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TRI-QUASI IDEALS AND FUZZY TRI-QUASI IDEALS OF SEMIGROUPS

MARAPUREDDY MURALI KRISHNA RAO*, RAJENDRA KUMAR KONA,
NOORBHASHA RAFI AND VENKATESWARLU BOLINENI

ABSTRACT. In this paper, we introduce the notion of a tri-quasi ideal and a fuzzy tri-quasi ideal as a further generalization of ideals, left ideals, right ideals, bi-ideals, quasi-ideals, and interior ideals. We characterize the regular semigroup in terms of tri-quasi ideals, fuzzy tri-quasi ideals and study some of their properties. This generalization enables mathematicians to explore new relationships and enhancing the understanding of these structures. We establish that, a semigroup is a regular semigroup if and only if $B \cap I \cap L \subseteq BIL$, for any tri-quasi ideal B, ideal I and left ideal L of a semigroup, and for a semigroup, if μ is a fuzzy left tri-ideal of a semigroup then μ is a fuzzy tri-quasi ideal.

1. INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide range of applications. Semigroup as the basic algebraic structure used in the areas of theoretical computer science, formal languages, solutions of graph theory, optimization theory, and coding theory. In 1981, Sen introduced the notion of a $\Gamma-$semigroup as a generalization of semigroup. The notion of ideals was introduced by Dedekind for the theory of algebraic numbers, further generalized by Noether for associative rings. In 1952, the concept of bi-ideals was introduced by Good and Hughes[2] for semigroups. Quasi-ideals are generalization of right-ideals and left-ideals whereas bi-ideals are generalization of quasi-ideals. In 1976, the concept of interior-ideals was introduced by Lajos[5] for semigroups. The notion of bi-ideals in rings and semigroups was introduced by Lajos and Szasz[4]. Bi-ideal is a special case of (m-n) ideal. Steinfeld[18] first introduced the notion of quasi-ideals for semigroups and then for rings. Iseki introduced the concept of quasi-ideal for semigroup.

During 1950-1980, the concepts of bi-ideals, quasi ideals and interior ideals were studied by many mathematicians and during 1950-2019, the applications of these ideals only studied by mathematicians. Between 1980 and 2016 there have been no new generalization of these ideals of algebraic structures. The author[7, 8, 9, 10] introduced and studied weak interior ideals, bi-interior ideals, bi-quasi ideals, quasi-interior ideals and bi-quasi interior ideals of $\Gamma-$semirings, semirings, $\Gamma-$semigroups, semigroups as a generalization

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*Corresponding author.

of bi-ideal, quasi-ideal and interior-ideal of algebraic structures and charecterized regular algebraic structures as well as simple algebraic structures using these ideals.

The fuzzy set theory was developed by Zadeh[19] in 1965. The fuzzification of algebraic structure was introduced by Rosenfeld[17] and studied fuzzy subgroups in 1971. Many papers on fuzzy sets appeared, showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology,and measure theory. N. Kuroki[3] studied fuzzy ideals in semigroups. Murali Krishna Rao[2, 13, 14, 15, 16] studied fuzzy bi-quasi ideals, fuzzy bi-interior ideals, fuzzy quasi-interior ideals as a generalization of fuzzy bi-ideals, fuzzy quasi-ideals, and fuzzy interior ideals of semirings, $\Gamma-$semirings and semigroups. In this paper, we introduce the notion of a tri-quasi ideal, a fuzzy tri-quasi ideal of a semigroup by studying their properties and we characterize the regular semigroup in terms of fuzzy tri-quasi ideals.

2. Preliminaries

In this section we will recall some fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. [10] A semigroup is an algebraic system $(M, .)$ consisting of a non-empty set M together with an associative binary operation $'' \cdot ''$.

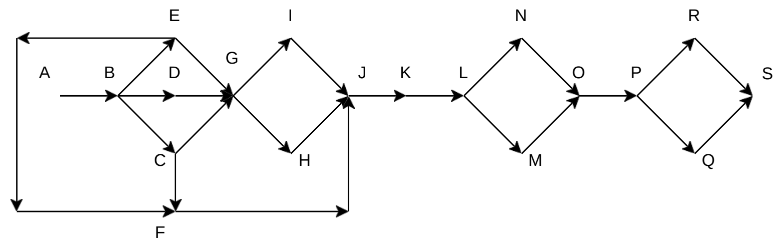
Definition 2.2. [10] A non-empty subset B of a semigroup M is called

(i) a subsemigroup of M if $BB \subseteq B$.
(ii) a quasi ideal of M if B is a subsemigroup of M and $BM \cap MB \subseteq B$.
(iii) a bi-ideal of M if B is a subsemigroup of M and $BMB \subseteq B$.
(iv) an interior ideal of M if $MBM \subseteq B$.
(v) a left (right) ideal of M if $MB \subseteq B(BM \subseteq B)$.
(vi) an ideal if $BM \subseteq B$ and $MB \subseteq B$.
(vii) a left bi-quasi ideal (right bi-quasi ideal) of M if B is a subsemigroup of M and $MB \cap BMB \subseteq B$ $(BM \cap BMB \subseteq B)$. A bi-quasi ideal of M if B is a subsemigroup of M and B is a left and right bi-quasi ideal of M.
(viii) a left quasi-interior ideal (right quasi-interior ideal) of M if B is a subsemigroup of M and $MBMB \subseteq B$ $(BMBM \subseteq B)$. A quasi-interior of M if B is a subsemigroup of M and B is a left quasi-interior ideal and a right quasi-interior ideal of M.
(ix) a bi-quasi-interior ideal of M if B is a subsemigroup of M and $BMBMB \subseteq B$.
(x) a left tri-ideal (right tri-ideal) of M if B is a subsemigroup of M and $BMBB \subseteq B$ $(BBMB \subseteq B)$. A tri-ideal of M if B is a subsemigroup of M, $BMBB \subseteq B$ and $BBMB \subseteq B$.

