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SOME RESULTS ON MULTIDIMENSIONAL FIXED POINT THEOREMS IN PARTIALLY ORDERED GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

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ABSTRACT. In this paper, using the idea of a coincidence point for nonlinear mappings in any number of variables, we study a fuzzy contractivity condition to ensure the existence of coincidence points in the framework of generalized intuitionistic fuzzy metric spaces. Recently, many authors have conducted in-depth research on coupling, triple and quadruple fixed point theorems in the context of partially ordered complete metric spaces with different contractive conditions. In partially ordered generalized intuitionistic fuzzy metric spaces, we demonstrate several theorems regarding multidimensional co-incidence points and common fixed points for ϕ -compatible systems.

1. INTRODUCTION

The fuzzy set was released in 1965 by the pioneer scientist Zadeh [18] as a class of objects with a continuum of grades of membership. After Zadeh's paper [18], many scientists employed the notion of fuzzy sets in many subjects of sciences such as fuzzy metric space, fuzzy topology, fuzzy decisions, fuzzy set theory etc. One of the most hot topics in mathematics is fuzzy metric spaces, which is introduced by George and Veeramani [4]. In 2004, Park [9] released the notion of intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces in sense of George and Veeramani [4]. While, Sun and Yang [12] extended the definition of fuzzy metric space to the notion of a generalized fuzzy metric space. Abbas et al. [1] introduced a new concept, called A - metric spaces.

Moreover Abbas et al. [1] studied the properties of A-metric spaces and investigated many interesting fixed point theorems. While, Gupta and Kanwar [13] extended the notion of A - metric spaces to the notion of V-fuzzy metric spaces. Moreover, Gupta and Kanwar [13] employed the notion of partially ordered V-fuzzy metric spaces to invistigate some coupled fixed point theorems in sense of Bhaskar and Lakshmikantham [17]. Very recently, Gupta et al. [14] studied some coupled fixed point theorem for mappings satisfying $CLR_{\Omega}-$prperty on the notion of V-fuzzy metric spaces. In 2016, Jeyaraman at el. [5] brought the notion of generalized intuitionistic fuzzy metric spaces and stated

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their properties. In 2014, Roldan, Martinez-Moreno et al. [6, 7] studied some multidimensional coincidence point results in partially ordered fuzzy metric spaces for compatible mappings. For some work in fuzzy metric space, we refer the reader to [2, 3, 6, 10, 11, 15]. We study and establish a few standard fixed point theorems for ϕ-compatible in extended intuitionistic fuzzy metric spaces in this paper. Additionally, we demonstrate some findings about multidimensional coincidenceand established fixed point theorems for ϕ-compatible in partially ordered generalized intuitionistic fuzzy metric spaces.

2. Preliminaries

In order to state and prove our results, we will use the following notions. These notions can be found in [7]. Consider a partition $\{A, B\}$ of $\wedge_p = 1, 2, \dots, p$; that is, $A \cup B = \wedge_p$ and $A \cap B = \emptyset$ such that A and B are non-empty sets. Define
$\Omega_{A,B} = \{\sigma : \wedge_p \to \wedge_p : \sigma(A) \subseteq A$ and $\sigma(B) \subseteq B\}$ and
$\Omega^{'}_{A,B} = \{\sigma : \wedge_p \to \wedge_p : \sigma(A) \subseteq B$ and $\sigma(B) \subseteq A\}$.

Let $(X, \preceq)$ be a partially ordered set, $a, b \in X$ and $i \in \wedge_p$. In the present paper, we will use the following notation:

$$a \preceq_i b \Leftrightarrow \begin{cases} a \preceq b & \text{if } i \in A \\ a \succeq b & \text{if } i \in B. \end{cases}$$

Let $a_1, a_2, \dots, a_n \in [0, 1]$. In the rest of this paper, we use the following notations:
$*_{i=1}^{n} a_i = a_1 * a_2 * \dots * a_n$ and $\diamond_{i=1}^{n} a_i = a_1 \diamond a_2 \diamond \dots \diamond a_n$.

Definition 2.1 [5]
Let X be a nonempty set and V, W be fuzzy sets on $X^n \times (0, \infty)$.
For all $a_1, a_2, a_3, \dots, a_n, l \in X$ and $t, s > 0$, assume the following conditions hold:

