is closest to the PIS and farthest from the NIS should be selected. The DFSs for the PIS and NIS are:

$$PIS = \ max\left(A_{t_j}^{x_i}, B_{t_j}^{x_i}\right), \qquad NIS = \min\left(A_{t_j}^{x_i}, B_{t_j}^{x_i}\right).$$

If we utilize the similarity metric in TOPSIS instead of the distance measure, the chosen option should have the highest similarity to PIS and the lowest similarity to NIS. However, as the following examples show, there is not even the slightest similarity between NIS and the TOPSIS-selected option.

Example 5.1. Consider a DFS $M_1\ and\ M_2$

$$M_1 = \begin{pmatrix} 0.21 & 0.76 & 0.43 \\ 0.45 & 0.87 & 0.67 \\ 0.12 & 0.67 & 0.53 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0.41 & 1.16 & 0.42 \\ 0.15 & 1.17 & 0.67 \\ 0.62 & 0.67 & 0.41 \end{pmatrix}$$

Then the PIS M^+ and NIS M^- are given below:

$$M^+ \ = \begin{pmatrix} 0.41 & 1.16 & 0.43 \\ 0.45 & 1.17 & 0.67 \\ 0.62 & 0.67 & 0.53 \end{pmatrix}$$

$$M^- = \begin{pmatrix} 0.21 & 0.76 & 0.42 \\ 0.15 & 0.87 & 0.67 \\ 0.12 & 0.67 & 0.41 \end{pmatrix}$$

The similarity of each alternative with M^+ i.e., $S_{G1}(M_j,\ M^+)$ and M^- i.e., $S_1(M_j,\ M^-)$ along with their closeness coefficient $\delta_j = \frac{S_1\left(M_j,\ M^+\right)}{S_1(M_j,\ M^+)+S_1(M_j,\ M^-)},\ j = 1,\ 2$ are shown in Table 12 and Fig 3. Also, the final ranking of alternatives is shown in the same Table 12.

$$S_1(\mathcal{A},\mathcal{B}) = 1 - \frac{1}{2mn}\left|\left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(A_{t_j}^{x_i}\right)\right)\right) - \left(\sum_{j=1,}^{m}\left(\sum_{i=1}^{n}\left(B_{t_j}^{x_i}\right)\right)\right)\right|$$

$$= 1 - \frac{1}{\sqrt{18}}\left(|4.71 - 6.11|\right) = 0.5427,$$

By the fashion

TABLE 12. Computed values regarding Example 7

	$S_1(M_j,\ M^+)$	$S_1(M_j,\ M^-)$	δ_j	Ranking
M_1	0.5427	0.8986	0.3765	2
M_2	0.8.986	0.67001	0.5728	1

From Table 12, it is clear that the best alternative M_2 due to the DFS TOPSIS method. Table 12 makes it evident that the optimal choice M_2, as a result of the DFS *TOPSIS* approach.

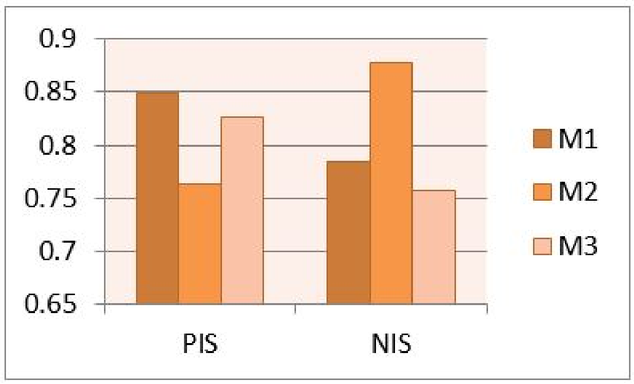


FIGURE 3. it is clear that the best alternative M_2 due to the DFS TOPSIS method.

6. CONCLUSIONS AND FUTURE DIRECTIONS

The following sums up the main contributions made in this paper:

(1) We determine how to set a new fuzzy similarity measure standard.
(2) In a scenario including multiple criteria for decision-making, fuzzy entropy measurements are essential for determining the weights of the criterion.
(3) Using suggested similarity metrics, we created a few new entropy measures for Dynamics fuzzy collections.
(4) A new multi-attribute decision-making technique called ”A Dynamics fuzzy environment” is designed to address a major shortcoming of the well-known decision-making approach.

In the future, to make decisions using Pythagorean fuzzy sets, we will apply more sophisticated theories.

7. CONCLUSIONS

A membership function of a fuzzy set that is subjectively provided by a specific amount of knowledge may vary with time in a real-world application. As a result, knowledge-based dynamic fuzzy sets—a expanded concept of fuzzy sets—were presented in this study. The idea is a combination of knowledge-based fuzzy sets, as presented and debated by Intan and Mukaidono, and dynamic fuzzy sets, as suggested by Wang [20]. One application of two-dimensional fuzzy multisets that deals with time and knowledge variables is the idea of knowledge-based dynamic fuzzy sets. Using aggregation functions, three types of summary fuzzy sets are proposed and discussed: knowledge-based summary fuzzy sets, time-based summary fuzzy sets, and general summary fuzzy sets. There are definitions for a few fundamental operations, including complementation, union, intersection, equality, and containment. Their characteristics are checked and validated.

8. Conflict of interest

The authors declare there is no conflict of interest.

9. ACKNOWLEDGEMENTS

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