Figure(1): The interrelationships between some generalization of the ideal mentioned before are visualized in Figure (1). (Arrows indicate proper inclusions. That is if X and Y are ideals, then $X \to Y$ means $X \subset Y$.)

Definition 2.3. [10] Let M be a semigroup. An element $1 \in M$ is said to be unity if for each $a \in M$ such that $a1 = 1a = a$.

Definition 2.4. [10] A semigroup M is said to be commutative if $ab = ba$, for all $a, b \in M$.



A:Algebraic structure
B:Ideal
C:Left ideal
D:Interior ideal
E:Right-ideal
F:Quasi-ideal
G:Quasi-interior ideal
H:Left Quasi-interior ideal
I:Right Quasi- interior ideal
J:Bi-ideal
K:Bi-interior ideal
L:Bi-quasi ideal
M:Left Bi-quasi ideal
N:Right Bi-quasi ideal
O:Bi-quasiinterior ideal
P:Tri-ideal
Q:LeftTri-ideal
S:RightTri-ideal

Definition 2.5. [10] Let M be a semigroup. An element $a \in M$ is said to be an idempotent of M if $a = aa$.

Definition 2.6. [10] Let M be a semigroup. If every element of M is an idempotent of M, then semigroup M is said to be band.

Definition 2.7. [10] Let M be a semigroup. An element $a \in M$ is said to be regular element of M if there exist $x \in M$, such that $a = axa$.

Definition 2.8. [10] Let M be a semigroup. Every element of M is a regular element of M then M is said to be a regular semigroup.

Definition 2.9. [13] Let M be a non-empty set. A mapping $\mu : M \to [0, 1]$ is called a fuzzy subset of M.

Definition 2.10. [13] If μ is a fuzzy subset of M, for $t \in [0, 1]$ then the set $\mu_t = \{x \in M \mid \mu(x) \geq t\}$ is called a level subset of M with respect to a fuzzy subset μ.

Definition 2.11. [13] For any two fuzzy subsets λ and μ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in M$.

Definition 2.12. [13] Let A be a non-empty subset of M. The characteristic function of A is a fuzzy subset of M, defined by $\chi_{_A}(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A. \end{cases}$

Definition 2.13. [13] Let μ and λ be fuzzy subsets of M. Then

(i) $\mu \cup \lambda$ is a fuzzy subset of M defined by $\mu \cup \lambda(x) = \max\{\mu(x), \lambda(x)\}$, for all $x \in M$.

(ii) $\mu \cap \lambda$ is a fuzzy subset of M defined by $\mu \cap \lambda(x) = \min\{\mu(x), \lambda(x)\}$ for all $x \in M$.

$\mu \circ \lambda$ is defined by

$$\mu \circ \lambda(z) = \begin{cases} \sup\limits_{z=xy,\ x,y \in M} & \{\min\{\mu(x), \lambda(y)\}\}, \\ 0 & \text{otherwise} \end{cases}, \text{ for all } z \in M.$$

Definition 2.14. [13] A fuzzy subset μ of a semigroup M is called

(i) a fuzzy subsemigroup of M, if $\mu(xy) \geq min\ \{\mu(x), \mu(y)\}$, for all $x, y \in M$.
(ii) a fuzzy left (right) ideal of M, if $\mu(xy) \geq \mu(y)\ (\mu(x))$, for all $x, y \in M$.
(iii) a fuzzy ideal of M, if $\mu(xy) \geq max\ \{\mu(x), \mu(y)\}$, for all $x, y \in M$.
(iv) a fuzzy bi-ideal of M, if μ is a fuzzy subemigroup of M and $\mu \circ \chi_M \circ \mu \subseteq \mu$,
(v) a fuzzy quasi -ideal of M, if μ is a fuzzy subemigroup of M and $\mu \circ \chi_M \cap \chi_M \circ \mu \subseteq \mu$,
(vi) a fuzzy interior ideal of M, if μ is a fuzzy subemigroup of M and $\chi_M \circ \mu \circ \chi_M \subseteq \mu$,
(vii) a fuzzy bi-quasi ideal of M, if μ is a fuzzy subemigroup of M and $\chi_M \circ \mu \cap \mu \circ \chi_M \circ \mu \subseteq \mu$,
(viii) a fuzzy tri-ideal of M, if μ is a fuzzy subemigroup of M and $\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$,
(ix) a fuzzy quasi-interior ideal of M, if μ is a fuzzy subemigroup of M and $\chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu$,
(x) a fuzzy bi-interior ideal of M, if $\chi_M \circ \mu \circ \chi_M \cap \mu \circ \chi_M \circ \mu \subseteq \mu$,
(xi) a fuzzy bi-quasi interior ideal of M, if $\mu \circ \chi_M \circ \mu \circ \chi_M \circ \mu \subseteq \mu$.

3. Tri- quasi ideals of Semigroups

In this section, we introduce the notion of tri-quasi ideal as a generalization of bi-ideal, quasi-ideal and interior ideal of a semigroup and study the properties of tri-quasi ideal of a semigroup. Throughout this paper, M is a semigroup with unity element.

Definition 3.1. A non-empty subset B of a semigroup M is said to be a tri-quasi ideal of M, if B is a subsemigroup of M and $BBMBB \subseteq B$.

Example 3.2. Let $M = \{0, \alpha, \beta, \gamma\}$. Define the binary operation " $\cdot$ " on M, by the following table

$\cdot$	0	α	β	γ
0	0	0	0	0
α	0	α	β	γ
β	0	β	β	β
γ	0	γ	γ	γ

Then M is a semigroup, and $B = \{0, \beta\}$ is a tri-quasi ideal of M.

Every tri-quasi ideal of a semigroup M need not be bi-ideal, quasi-ideal, interior ideal, bi-interior ideal, and bi-quasi ideals of M.

In the following theorem, we mention some properties and omit the proofs since proofs are straight forward.