(i) $V(a_1, a_2, a_3, \dots, a_n, t) + W(a_1, a_2, a_3, \dots, a_n, t) \leq 1$,
(ii) $V(a, a, a, \dots, a, b, t) > 0$ for all $a, b \in X$ with $a \neq b$,
(iii) $V(a_1, a_1, a_1, ..., a_1, a_2, t) \geq V(a_1, a_2, a_3, \dots, a_n, t)$,
for all $a_1, a_2, a_3, \dots, a_n \in X$ with $a_2 \neq a_3 \neq \dots \neq a_n$,
(iv) $V(a_1, a_2, a_3, \dots, a_n, t) = 1$ if and only if $a_1 = a_2 = a_3 = \dots = a_n$,
(v) $V(a_1, a_2, a_3, ..., a_n, t) = V(p(a_1, a_2, a_3, ..., a_n), t)$ where p is a permutation function,
(vi) $V(a_1, a_2, a_3, ..., a_n, t + s) \geq V(a_1, a_2, a_3, ..., a_{n-1}, l, t) * V(l, l, l, ..., a_n, s)$,
(vii) $V(a_1, a_2, a_3, ..., a_n, .) : (0, \infty) \to [0, 1]$ is continuous,
(viii) V is a non-decreasing function on R^+,
$\lim_{t\to\infty} V(a_1, a_2, a_3, ..., a_n, t) = 1$ and
$\lim_{t\to 0} V(a_1, a_2, a_3, ..., a_n, t) = 0$
for all $a_1, a_2, a_3, \dots, a_n \in X$, $t > 0$,
(ix) $W(a, a, a, \dots, a, b, t) < 1$ for all $a, b \in X$ with $a \neq b$,
(x) $W(a_1, a_1, a_1, ..., a_1, a_2, t) \leq W(a_1, a_2, a_3, \dots, a_n, t)$ for all $a_1, a_2, a_3, \dots, a_n \in X$ with $a_2 \neq a_3 \neq \dots \neq a_n$,
(xi) $W(a_1, a_2, a_3, \dots, a_n, t) = 0$ if and only if $a_1 = a_2 = a_3 = \dots = a_n$,
(xii) $W(a_1, a_2, a_3, ..., a_n, t) = W(p(a_1, a_2, a_3, ..., a_n), t)$ where p is a permutation function,
(xiii) $W(a_1, a_2, a_3, ..., a_n, t + s) \geq$
$W(a_1, a_2, a_3, ..., a_{n-1}, l, t) \diamond W(l, l, l, ..., a_n, s)$,
(xiv) $W(a_1, a_2, a_3, ..., a_n, .) : (0, \infty) \to [0, 1]$ is continuous,

(xv) W is a non-increasing function on R^+,
$\lim_{t\to\infty} W(a_1, a_2, a_3, ..., a_n, t) = 0$ and $\lim_{t\to 0} V(a_1, a_2, a_3, ..., a_n, t) = 1$
for all $a_1, a_2, a_3, \ldots, a_n \in X$, $t > 0$.

Then the triple (*X, V, W,*$*$, $\diamond$) is said to be a generalized intuitionistic fuzzy metric space (shortly GIFMS) and the pair (V, W) is called a generalized intuitionistic fuzzy metric spaces.

Example 2.2[5]
Let (X, A) be an A - metric space. For all $a_1, a_2, a_3, \ldots, a_n \in X$ and every $t > 0$. Define the fuzzy sets V, W on $X^n \times \ (0, \infty)$ by
$V(a_1, a_2, a_3, \ldots\ , a_n, t) = \frac{t}{t+A(a_1,a_2,a_3,\ldots,a_n)}$, and
$W(a_1, a_2, a_3, \ldots, a_n, t) = \frac{A(a_1,a_2,a_3,\ldots,a_n)}{t+A(a_1,a_2,a_3,\ldots,a_n)}$. Also, define $*$ and $\diamond$ via $a * b = ab$ and $a \diamond b = min\{a + b, 1\}$. Then $(X, V, W, *, \diamond)$ is a generalized intuitionistic fuzzy metric spaces.

Definition 2.3[16]
Consider the two self mappings R and S on a partially ordered set $(X^p, \preceq)$. We say that R is $S-$ isotone map, if for any $A_1, A_2 \in X^p, S(A_1) \preceq S(A_2) \Rightarrow R(A_1) \preceq R(A_2)$.

Definition 2.4[8]
Consider the two mappings $F : X^p \to X$ and $g : X \to X$ on a partially ordered set $(X, \preceq)$. We say that F has the mixed g-monotone property if it satisfies the following property:
For all $a_1, a_2, \ldots, a_p, b, c \in X$ and for every i,
$g(b) \preceq g(c) \Rightarrow F(a_1, \ldots, a_{i-1}, b, a_{i+1}, \ldots, a_p) \preceq_i F(a_1, \ldots, a_{i-1}, c, a_{i+1}, \ldots, a_p)$.

Definition 2.5
Let $(X, V, W, *, \diamond)$ be a GIFMS. The two self mappings R and S on X are said to self-mappings if and only if

$$\lim_{p\to\infty} V(R(S(x_p)), \ldots, R(S(x_p)), S(R(x_p)), t) = 1$$

and

$$\lim_{p\to\infty} W(R(S(x_p)), \ldots, R(S(x_p)), S(R(x_p)), t) = 0$$

for all $t > 0$, whenever $\{x_p\} \in X$ such that $\lim_{p\to\infty} R(x_p) = \lim_{p\to\infty} S(x_p) = x$ for some $x \in X$.

Definition 2.6
Let $(X, V, W, *, \diamond)$ be a GIFMS and $(X, \preceq)$ be a partially ordered set. Let $\phi = (\sigma_1, \sigma_2, \ldots, \sigma_p)$ be an $p-$tuple mappings from $\{1, 2, \ldots, p\}$ to itself. The mappings $F : X^p \to X$ and $g : X \to X$ are said to be $\phi-$ compatible if $\{a_{r1}\}, \{a_{r2}\}, \ldots, \{a_{rp}\}$ are monotonic sequences in X with

$$\lim_{r\to\infty} F(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}) = \lim_{r\to\infty} ga_{ri} \in X$$

then

$$\lim_{r\to\infty} V \begin{bmatrix} gF(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}), \ldots, \\ gF(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}), \\ F(ga_{r\sigma_i(1)}, ga_{r\sigma_i(2)}, \ldots, ga_{r\sigma_i(p)}), t \end{bmatrix} = 1$$

and

$$\lim_{r\to\infty} W \begin{bmatrix} gF(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}), \ldots, \\ gF(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}), \\ F(ga_{r\sigma_i(1)}, ga_{r\sigma_i(2)}, \ldots, ga_{r\sigma_i(p)}), t \end{bmatrix} = 0$$

for all $t > 0$ and $i \in \{1, 2, \ldots, p\}$.