Theorem 3.1. *Let M be a semigroup. Then the following hold.*

(1) *Every (quasi, bi, interior, bi-interior, bi-quasi, left, right) ideal is a tri-quasi ideal of M.*
(2) *The intersection of a right ideal and a left ideal of M is a tri-quasi ideal of M.*
(3) *If L is a left ideal and R is a right ideal of a semigroup M then $B = RL$ is a tri-quasi ideal of M.*
(4) *Let B be a bi-ideal of a semigroup M and I be an interior ideal of M. Then $B \cap I$ is a tri-quasi ideal of M.*

Theorem 3.2. *The intersection of a tri-quasi ideal B of a semigroup M and a right ideal A of M is always a tri-quasi ideal of M.*

Proof. Suppose $C = B \cap A$. Then
$CCMCC \subseteq BBMBB \subseteq B$ and $CCMCC \subseteq AAMAA \subseteq A$,
Since A is a right ideal of M. Therefore $CCMCC \subseteq B \cap A = C$.
Hence the intersection of a tri-quasi ideal B of M and a right ideal A of M is always a tri-quasi ideal of M. □

Theorem 3.3. *Let M be a semigroup. Then B is a tri-quasi ideal of M if and only if B is a left ideal of some right ideal of M.*

Proof. Let B be a tri-quasi ideal of the semigroup M. Then $BBMBB \subseteq B$. Therefore BB is a left ideal of right ideal BBM of a semigroup M.
Conversely suppose that B is a left ideal of some right ideal R of the semigroup M. Then $RB \subseteq B, RM \subseteq R$. Hence $BBMBB \subseteq BMB \subseteq RMB \subseteq RB \subseteq B$. Therefore B is a tri-quasi ideal of the semigroup M. □

Theorem 3.4. *Let M be a semigroup. B is a tri-quasi ideal of M and $BB = B$ if and only if there exist a left ideal L and a right idealR such that $RL \subseteq B \subseteq R \cap L$.*

Proof. Suppose B is a tri-quasi ideal of the semigroup M.
Then $BBMBB \subseteq B$. Let $R = BM$ and $L = MB$.
Then L and R are left and right ideals of M respectively.
Therefore $RL \subseteq B \subseteq R \cap L$.
Conversely suppose that there exist L and R are left and right ideals of M respectively such that $RL \subseteq B \subseteq R \cap L$. Then
$BBMBB \subseteq (R \cap L)(R \cap L)M(R \cap L)(R \cap L)$
$\subseteq (R)RML(L)$
$\subseteq RL \subseteq B$.
Hence B is a tri-quasi ideal of the semigroup M. □

Theorem 3.5. *Let M be a semigroup and T be a non-empty subset of M. Then every subsemigroup of T containing $TTMTT$ is a tri-quasi ideal of M.*

Proof. Let B be a subsemigroup of T containing $TTMTT$. Then
$BBMBB \subseteq TTMTT \subseteq B$. Therefore $BBMBB \subseteq B$. Hence B is a tri-quasi ideal of M. □

Theorem 3.6. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ tri-quasi ideals of a semigroup M is a tri-quasi ideal of M.*

Proof. Let $B = \bigcap_{\lambda \in A} B_\lambda$. Then B is a subsemigroup of M.
Since B_λ is a tri-quasi ideal of M, we have

$$B_\lambda B_\lambda M B_\lambda B_\lambda \subseteq B_\lambda, \text{ for all } \lambda \in A$$
$$\Rightarrow BBMBB \subseteq B.$$

Hence B is a tri-quasi ideal of M. □

Theorem 3.7. *Let M be a semigroup. If $BBMBB = B$, for all tri-quasi ideals B of M, then M is a regular semigroup.*

Proof. Suppose that $BBMBB = B$, for all tri-quasi ideals B of M.
Let $B = R \cap L$, where R is a right ideal and L is a left ideal of M.
Then B is a tri-quasi ideal of M.
Therefore $(R \cap L)(R \cap L)M(R \cap L)(R \cap L) = R \cap L$

$$\begin{aligned} R \cap L &= (R \cap L)(R \cap L)M(R \cap L)(R \cap L) \\ &\subseteq RRMLL \\ &\subseteq RL \\ &\subseteq R \cap L \text{ (since } RL \subseteq L \text{ and } RL \subseteq R). \end{aligned}$$

Therefore $R \cap L = RL$. Hence M is a regular semigroup. □

Theorem 3.8. *M is a regular semigroup if and only if $B \cap I \cap L \subseteq BIL$, for any tri-quasi ideal B, ideal I and left ideal L of M.*

Proof. Let M be a regular semigroup, B, I and L are a tri-quasi ideal, an ideal and a left ideal of M respectively.
Let $a \in B \cap I \cap L$. Then $a \in aMa$, since M is regular.

$$\begin{aligned} a \in aMa &\subseteq aMaMa \\ &\subseteq BIL. \end{aligned}$$

Hence $B \cap I \cap L \subseteq BIL$.
Conversely suppose that $B \cap I \cap L \subseteq BIL$, for any tri-quasi ideal B, ideal I and left ideal L of M. Let R be a right ideal and L be left ideal of M. Then by assumption, $R \cap L = R \cap M \cap L \subseteq RML \subseteq RL$. We have $RL \subseteq R$, $RL \subseteq L$. Therefore $RL \subseteq R \cap L$. Hence $R \cap L = RL$.
Thus M is a regular semigroup. □

Theorem 3.9. *Let M be a regular idempotent semigroup. Then B is a tri-quasi ideal of M if and only if $BBMBB = B$, for all tri-quasi ideals B of M.*

Proof. Suppose M is a regular semigroup, B is a tri-quasi ideal of M and $x \in B$. Then $BBMBB \subseteq B$ and there exist $y \in M$, such that $x = xxyxx \in BBMBB$. Therefore $x \in BBMBB$.
Hence $BBMBB = B$.

Conversely suppose that $BBMBB = B$, for all tri-quasi ideals B of M.
Let $B = R \cap L$, where R is a right ideal and L is a left ideal of M.
Then B is a tri-quasi ideal of M.
Therefore $(R \cap L)M(R \cap L)M(R \cap L) = R \cap L$

$$\begin{aligned} R \cap L &= (R \cap L)(R \cap L)MM(R \cap L)(R \cap L) \\ &\subseteq RMLML \\ &\subseteq RL \\ &\subseteq R \cap L \text{ (since } RL \subseteq L \text{ and } RL \subseteq R). \end{aligned}$$

Therefore $R \cap L = RL$. Hence M is a regular semigroup. □

Theorem 3.10. *Let M be a regular commutative semigroup. Then every tri-quasi ideal of M is an ideal of M.*

Proof. Let B be a tri-quasi ideal of M.and $C = BBMBB$.
Then $C = BBMBB = B$
$\Rightarrow BM = CM \subseteq CMC$, since M is regular

$\Rightarrow BM \subseteq BBMBBMBBMBB \subseteq B.$

□

Theorem 3.11. *Let B be a subsemigroup of a regular idempotent semigroup M. B can be represented as $B = RL$, where R is a right ideal and L is a left ideal of M if and only if B is a tri-quasi ideal of M.*

Proof. Suppose $B = RL$, where R is right ideal of M and L is a left ideal of M.

$$\begin{aligned} BBMBB &= RLRLMRLRL \\ &\subseteq RL = B. \end{aligned}$$

Hence B is a tri-quasi ideal of the semigroup M.

Conversely, suppose that B is a tri-quasi ideal of the regular idempotent semigroup M.Then $BBMBB = B$. Let $R = BM$ and $L = MB$.
Then $R = BM$ is a right ideal of M and $L = MB$ is a left ideal of M.

$$\begin{aligned} &BM \cap MB \subseteq BBMBB = B \\ \Rightarrow &BM \cap MB \subseteq B \\ \Rightarrow &R \cap L \subseteq B. \\ \text{We have } &B \subseteq BM = R \text{ and } B \subseteq MB = L \\ \Rightarrow &B \subseteq R \cap L \\ \Rightarrow &B = R \cap L = RL, \text{ since } M \text{ is a regular semigroup.} \end{aligned}$$

Hence, B can be represented as RL, where R is the right ideal and L is the left ideal of M. □

4. Fuzzy tri-quasi ideals of Semigroups

In this section, we introduce the notion of a fuzzy tri-quasi ideal as a generalization of a fuzzy bi-ideal, a fuzzy quasi-ideal and a fuzzy interior-ideal of a semigroup and study the properties of fuzzy tri-quasi ideals.

Definition 4.1. A fuzzy subset μ of a semigroup M is called a fuzzy tri-quasi ideal of M if μ is a fuzzy subemigroup of M and $\mu \circ \mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$.

Example 4.2. Let $M = \{0, \alpha, \beta, \gamma\}$. Define the binary operation " $\cdot$ " on M, by the following table

$\cdot$	0	α	β	γ
0	0	0	0	0
α	0	α	β	γ
β	0	β	β	β
γ	0	γ	γ	γ

Let $B = \{0, \beta\}$. (i) Define a fuzzy subset μ of M by

$$\mu(x) = \begin{cases} 1, & \text{if } x \in B; \\ 0, & \text{if } x \notin B. \end{cases}$$

Then μ is a fuzzy tri-quasi ideal of M.

Theorem 4.1. *Let M be a semgroup. A fuzzy set μ of M is a fuzzy subsemigroup if and only if $\mu \circ \mu \subseteq \mu$.*

Proof. Suppose μ is a fuzzy subsemigroup of M and $z, l, m \in M$. Then

$$\begin{aligned}\mu\circ\mu(z) &= \sup_{z\,=\,lm}\{\min\{\mu(l),\mu(m)\}\}\\ &\le \sup_{z\,=\,lm}\mu(lm), \text{ since } \mu(lm)\ge\min\{\mu(l),\mu(m)\}\\ &=\mu(z).\end{aligned}$$

If $l, m \in M$, does not exist such that $z = lm$, then $\mu\circ\mu(z) = 0 \le \mu(z)$, for all $z \in M$. Thus $\mu\circ\mu \subseteq \mu$.

Conversely, suppose that $\mu\circ\mu\subseteq\mu$ and $x, y \in M$. Then

$$\begin{aligned}\mu(xy) &\ge \mu\circ\mu(xy)\\ &=\sup\{\min\{\mu(x),\mu(y)\}\}\\ &\ge \min\{\mu(x),\mu(y)\}.\end{aligned}$$

Hence, μ is a fuzzy subsemigroup of M. □

Theorem 4.2. *Every fuzzy left ideal of a semigroup M is a fuzzy tri-quasi ideal of M.*

Proof. Let μ be a fuzzy left ideal of the semigroup M and $x \in M$.

$$\begin{aligned}\chi_M\circ\mu\circ\mu(x) &= \sup_{x\,=\,lm}\{\min\{\chi_M(l),\mu\circ\mu(m)\}\}, l, m\in M.\\ &= \sup_{x\,=\,lm}\mu(m)\\ &\le \sup_{x\,=\,lm}\mu(lm)\\ &=\mu(x).\end{aligned}$$

Therefore $\chi_M\circ\mu\circ\mu(x)\le\mu(x)$.

$$\begin{aligned}\text{Now } \mu\circ\mu\circ\chi_M\circ\mu\circ\mu(x) &= \sup_{x\,=\,lm}\{\min\{\mu\circ\mu(l),\chi_M\circ\mu\circ\mu(m)\}\}\\ &\le \sup_{x\,=\,lm}\{\min\{\mu(l),\mu(m)\}\}\\ &=\mu(x).\end{aligned}$$

Hence, μ is a fuzzy tri-quasi ideal of M. □

Corollary 4.3. *Every fuzzy ideal of a semigroup M is a fuzzy tri-quasi ideal of M.*

Theorem 4.4. *Let M be a semigroup and μ be a non-empty fuzzy subset of M. A fuzzy subset μ is a fuzzy tri-quasi ideal of a semigroup M if and only if the level subset μ_k of μ is a tri-quasi ideal of a semigroup M for every $k \in [0, 1]$, where $\mu_k \neq \phi$.*