3. Main Results

In this section, we set inaugurate main results and make use of these results to acquire the multidimensional results in partially ordered GIFMS.

Lemma 3.1

Let $(X, V, W, *, \diamond)$ be a GIFMS and $\{v_p\}$ a sequence in $(X, V, W, *, \diamond)$. Assume there exists a function $\phi \in \Phi_w$ be such that

(3.1.1) $\phi(t) > 0$, for all $t > 0$,
(3.2.2) $V(v_p, \ldots, v_p, v_{p+1}, \phi(t)) \geq V(v_{p-1}, \ldots, v_{p-1}, v_p, t)$ and
$W(v_p, \ldots, v_p, v_{p+1}, \phi(t)) \leq W(v_{p-1}, \ldots, v_{p-1}, v_p, t)$ for every $p \in N$.

Then $\{v_p\}$ is Cauchy.

Proof. Let $(X, V, W, *, \diamond)$ be a GIFMS. We have,

$$\lim_{t\to\infty} V(v_1, v_2, v_3, \ldots, v_n, t) = 1, \ \lim_{t\to\infty} W(v_1, v_2, v_3, \ldots, v_n, t) = 0.$$

So, for $\varepsilon > 0$, there exist $t_0 > 0$ such that
$V(v_0, v_0, \ldots, v_0, v_1, t_0) > 1 - \varepsilon$ and $W(v_0, v_0, \ldots, v_0, v_1, t_0) < \varepsilon$.

Since $\phi \in \Phi_w$, there exists a real number t_1 with $t_1 \geq t_0$ such that $\lim_{p\to\infty} \phi^p(t_1) = 0$.

Therefore, for $t > 0$, there is $p_0 \in N$ such that $\lim_{p\to\infty} \phi^p(t_1) \leq t$ for all $p \geq p_0$. Condition (3.1.1) implies that $\phi^p(t) > 0$ for all $p \in N$ and $t > 0$.

By induction and condition (3.1.2), we have
$V(v_p, v_p, \ldots, v_p, v_{p+1}, \phi^p(t)) \geq V(v_0, v_0, \ldots, v_0, v_1, t)$ and
$W(v_p, v_p, \ldots, v_p, v_{p+1}, \phi^p(t)) \leq W(v_0, v_0, \ldots, v_0, v_1, t)$ for all $p \in N$ and $t > 0$.
Since V and W are non-decreasing and non-increasing, we have

$$\begin{aligned}
V(v_p, v_p, \ldots, v_p, v_{p+1}, t) &\geq V(v_p, v_p, \ldots, v_p, v_{p+1}, \phi^p(t_1)) \\
&\geq V(v_0, v_0, \ldots, v_0, v_1, t_1) \\
&\geq V(v_0, v_0, \ldots, v_0, v_1, t_0) \\
&> 1 - \varepsilon \text{ and} \\
W(v_p, v_p, \ldots, v_p, v_{p+1}, t) &\leq W(v_p, v_p, \ldots, v_p, v_{p+1}, \phi^p(t_1)) \\
&\leq W(v_0, v_0, \ldots, v_0, v_1, t_1) \\
&\leq W(v_0, v_0, \ldots, v_0, v_1, t_0) \\
&< \varepsilon
\end{aligned}$$

that is, as $p \to \infty$, we have
$V(v_p, v_p, \ldots, v_p, v_{p+1}, t) \to 1$ and $W(v_p, v_p, \ldots, v_p, v_{p+1}, t) \to 0$ for any $\varepsilon > 0$ and

$t > 0$. For $q \in N$ and $t > 0$ we have

$$\begin{aligned} V(v_p, \ldots, v_p, v_{p+q}, t) &\geq V\left(v_p, \ldots, v_p, v_{p+1}, \frac{t}{q}\right) \\ &* V\left(v_{p+1}, \ldots, v_{p+1}, v_{p+2}, \frac{t}{q}\right) \\ &* \cdots * V\left(v_{p+q-1}, \ldots, v_{p+q-1}, v_{p+q}, \frac{t}{q}\right) \text{ and} \\ W(v_p, \ldots, v_p, v_{p+q}, t) &\leq W\left(v_p, \ldots, v_p, v_{p+1}, \frac{t}{q}\right) \\ &\diamond W\left(v_{p+1}, \ldots, v_{p+1}, v_{p+2}, \frac{t}{q}\right) \\ &\diamond \cdots \diamond W\left(v_{p+q-1}, \ldots, v_{p+q-1}, v_{p+q}, \frac{t}{q}\right). \end{aligned}$$

Letting $p \to \infty$, we get
$V(v_p, v_p, \ldots, v_p, v_{p+q}, t) \geq 1 * 1 * \cdots * 1 = 1$ and
$W(v_p, v_p, \ldots, v_p, v_{p+q}, t) \leq 0 \diamond 0 \diamond \cdots \diamond 0 = 0$.
Hence the sequence $\{v_p\}$ is a Cauchy sequence.

Theorem 3.2

Let $(X, V, W, *, \diamond)$ be a complete GIFMS, $(X, \preceq)$ be a partially ordered set. Assume R and S be two mappings on a set X be such that

(3.2.1) $R(X) \subseteq S(X)$.
(3.2.2) R is a S-isotone mapping.
(3.2.3) If there exists a function $\phi \in \Phi_w$, such that
$V(R(x), \ldots, R(x), R(y), \phi(t) \geq V(S(x), \ldots, S(x), S(y), t)$ and
$W(R(x), \ldots, R(x), R(y), \phi(t) \leq W(S(x), \ldots, S(x), S(y), t)$ for all $x, y \in X$, $t > 0$ and $S(x) \preceq S(y)$.
(3.2.4) R and S are continuous and compatible maps.