Proof. Let M be a semigroup and μ be a non-empty fuzzy subset of M.
Suppose μ is a fuzzy tri-quasi ideal of a semigroup M, $\mu_k \neq \phi, k \in [0, 1]$ and $a, b \in \mu_k$, Then

$$\begin{aligned}&\mu(a)\ge k, \mu(b)\ge k\\ \Rightarrow&\mu(ab)\ge\min\{\mu(a),\mu(b)\}\ge k\\ \Rightarrow&ab\in\mu_k.\end{aligned}$$

Let $x \in \mu_k\mu_k M\mu_k\mu_k$. Then $x = bcade$, where
$a \in M, b, c, d, e \in \mu_k$,Then

$$\mu \circ \mu \circ \chi_M \circ \mu \circ \mu(x) \geq k$$
$$\Rightarrow \mu(x) \geq \mu \circ \mu \circ \chi_M \circ \mu \circ \mu(x) \geq k$$
$$\text{Therefore } x \in \mu_k.$$

Hence μ_k is a tri-quasi ideal of M.

Conversely suppose that μ_k is a tri-quasi ideal of M, for all $k \in Im(\mu)$.
Let $x, y \in M, \mu(x) = k_1, \mu(y) = k_2$ and $k_1 \geq k_2$. Then $x, y \in \mu_{k_2}$.

$$\Rightarrow xy \in \mu_{k_2}$$
$$\Rightarrow \mu(xy) \geq k_2 = \min\{k_1, k_2\} = \min\{\mu(x), \mu(y)\}$$
$$\text{Therefore } \mu(xy) \geq k_2 = \min\{\mu(x), \mu(y)\}.$$

We have $\mu_l\mu_l M\mu_l\mu_l \subseteq \mu_k$, for all $l \in Im(\mu)$.
Suppose $k = \min\{Im(\mu)\}$. Then $\mu_k\mu_k M\mu_k\mu_k \subseteq \mu_k$.
Therefore $\mu \circ \mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$.
Hence μ is a fuzzy tri-quasi ideal of M. □

Theorem 4.5. *Let I be a non-empty subset of a semigroup M and χ_I be the characteristic function of I. Then I is a tri-quasi ideal of a semigroup M if and only if χ_I is a fuzzy tri-quasi ideal of a semigroup M.*

Proof. Let I be a non-empty subset of a semigroup M and χ_I be the characteristic function of I. Suppose I is a tri-quasi ideal of M.

Obviously χ_I is a fuzzy subsemigroup of M. We have $IIMII \subseteq I$. Then

$$\begin{aligned}\chi_I \circ \chi_I \circ \chi_M \circ \chi_I \circ \chi_I &= \chi_{IIMII}\\ &= \chi_{IIMII}\\ &\subseteq \chi_I.\end{aligned}$$

Therefore χ_I is a fuzzy tri-quasi ideal of M.
Conversely suppose that χ_I is a fuzzy tri-quasi ideal of M.
Then I is a subsemigroup of M. We have

$$\chi_I \circ \chi_I \circ \chi_M \circ \chi_I \circ \chi_I \subseteq \chi_I$$
$$\Rightarrow \chi_{IIMII} \subseteq \chi_I$$
$$\text{Therefore } IIMII \subseteq I.$$

Hence I is a tri-quasi ideal of M. □

Theorem 4.6. *If μ and λ are fuzzy tri-quasi ideals of a semigroup M, then $\mu \cap \lambda$ is a fuzzy tri-quasi ideal of M.*

Proof. Let μ and λ be fuzzy tri-quasi ideals of M. $l, m \in M$. Then

$$\begin{aligned}\mu \cap \lambda(lm) &= \min\{\mu(lm), \lambda(lm)\}\\ &\geq \min\{\min\{\mu(l), \mu(m)\}, \min\{\lambda(l), \lambda(m)\}\}\\ &= \min\{\min\{\mu(l), \lambda(l)\}, \min\{\mu(m), \lambda(m)\}\}\\ &= \min\{\mu \cap \lambda(l), \mu \cap \lambda(m)\}\end{aligned}$$

$$\begin{aligned}\chi_M \circ \mu \cap \lambda(x) &= \sup_{x=pq}\{\min\{\chi_M(p),\mu \cap \lambda(q)\}\}\\ &= \sup_{x=pq}\{\min\{\chi_M(p), \min\{\mu(q), \lambda(q)\}\}\}\\ &= \sup_{x=pq}\{\min\{\min\{\chi_M(p), \mu(q)\}, \min\{\chi_M(p), \lambda(q)\}\}\}\\ &= \min\{\sup_{x=pq}\{\min\{\chi_M(p), \mu(q)\}\}, \sup_{x=pq}\{\min\{\chi_M(p), \lambda(q)\}\}\\ &= \min\{\chi_M \circ \mu(x),\chi_M \circ \lambda(x)\}\}\\ &= \chi_M \circ \mu \cap \chi_M \circ \lambda(x).\end{aligned}$$

Therefore $\chi_M \circ \mu \cap \chi_M \circ \lambda = \chi_M \circ \mu \cap \lambda$. And

$$\begin{aligned}((\mu \cap \lambda) \circ (\mu \cap \lambda))(x) &= \sup_{x=ab}\{\min\{(\mu \cap \lambda)(a), (\mu \cap \lambda)(b)\}\}\\ &= \sup_{x=ab}\{\min\{\min\{\mu(a), \lambda(a)\}, \min\{\mu(b), \lambda(b)\}\}\}\\ &= \sup_{x=ab}\{\min\{\min\{\mu(a), \lambda(a)\}, \sup_{x=ab} \min\{\min\{\mu(b), \lambda(b)\}\}\\ &= \min\{\sup_{x=ab}\{\min\{\mu(a), \mu(b)\}\}, \sup_{x=ab}\{\min\{\lambda(a), \lambda(b)\}\}\}\\ &= \min\{\mu \circ \mu(x), \lambda \circ \lambda(x)\}\\ &= (\mu \circ \mu) \cap (\lambda \circ \lambda(x).\end{aligned}$$

$\chi_M \circ \mu \cap \lambda \circ \mu \cap \lambda(x) = \chi_M \circ \mu \circ \mu \cap \chi_M \circ \lambda \circ \lambda.$

$$\begin{aligned}&(\mu \cap \lambda \circ \mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda \circ \mu \cap \lambda)(x)\\ &= \sup_{x=ab}\{\min\{\mu \circ \mu \cap \lambda \circ \lambda(a), \chi_M \circ \mu \cap \lambda \circ \mu \cap \lambda(b)\}\}\\ &= \sup_{x=ab}\{\min\{\mu \circ \mu(a), \lambda \circ \lambda(a), min\{\chi_M \circ \mu \circ \mu(b), \chi_M \circ \lambda \circ \lambda(b)\}\}\}\\ &= \sup_{x=ab}\{\min\{\min\{\mu \circ \mu(a), \chi_M \circ \mu \circ \mu(b)\}\}, \min\{\lambda \circ \lambda(a), \chi_M \circ \lambda \circ \lambda(b)\}\}\\ &= \min\{\sup_{x=ab}\{\min\{\mu \circ \mu(a), \chi_M \circ \mu \circ \mu(b)\}\},\\ &\qquad \sup_{x=ab}\{\min\{\lambda \circ \lambda(a),\chi_M \circ \lambda \circ \lambda(b)\}\}\}\\ &= \min\{\mu \circ \mu \circ \chi_M \circ \mu \circ \mu(x), \lambda \circ \lambda \circ \chi_M \circ \lambda \circ \lambda(x)\}\\ &= \mu \circ \mu \circ \chi_M \circ \mu \circ \mu \cap \lambda \circ \lambda \circ \chi_M \circ \lambda \circ \lambda(x).\end{aligned}$$

Therefore $\mu \cap \lambda \circ \mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda \circ \mu \cap \lambda = \mu \circ \mu \circ \chi_M \circ \mu \circ \mu \cap \lambda \circ \lambda \circ \chi_M \circ \lambda \circ \lambda$
Hence $\mu \cap \lambda \circ \mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda \circ \mu \cap \lambda = \mu \circ \mu \circ \chi_M \circ \mu \circ \mu \cap \lambda \circ \lambda \circ \chi_M \circ \lambda \circ \lambda \subseteq \mu \cap \lambda$.
Hence, $\mu \cap \lambda$ is a fuzzy tri-quasi ideal of M. □

Theorem 4.7. *Let M be a semigroup. If μ and λ are fuzzy tri-quasi ideals of M, then $\mu \cup \lambda$ is a fuzzy tri-quasi ideal of M.*

Proof. Let μ and λ be fuzzy tri-quasi ideals of M. $l, m \in M$. Then

$$\begin{aligned}\mu \cup \lambda(xy) &= \max\{\mu(xy), \lambda(xy)\}\\ &\geq \max\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\}\\ &\geq \min\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\}\\ &= \min\{\mu \cup \lambda(x), \mu \cup \lambda(y).\}\end{aligned}$$

$$\begin{aligned}
\chi_M \circ (\mu \cup \lambda)(x) &= \sup_{x=ab}\{\min\{\chi_M(a), \mu(b) \cup \lambda(b)\}\}\\
&= \sup_{x=ab}\{\min\{\chi_M(a), \max\{\mu(b), \lambda(b)\}\}\}\\
&= \sup_{x=ab}\{\max\{\min\{\chi_M(a), \mu(b)\}, \min\{\chi_M(a), \lambda(b)\}\}\}\\
&\geq \max\{\sup_{x=ab}\{\min\{\chi_M(a), \mu(b)\}\}, \sup_{x=ab}\{\min\{\chi_M(a), \lambda(b)\}\}\}\\
&\geq \max\{\chi_M \circ \mu(x), \chi_M \circ \lambda(x)\}\\
&= (\chi_M \circ \mu \cup \chi_M \circ \lambda(x).
\end{aligned}$$

Thus $\chi_M \circ \mu \cup \chi_M \circ \lambda \leq \chi_M \circ \mu \cup \lambda$.

$$\begin{aligned}
((\mu \cup \lambda) \circ (\mu \cup \lambda))(x) &= \sup_{x=ab}\{\min\{(\mu \cup \lambda)(a), (\mu \cup \lambda)(b)\}\}\\
&= \sup_{x=ab}\{\min\{(\mu \circ \mu)(a) \cup (\lambda \circ \lambda)(b)\}\}\\
&= \sup_{x=ab}\{\min\{\max\{\mu(a), \lambda(a)\}, \{\max\{\mu(b), \lambda(b)\}\}\}\\
&\geq \sup_{x=ab}\{\min\{\max\{\mu(a), \lambda(a)\}\}, \sup_{x=ab}\{\min\{\max\{\mu(b), \lambda(b)\}\}\}\\
&= \max\{\sup_{x=ab}\{\min\{\mu(a), \mu(b)\}, \sup_{x=ab}\{\min\{\lambda(a), \lambda(b)\}\}\\
&= \max\{\mu \circ \mu(x)\}, \{\lambda \circ \lambda(x)\}\\
&= ((\mu \circ \mu) \cup (\lambda \circ \lambda))(x).
\end{aligned}$$