If there exists $x_0 \in X$ such that $S(x_0) \approx R(x_0)$, then R and S have a coincidence point.

Proof. Let $x_0 \in X$ be a point such that $S(x_0) \approx R(x_0)$.
Since $R(X) \subseteq S(X)$, we choose $x_1 \in X$ be such that
$S(x_1) = R(x_0)$.
Proceeding like this way, we construct a sequence $\{x_p\} \in X$ be such that
$S(x_{p+1}) = R(x_p)$ for $p \in N \cup \{0\}$. Without loss of generality and in view of $S(x_0) \approx R(x_0)$, we assume that $S(x_0) \preceq R(x_0)$.

Assume that $S(x_{p-1}) \preceq S(x_p)$. The property of S-isotone for the mapping R implies that $R(x_{p-1}) \preceq R(x_p)$. We set $S(x_0) = v_0 \preceq R(x_0) = v_1$ and
$R(x_{p-1}) = v_p \preceq R(x_p) = v_{p+1}$.

Thus, the sequence $\{v_p\}$ is an increasing sequence.

From (3.2.3), we get

$$\begin{aligned} V(v_p,\ldots,v_p,v_{p+1},\phi(t)) &= V(R(x_{p-1}),\ldots,R(x_{p-1}),R(x_p),\phi(t)) \\ &\geq V(S(x_{p-1}),\ldots,S(x_{p-1}),S(x_p),t) \\ &= V(v_{p-1},\ldots,v_{p-1},v_p,t) \text{ and} \\ W(v_p,\ldots,v_p,v_{p+1},\phi(t)) &= W(R(x_{p-1}),\ldots,R(x_{p-1}),R(x_p),\phi(t)) \\ &\leq W(S(x_{p-1}),\ldots,S(x_{p-1}),S(x_p),t) \\ &= W(v_{p-1},\ldots,v_{p-1},v_p,t) \end{aligned}$$

for all $p \in N \cup \{0\}$ and $t > 0$. Clearly $\phi(t) > 0$ for all $t > 0$. From Lemma (3.1.), we conclude that $\{v_p\}$ is a Cauchy sequence.

Since $(X,V,W,*,\diamond)$ is a complete GIFMS, there exists a point $v \in X$ such that $\lim_{p\to\infty} v_p = v$ that is, $\lim_{p\to\infty} R(x_p) = \lim_{p\to\infty} S(x_p) = v$.

Since R and S are compatible, we have

$$\lim_{p\to\infty} V(R(S(x_p)),\ldots,R(S(x_p)),S(R(x_p)),t) = 1,$$

$$\lim_{p\to\infty} W(R(S(x_p)),\ldots,R(S(x_p)),S(R(x_p)),t) = 0$$

for all $t > 0$.

Since R and S are both continuous mappings,
$V(R(v),R(v),\ldots,R(v),S(v),t) = 1$ and
$W(R(v),R(v),\ldots,R(v),S(v),t) = 0$
for all $t > 0$, which implies that, $S(v) = R(v)$.
Thus v is a coincidence point of R and S in X.

Theorem 3.3

Let $(X,\preceq)$ be a partially ordered set, $(X,V,W,*,\diamond)$ be a complete GIFMS. Assume the two self mappings R and S on X satisfying the following conditions:

(3.3.1) $R(X) \subseteq S(X)$.
(3.3.2) R is a S-isotone mapping.
(3.3.3) Assume that there exists a function $\phi \in \Phi_w$, such that
$V(R(x),\ldots,R(x),R(y),\phi(t) \geq V(S(x),\ldots,S(x),S(y),t)$ and
$W(R(x),\ldots,R(x),R(y),\phi(t) \leq W(S(x),\ldots,S(x),S(y),t)$ for all $x,y \in X$, $t > 0$ and $S(x) \preceq S(y)$.
(3.3.4) X has the following property,
  (a) If $\{x_p\}$ is a non-decreasing sequence such that $x_p \to x$ then $x_p \leq x$ for all $p \in N$.
  (b) If $\{x_p\}$ is a non-increasing sequence such that $x_p \to x$ then $x_p \geq x$ for all $p \in N$.
(3.3.5) $S(X)$ is closed.

If there exists $x_0 \in X$ such that $S(x_0) \approx R(x_0)$, then R and S have a coincidence point.

Proof. Let $x_0 \in X$ be a point such that $S(x_0) \approx R(x_0)$. In view of $R(X) \subseteq S(X)$, we choose $x_1 \in X$ such that $S(x_1) = R(x_0)$.
Proceeding like this way, we construct a sequence $\{x_p\} \in X$ such that $S(x_{p+1}) = R(x_p)$ for $p \in N \cup \{0\}$. Without loss of generality and in view of $S(x_0) \approx R(x_0)$, we may assume that $S(x_0) \preceq R(x_0)$.

Assume that $S(x_{p-1}) \preceq S(x_p)$. The property of S-isotone for the mapping R implies that $R(x_{p-1}) \preceq R(x_p)$.