$$\begin{aligned}
&\chi_M \circ (\mu \cup \lambda) \circ (\mu \cup \lambda)(x)\\
&= \sup_{x=ab}\{\min\{\chi_M(a), (\mu \cup \lambda) \circ (\mu \cup \lambda)(b)\}\}\\
&= \sup_{x=ab}\{\min\{\chi_M(a), (\mu \circ \mu)(b) \cup (\lambda \circ \lambda)(b)\}\}\\
&= \sup_{x=ab}\{\min\{\chi_M(a), \max\{(\mu \circ \mu), (\lambda \circ \lambda)(b)\}\}\\
&= \sup_{x=ab}\{\min\{\max\{\chi_M(a), \chi_M(a)\}, \{\max\{(\mu \circ \mu)(b), (\lambda \circ \lambda)(b)\}\}\}\\
&\geq \sup_{x=ab}\{\max\{\min\{\chi_M(a), (\mu \circ \mu)(b)\}, \min\{\chi_M(a), (\lambda \circ \lambda)(b)\}\}\}\\
&\geq \max\{\sup_{x=ab}\{\min\{\chi_M(a), (\mu \circ \mu)(b)\}\}, \sup_{x=ab}\{\min\{\chi_M(a), (\lambda \circ \lambda)(b)\}\}\}\\
&= \max\{\chi_M \circ \mu \circ \mu(x), \chi_M \circ \lambda \circ \lambda(x)\}\\
&= ((\chi_M \circ \mu \circ \mu) \cup (\chi_M \circ \lambda \circ \lambda))(x).
\end{aligned}$$

Thus

$$(\chi_M \circ \mu \circ \mu) \cup (\chi_M \circ \lambda \circ \lambda) \leq \chi_M \circ (\mu \cup \lambda) \circ (\mu \cup \lambda).$$

Then, $(\mu \cup \lambda) \circ (\mu \cup \lambda) \circ \chi_M \circ (\mu \cup \lambda) \circ (\mu \cup \lambda)(x)$

$$\begin{aligned}
&\leq \sup_{x=abc} \min\{\mu \circ \mu \cup \lambda \circ \lambda(a), \chi_M \circ \mu \cup \lambda \circ \mu \cup \lambda(bc)\}\\
&= \sup_{x=abc} \max\{\min\{\mu \circ \mu(a), \chi_M \circ \mu \circ \mu(bc)\}, \min\{\lambda \circ \lambda(a), \chi_M \circ \lambda \circ \lambda(bc)\}\}\\
&= \max\Big\{\sup_{x=abc} \min\{\mu \circ \mu(a), \chi_M \circ \mu \circ \mu(bc),\}\\
&\sup_{x=abc} \min\{\lambda \circ \lambda(a), \chi_M \circ \lambda \circ \lambda(bc)\}\Big\}\\
&= \max\{\mu \circ \mu \circ \chi_M \circ \mu \circ \mu(x), \lambda \circ \lambda \circ \chi_M \circ \lambda \circ \lambda(x)\}
\end{aligned}$$

$= \mu \circ \mu \circ \chi_M \circ \mu \circ \mu \cup \lambda \circ \lambda \circ \chi_M \circ \lambda \circ \lambda(x)$.
Then, $\mu \cup \lambda \circ \mu \cup \lambda \circ \chi_M \circ \mu \cup \lambda \circ \mu \cup \lambda \subseteq \mu \circ \mu \circ \chi_M \circ \mu \circ \mu \cup \lambda \circ \lambda \circ \chi_M \circ \lambda \circ \lambda \subseteq \mu \cup \lambda$.
Hence, $\mu \cup \lambda$ is a fuzzy tri-quasi ideal of M. □

Theorem 4.8. *Let M be a semigroup. Then M is regular if and only if $\mu = \mu \circ \mu \circ \chi_M \circ \mu \circ \mu$, for any fuzzy tri-quasi ideal μ of M.*

Proof. Let μ be a fuzzy tri-quasi ideal of the regular semigroup M
and $x, y \in M$. Then $\mu \circ \mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$. And

$$\begin{aligned}
\mu \circ \mu \circ \chi_M \circ \mu \circ \mu(x) &= \sup_{x=xyx} \{\min\{\mu \circ \mu \circ (x), \chi_M \circ \mu \circ \mu(yx)\}\} \\
&\geq \sup_{x=xyx} \{\min\{\mu(x), \mu(yx)\}\} \\
&\geq \mu(xyx) \\
&= \mu(x).
\end{aligned}$$

Therefore $\mu \subseteq \mu \circ \mu \circ \chi_M \circ \mu \circ \mu$. Hence, $\mu \circ \mu \circ \chi_M \circ \mu \circ \mu = \mu$.

Conversely, suppose that $\mu = \mu \circ \mu \circ \chi_M \circ \mu \circ \mu$, for any fuzzy tri-quasi ideal μ of M.
Let B be a tri-quasi ideal of the semigroup M.
Then, χ_B is a fuzzy tri-quasi ideal of M.

$$\begin{aligned}
\text{Therefore } \chi_B &= \chi_B \circ \chi_B \circ \chi_M \circ \chi_B \circ \chi_B \\
&= \chi_{BBMBB} \\
B &= BBMBB.
\end{aligned}$$

Therefore, M is a regular semigroup. □

Theorem 4.9. *Let M be a semigroup. If μ is a fuzzy left tri-ideal of M, then μ is a fuzzy tri-quasi ideal of M.*

Proof. Suppose μ is a fuzzy left tri-ideal of M, then
$\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$. Let $z \in M$.

$$\begin{aligned}
\mu \circ \mu \circ \chi_M \circ \mu \circ \mu(z) &= \sup_{z=lm} \{\min\{\mu(l), \mu \circ \chi_M \circ \mu \circ \mu(m)\}\}\ l, m \in M, \\
&\leq \sup_{z=lm} \{\min\{\mu(l), \mu(m)\}\} \\
&= \{\mu \circ \mu(z)\} \\
&\leq \mu(z).
\end{aligned}$$

Therefore, $\mu \circ \mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$. Hence, μ is a fuzzy tri-quasi ideal of M. □

Theorem 4.10. *Let M be a semigroup. If μ is a fuzzy (right) tri-ideal of M, then μ is a fuzzy tri-quasi ideal of M.*

Theorem 4.11. *Let M be a semigroup. If μ is a fuzzy left-ideal of M, then μ is a fuzzy bi-ideal of M.*