Putting $S(x_0) = v_0 \preceq R(x_0) = v_1$ and $R(x_{p-1}) = v_p \preceq R(x_p) = v_{p+1}$.
Thus, the sequence $\{v_p\}$ is an increasing sequence. From (3.3.3), we get

$$\begin{aligned} V(v_p, \ldots, v_p, v_{p+1}, \phi(t)) &= V(R(x_{p-1}), \ldots, R(x_{p-1}), R(x_p), \phi(t)) \\ &\geq V(S(x_{p-1}), \ldots, S(x_{p-1}), S(x_p), t) \\ &= V(v_{p-1}, \ldots, v_{p-1}, v_p, t) \text{ and} \\ W(v_p, \ldots, v_p, v_{p+1}, \phi(t)) &= W(R(x_{p-1}), \ldots, R(x_{p-1}), R(x_p), \phi(t)) \\ &\leq W(S(x_{p-1}), \ldots, S(x_{p-1}), S(x_p), t) \\ &= W(v_{p-1}, \ldots, v_{p-1}, v_p, t) \end{aligned}$$

for all $p \in N \cup \{0\}$ and $t > 0$. Clearly $\phi(t) > 0$ for all $t > 0$. Lemma (3.1), guarantee that $\{v_p\}$ is Cauchy.

Since $(X, V, W, *, \diamond)$ is a complete GIFMS and $S(X)$ is closed, there exists $v_0 \in X$ such that

$$\lim_{p \to \infty} R(x_p) = \lim_{p \to \infty} S(x_p) = S(v_0) = v.$$

Since $(S(x_p))$ is a non-decreasing sequence, we have $S(x_p) \leq S(v_0)$ for all $p \in N$.

In $(X, V, W, *, \diamond)$, V is non-decreasing and W is non-increasing with respect to t.

Also, for every $t > 0$ there exists a real number r with $r \geq t$ such that $\phi(r) < t$. Hence,

$$\begin{aligned} V(R(x_p), \ldots, R(x_p), R(v_0), t) &\geq V(R(x_p), \ldots, R(x_p), R(v_0), \phi(r)) \\ &\geq V(S(x_p), \ldots, S(x_p), S(v_0), r) \\ &\geq V(S(x_p), \ldots, S(x_p), S(v_0), t) \text{ and} \\ W(R(x_p), \ldots, R(x_p), R(v_0), t) &\leq W(R(x_p), \ldots, R(x_p), R(v_0), \phi(r)) \\ &\leq W(S(x_p), \ldots, S(x_p), S(v_0), r) \\ &\leq W(S(x_p), \ldots, S(x_p), S(v_0), t) \end{aligned}$$

for all $t > 0$ and $p \in N$. Taking $p \to \infty$ in above inequality, we get $R(x_p) \to R(v_0)$. The uniqueness of the limit indicate that $R(v_0) = S(v_0)$.

Hence v_0 is a coincidence point of R and S.

Theorem 3.4

In additional of the hypothesis of Theorem 3.2 and Theorem 3.3, suppose that X is a totally ordered set. Then R and S have a unique coincidence point. Moreover, if S is weakly compatible with R, then R and S have unique common fixed point.

Proof. Assume that $u, v \in X$ are coincidence points of R and S. Let $y \in X$ be such that $S(y)$ is comparable to $S(u)$ and $S(v)$. Starting with $y_0 = y$ to construct a sequence $S(y_p)$. The sequence $S(y_p)$ and its limit is defined in similar way as in Theorem 3.2 and Theorem 3.3 So we have $S(y_{p+1}) = R(y_p)$ and $S(y_1) = R(y_0)$. Also, we have

$$\lim_{t \to \infty} V(S(y), S(y), \ldots, S(y), S(v), t) = 1$$

and

$$\lim_{t \to \infty} W(S(y), S(y), \ldots, S(y), S(v), t) = 0,$$

which imply that for any $\varepsilon \in (0, 1)$, there exists t_1 such that
$V(S(y_0), S(y_0), \ldots, S(y_0), S(v), t_1) > 1 - \varepsilon$ and $W(S(y_0), S(y_0), \ldots, S(y_0), S(v), t_1) < \varepsilon$.

Referring to the fact that $\phi \in \Phi_w$, we choose $r \geq t_1$ be such that $\lim_{p\to\infty} \phi^p(r) = 0$. So, there exists $p_0 \in N$ such that $\phi^p(r) < t$ for all $p \geq p_0$ and $t > 0$. Consider,

$$\begin{aligned}
V(S(y_p), \ldots, S(y_p), S(v), t) &\geq V(S(y_p), \ldots, S(y_p), S(v), \phi^p(r)) \\
&= V(R(y_{p-1}), \ldots, R(y_{p-1}), R(v), \phi^p(r)) \\
&\geq V(S(y_{p-1}), \ldots, S(y_{p-1}), S(v), \phi^{p-1}(r)) \\
&\geq \cdots \geq V(S(y_0), \ldots, S(y_0), S(v), r) \\
&\geq V(S(y_0), \ldots, S(y_0), S(v), t_1) \\
&\geq 1 - \varepsilon \text{ and} \\
W(S(y_p), \ldots, S(y_p), S(v), t) &\leq W(S(y_p), \ldots, S(y_p), S(v), \phi^p(r)) \\
&= W(R(y_{p-1}), \ldots, R(y_{p-1}), R(v), \phi^p(r)) \\
&\leq W(S(y_{p-1}), \ldots, S(y_{p-1}), S(v), \phi^{p-1}(r)) \\
&\leq \cdots \leq W(S(y_0), \ldots, S(y_0), S(v), r) \\
&\leq W(S(y_0), \ldots, S(y_0), S(v), t_1) \\
&\leq \varepsilon
\end{aligned}$$

for all $p \geq p_0$ and $t > 0$.

Hence,$\{\lim_{p\to\infty} S(y_p)\} = S(v)$. Similarly, we can easily show that $\{\lim_{p\to\infty} S(y_p)\} = S(u)$. The uniqueness of limit guarantee that $S(v) = S(u)$.