Proof. Suppose μ is a fuzzy left-ideal of M. Then μ is a left-ideal of M. Let $z \in M$,

$$\begin{aligned}\chi_M \circ \mu(z) &= \sup_{z=lm} \{\min\{\chi_M(l), \mu(m)\}\} \\ &= \sup_{z=lm} \{\min\{1, \mu(m)\}\} \\ &= \sup_{z=lm} \{\mu(m)\} \\ &\leq \sup_{z=lm} \{\mu(lm)\} \\ &= \mu(z)\end{aligned}$$

$$\begin{aligned}\mu \circ \chi_M \circ \mu(z) &= \sup_{z=lmp} \{\min\{\mu(l), \chi_M \circ \mu(mp)\}\}\ l, m, p \in M. \\ &\leq \sup_{z=lmp} \{\min\{\mu(l), \mu(mp)\}\} \\ &= \mu(z)\end{aligned}$$

$$\Rightarrow \mu \circ \chi_M \circ \mu(z) \leq \mu(z)$$

Therefore, $\mu \circ \chi_M \circ \mu \subseteq \mu$. Hence, μ is a fuzzy bi-ideal of M. □

Theorem 4.12. *Let M be a semigroup. If μ is a fuzzy bi-ideal of M, then μ is a fuzzy tri-quasi ideal of M.*

Proof. Suppose μ is a fuzzy bi-ideal of M. Then
$\mu \circ \chi_M \circ \mu \subseteq \mu$. Let $z \in M$,

$$\begin{aligned}\mu \circ \chi_M \circ \mu \circ \mu(z) &= \sup_{z=lm} \{\min\{\mu \circ \chi_M \circ \mu(l), \mu(m)\}\}\ l, m \in M. \\ &\leq \sup_{z=lm} \{\min\{\mu(l), \mu(m)\}\} \\ &= \mu \circ \mu(lm) \leq \mu(z).\end{aligned}$$

Therefore, μ is a fuzzy left tri-ideal of M. Now

$$\begin{aligned}\mu \circ \mu \circ \chi_M \circ \mu \circ \mu(z) &= \sup_{z=lm} \{\min\{\mu(l), \mu \circ \chi_M \circ \mu \circ \mu(m)\}\}\ l, m \in M. \\ &\leq \sup_{z=lm} \{\min\{\mu(l), \mu(m)\}\} \\ &= \mu \circ \mu(lm) \leq \mu(z).\end{aligned}$$

Hence, μ is a fuzzy tri-quasi ideal of M. □

Theorem 4.13. *Let M be a regular semigroup. If μ is a fuzzy set of A then,*

$$\begin{aligned}&(i)\ \mu \circ \mu = \mu \\ &(ii)\ \chi_M \circ \mu = \mu \\ &(iii)\ \mu \circ \chi_M = \mu, \textit{ for all } a \in A.\end{aligned}$$

Proof. Suppose M is a regular semigroup, $z \in M$. Then there exists $x \in M$, such that $z = zxz$.

$$\begin{aligned}(i)\ \mu \circ \mu(z) &= \sup_{z=zxz} \{\min\{\mu(z), \mu(xz)\}\} \\ &= \mu(z).\end{aligned}$$

Therefore, $\mu \circ \mu = \mu$.

$$\begin{aligned}(ii)\ \chi_M \circ \mu(z) &= \sup_{z=zxz} \{\min\{\chi_M(zx), \mu(z)\}\} \\ &= \mu(z).\end{aligned}$$

Similarly, $\mu \circ \chi_M = \mu$. □

Theorem 4.14. *Let M be a regular semigroup. If μ is a fuzzy tri-quasi ideal of M, then μ is a fuzzy right tri-ideal of M.*

Proof. Suppose μ is a fuzzy tri-quasi ideal of M. Let M be a regular semigroup and $z \in M$. Then there exists $x \in M$, such that $z = zxz$.

$$\mu \circ \mu \circ \chi_M \circ \mu \circ \mu(z) \leq \mu(z).$$

$$\Rightarrow \sup_{z=zxz} \min\{\mu \circ \mu \circ \chi_M(zx), \mu \circ \mu(z)\} \leq \mu(z)$$

we have, $\mu \circ \mu(z) = \mu(z)$. *By Theorem* 4.13

$$\Rightarrow \sup_{z=zxz} \min\{\mu \circ \mu \circ \chi_M(zx), \mu(z)\} \leq \mu(z)$$

$$\Rightarrow \mu \circ \mu \circ \chi_M \circ \mu(z) \leq \mu(z).$$

Therefore, $\mu \circ \mu \circ \chi_M \circ \mu \subseteq \mu$.

Hence, μ is a fuzzy right tri-ideal of M. □

5. Conclusion

As a further generalization of ideals, we introduced the notion of tri-quasi ideals and fuzzy tri-quasi ideals of a semigroup. We characterized the regular semigroup in terms of fuzzy tri-quasi ideals of a semigroup and studied some of their algebraic properties. In the continuity of this paper, we study prime tri-quasi ideals, maximal and minimal tri-quasi ideals, and fuzzy soft tri-quasi ideals of semigroups.

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MARAPUREDDY MURALI KRISHNA RAO
DEPARTMENT OF MATHEMATICS, SANKETHIKA INSTITUTE OF TECH. AND MANAGEMENT, VISAKHAPATNAM-530 041, A.P., INDIA.
Email address: mmarapureddy@gmail.com

RAJENDRA KUMAR KONA
DEPT. OF MATHEMATICS, GIS, GITAM (DEEMED TO BE UNIVERSITY), VISAKHAPATNAM- 530 045, A.P., INDIA.
Email address: rkkona1972@gmail.com

NOORBHASHA RAFI
DEPARTMENT OF MATHEMATICS, BAPATLA ENGINEERING COLLEGE, BAPATLA-522 101, A. P., INDIA.
Email address: rafimaths@gmail.com

VENKATESWARLU BOLINENI
DEPT. OF MATHEMATICS, GIS, GITAM (DEEMED TO BE UNIVERSITY), BENGALURU-561 203, KARNATAKA, INDIA.
Email address: bvlmaths@gmail.com