Now, let $R(v) = S(v) = e$. Since R and S are weakly compatible mappings, we have $R(e) = R(S(v)) = S(R(v)) = S(e)$. So e is a coincidence point.

Hence $S(e) = S(v) = e$. Thus e is a common fixed point of R and S . Now suppose that there exists $e^{'}(\neq e) \in X$ such that $R\left(e^{'}\right) = S\left(e^{'}\right) = e^{'}$.

Then $e = S(e) = S\left(e^{'}\right) = e'$ which indicate the uniqueness of common fixed point of R and S.

Example 3.5

Let $X = [0,\ 1]$ and $(X, \preceq)$ be a partially ordered set. Let R and S be two self mappings on X defined by $R(x) = \frac{x^2}{2} + \frac{1}{2}$ and $S(x) = x$ for all $x \in X$.

It is clear that $R(X) \subseteq S(X)$ and *R* is an *S*-isotone mapping. Let $\phi(t) = \frac{t}{2}$ for all $t > 0$.

Define V, W on $X^p \times (0, \infty)$ via

$V(x_1,\ x_2,\ x_3,\ \ldots,\ x_p, t) = \frac{t}{t + A(x_1,\ x_2,\ x_3,\ \ldots,\ x_p)}$

and

$W(x_1, x_2, x_3, \ldots, x_p, t) = \frac{A(x_1, x_2, x_3, \ldots, x_p)}{t + A(x_1,\ x_2,\ x_3,\ \ldots, x_p)}$,

where

$$\mathrm{A}(x_1,\ x_2,\ x_3,\ \ldots,\ x_p) = \sum_{i=1}^{p} \sum_{i<j} |x_i - y_j|$$

for all $x_1,\ x_2,\ x_3,\ \ldots,\ x_p \in X$ and $t > 0$.

Let $x * y = \min\{x, y\}$ and $x \diamond y = \max\{x, y\}$ for all $x, y \in X$. Then $(X, V, W, *, \diamondsuit)$ is a complete GIFMS.

Now,

$$V\left(R\left(x\right),R\left(x\right),\ldots,R\left(x\right),R\left(y\right),\phi\left(t\right)\right)=\frac{\phi(t)}{\phi\left(t\right)+\left|R\left(x\right)-R(y)\right|}$$
$$=\frac{t}{t+\left|x^2-y^2\right|}\text{ and}$$
$$W\left(R\left(x\right),R\left(x\right),\ldots,R\left(x\right),R\left(y\right),\phi\left(t\right)\right)=\frac{\left|R\left(x\right)-R(y)\right|}{\phi\left(t\right)+\left|R\left(x\right)-R(y)\right|}$$
$$=\frac{\left|x^2-y^2\right|}{t+\left|x^2-y^2\right|}.$$

So $V\left(S\left(x\right),S\left(x\right),\ldots,S\left(x\right),S\left(y\right),t\right)=\frac{t}{t+|x-y|}$ and
$W\left(S\left(x\right),S\left(x\right),\ldots,S\left(x\right),S\left(y\right),t\right)=\frac{|x-y|}{t+|x-y|}$
for all $x,y\in X$ and $t>0$. Hence, we have

$$V\left(R\left(x\right),R\left(x\right),\ldots,R\left(x\right),R\left(y\right),\phi\left(t\right)\right)\geq V\left(S\left(x\right),S\left(x\right),\ldots,S\left(x\right),S\left(y\right),t\right)\text{ and}$$
$$W\left(R\left(x\right),R\left(x\right),\ldots,R\left(x\right),R\left(y\right),\phi\left(t\right)\right)\leq W\left(S\left(x\right),S\left(x\right),\ldots,S\left(x\right),S\left(y\right),t\right).$$

Taking $x_0=0$. We have $S\left(x_0\right)=0\leq R(x_0)$.
Now, construct the sequence $v_0=S(x_0)$ and $x_{p+1}=S\left(x_{p+1}\right)=R(x_p)$ for $p\in N\cup\{0\}$.
So $\{v_p\}=\{v_0=0,v_1=\frac{1}{2},v_2=\frac{5}{8},v_3=\frac{89}{128},\ldots\}$ this sequence is non-trivial.
By Theorem 3.4, we get R and S have a unique common fixed point v.

Lemma 3.6

Let $(X,V,W,*,\diamond)$ be a GIFMS such that $*$ is a continuous *t*-norm and $\diamond$ is a continuous *t*-conorm. Define the fuzzy sets

$$V^p,W^p:\ X^p\times\ldots n-times\times X^p\times(0,\infty)\rightarrow[0,1]$$

such that
$V^p\left(A_1,A_2,\ldots,A_n,t\right)=*_{i=1}^{p}V(a_{1i},a_{2i},\ldots,a_{ni},t)$ and
$W^p\left(A_1,A_2,\ldots,A_n,t\right)=\diamond_{i=1}^{p}W(a_{1i},a_{2i},\ldots,a_{ni},t)$
for all $A_i=(a_{i1},a_{i2},\ldots,a_{ip})\in X^p$ and
for all $t>0$, where $i\in\{1,2,\ldots,n\}$. Then the following properties hold:

(3.6.1) $\left(X^p,V^p,W^p,\ *,\ \diamond\right)$ is also a GIFMS.
(3.6.2) Let $\{A_r=(a_{r1},a_{r2},\ldots,a_{rp})\}$ be a sequence on X^p and a point $A=(a_1,a_2,\ldots,a_p)\in X^p$. Then $\{A_r\}\rightarrow A$ if and only if $\{a_{ri}\}\rightarrow a_i$ for all $i\in\{1,2,\ldots,p\}$.
(3.6.3) If (*X, V, W*, $*$, $\diamond$) is complete, then $\left(X^p,V^p,W^p,\ *,\ \diamond\right)$ is also complete.

Proof. (3.6.1) Suppose that $(X,\ V,W,\ *,\ \diamond)$ is a GIFMS. Then, $\left(X^p,V^p,W^p,\ *,\ \diamond\right)$ satisfies all properties of the definition of generalized intuitionistic fuzzy metric space. Hence $\left(X^p,V^p,W^p,\ *,\ \diamond\right)$ is also a generalized intuitionistic fuzzy metric space.

(3.6.2) Suppose $\{A_r\}\rightarrow A$ as $r\rightarrow\infty$. For $\varepsilon\in(0,1)$, there exists $n_0\in N$ such that for all $r\geq n_0$, we have $V^p\left(A_r,A_r,..,A_r,A,t\right)\geq1-\varepsilon$ and $W^p\left(A_r,A_r,..,A_r,A,t\right)\leq\varepsilon$ for all $t>0$.
Consider,

$$\min_{1\leq i\leq p}V\left(a_{ri},a_{ri},\ldots,a_{ri},a_i,t\right)$$

$$\begin{aligned}
&= V(a_{r1}, a_{r1}, \ldots, a_{r1}, a_1, t) \\
&\quad * V(a_{r2}, a_{r2}, \ldots, a_{r2}, a_2, t) \\
&\quad * \cdots * V(a_{rp}, a_{rp}, \ldots, a_{rp}, a_p, t) \\
&= *_{i=1}^{p} V(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) \\
&= V^p(A_r, A_r, \ldots, A_r, A, t) \\
&\geq 1 - \varepsilon
\end{aligned}$$

and

$$\begin{aligned}
&\max_{1 \leq i \leq p} W(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) \\
&= W(a_{r1}, a_{r1}, \ldots, a_{r1}, a_1, t) \\
&\quad \diamond W(a_{r2}, a_{r2}, \ldots, a_{r2}, a_2, t) \\
&\quad \diamond \cdots \diamond W(a_{rp}, a_{rp}, \ldots, a_{rp}, a_p, t) \\
&= \diamond_{i=1}^{p} W(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) \\
&= W^p(A_r, A_r, \ldots, A_r, A, t) \\
&\leq \varepsilon
\end{aligned}$$

for all $r \geq n_0$ and $t > 0$.
Thus, for all $r \geq n_0$, $V(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) \geq 1 - \varepsilon$ and $W(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) \leq \varepsilon$ for all $i \in \{1, 2, \ldots, p\}$.
Hence, $\{a_{ri}\} \to \{a_i\}$ as $r \to \infty$.

Conversely suppose that $\{a_{ri}\} \to \{a_i\}$ as $r \to \infty$ for all $i \in \{1, 2, \ldots, p\}$ and $*, \diamond$ are continuous mapping. From the definitions of V^p and W^p, we get

$$\begin{aligned}
&\lim_{r \to \infty} V^p(A_r, A_r, \ldots, A_r, A, t) \\
&= \lim_{r \to \infty} *_{i=1}^{p} V(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) \\
&= *_{i=1}^{p} \lim_{r \to \infty} V(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) = 1
\end{aligned}$$

and

$$\begin{aligned}
&\lim_{r \to \infty} W^p(A_r, A_r, \ldots, A_r, A, t) \\
&= \lim_{r \to \infty} \diamond_{i=1}^{p} W(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) \\
&= \diamond_{i=1}^{p} \lim_{r \to \infty} W(a_{ri}, a_{ri}, \ldots, a_{ri}, a_i, t) = 0.
\end{aligned}$$

Hence $\{A_r\} \to A$ as $r \to \infty$.

(3.6.3) Suppose that $\{A_r\}$ is a Cauchy sequence in $(X^p, V^p, W^p, *, \diamond)$; that is, for every $\varepsilon \in (0, 1)$, there exists $n_0 \in N$ such that
$V^p(A_r, A_r, \ldots, A_r, A_n, t) \geq 1 - \varepsilon$ and
$W^p(A_r, A_r, \ldots, A_r, A_n, t) \leq \varepsilon$
for all $r, n \geq n_0$ and $t > 0$. Thus,

$$\min_{1 \leq i \leq p} V(a_{ri}, a_{ri}, \ldots, a_{ri}, a_{ni}, t)$$

$$
\begin{aligned}
&= V\left(a_{r1}, a_{r1}, \ldots, a_{r1}, a_{n1}, t\right) \\
&\quad * V\left(a_{r2}, a_{r2}, \ldots, a_{r2}, a_{n2}, t\right) \\
&\quad * \cdots * V\left(a_{rp}, a_{rp}, \ldots, a_{rp}, a_{np}, t\right) \\
&= *_{i=1}^{p} V\left(a_{ri}, a_{ri}, \ldots, a_{ri}, a_{ni}, t\right) \\
&= V^p(A_r, A_r, \ldots, A_r, A_n, t) \\
&\geq 1-\varepsilon
\end{aligned}
$$

and

$$
\begin{aligned}
&\max_{1\leq i\leq p} W\left(a_{ri}, a_{ri}, \ldots, a_{ri}, a_{ni}, t\right) \\
&= W\left(a_{r1}, a_{r1}, \ldots, a_{r1}, a_{n1}, t\right) \\
&\quad \diamond W\left(a_{r2}, a_{r2}, \ldots, a_{r2}, a_{n2}, t\right) \\
&\quad \diamond \cdots \diamond W\left(a_{rp}, a_{rp}, \ldots, a_{rp}, a_{np}, t\right) \\
&= \diamond_{i=1}^{p} W\left(a_{ri}, a_{ri}, \ldots, a_{ri}, a_{ni}, t\right) \\
&= W^p(A_r, A_r, \ldots, A_r, A_n, t) \\
&\leq \varepsilon
\end{aligned}
$$

for all $r \geq n_0$ and $t > 0$. Hence. we deduce that $\{a_{ri}\}$ is a Cauchy sequence in $(X,\ V,\ W,\ *,\ \diamond)$ for all $i \in \{1, 2, \ldots, p\}$. Now, let $(X,\ V,\ W,\ *,\ \diamond)$ be a complete generalized intuitionistic fuzzy metric space,

That is, $\{a_{ri}\} \to \{a_i\}$ for every $i \in \{1, 2, \ldots, p\}$ and $a_i \in X$ which implies that sequence $\{A_r\}$ converges to a point A on X^p. Hence $\left(X^p, V^p, W^p,\ *,\ \diamond\right)$ is also a complete generalized intuitionistic fuzzy metric space.

Lemma 3.7

Let $(X,\ V,\ W,\ *,\ \diamond)$ be a GIFMS. If the mappings $F : X^p \to X$ and $g : X \to X$ are $\phi-$compatible mappings on $(X,\ V,\ W,\ *,\ \diamond)$, then $R :\ X^p \to X^p$ and $S : X^p \to X^p$ are also compatible in $\left(X^p, V^p, W^p,\ *,\ \diamond\right)$, where R and S are defined as

$$
\begin{aligned}
R(A) = \Big(& F(a_{\sigma_1(1)}, a_{\sigma_1(2)}), \ldots, a_{\sigma_1(p)}), \ldots, \\
& F\left(a_{\sigma_i(1)}, a_{\sigma_i(2)}, \ldots, a_{\sigma_i(p)}\right), \ldots, \\
& F\left(a_{\sigma_p(1)}, a_{\sigma_p(2)}, \ldots, a_{\sigma_p(p)}\right) \Big) \text{ and}
\end{aligned}
$$

$S\left(A\right) = \left(ga_1, ga_2, \ldots, ga_p\right)$
for all $A = (a_1, a_2, \ldots, a_p) \in X^p$ and $a_i \in X$, $i \in \{1, 2, \ldots, p\}$.

Proof. Let $\{a_{r1}\}, \{a_{r2}\}, \ldots, \{a_{rp}\}$ are monotonic sequence in X such that

$$\lim_{r\to\infty} ga_{ri} = \lim_{r\to\infty} F(a_{r\sigma_i(1)},\ a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)})$$

for all $a_i \in X$ and $i \in \{1, 2, \ldots, p\}$. By Lemma 3.6. $\lim_{r\to\infty} S(A_r) = \{\lim_{r\to\infty} R(A_r)\}$, where $A_r = (\{a_{r1}\}, \{a_{r2}\}, \ldots, \{a_{rp}\}) \in X^p$. Since F and g are $\phi-$compatible and $*,\ \diamond$ are continuous mapping, we get

$$\lim_{r\to\infty} V^p\left(S\left(R\left(A_r\right)\right), , \ldots, S\left(R\left(A_r\right)\right), R\left(S\left(A_r\right)\right), t\right)$$

$$\begin{aligned}
&= \lim_{r\to\infty} *_{i=1}^{p} V\Big(gF\left(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}\right), \\
&\qquad \ldots, gF\left(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}\right), \\
&\qquad F\left(ga_{r\sigma_i(1)}, ga_{r\sigma_i(2)}, , \ldots, ga_{r\sigma_i(p)}\right), t\Big) \\
&= *_{i=1}^{p} \lim_{r\to\infty} V\Big(gF\left(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}\right), \\
&\qquad \ldots, gF\left(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}\right), \\
&\qquad F\left(ga_{r\sigma_i(1)}, ga_{r\sigma_i(2)}, , \ldots, ga_{r\sigma_i(p)}\right), t\Big) \\
&= 1
\end{aligned}$$

and

$$\begin{aligned}
&\lim_{r\to\infty} W^p\left(S\left(R\left(A_r\right)\right), , \ldots, S\left(R\left(A_r\right)\right), R\left(S\left(A_r\right)\right), t\right) \\
&= \lim_{r\to\infty} \diamond_{i=1}^{p} W\Big(gF\left(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}\right), \\
&\qquad \ldots, gF\left(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}\right), \\
&\qquad F\left(ga_{r\sigma_i(1)}, ga_{r\sigma_i(2)}, , \ldots, ga_{r\sigma_i(p)}\right), t\Big) \\
&= \diamond_{i=1}^{p} \lim_{r\to\infty} W\Big(gF\left(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}\right), \\
&\qquad \ldots, gF\left(a_{r\sigma_i(1)}, a_{r\sigma_i(2)}, \ldots, a_{r\sigma_i(p)}\right), \\
&\qquad F\left(ga_{r\sigma_i(1)}, ga_{r\sigma_i(2)}, , \ldots, ga_{r\sigma_i(p)}\right), t\Big) \\
&= 0 \quad for\ all \quad t > 0.
\end{aligned}$$

Hence R and S are compatible mapping in
$(X^p, V^p, W^p, *, \diamond)$.

Theorem 3.8

Let $(X,\ V,\ W,\ *,\ \diamond)$ be a GIFMS and $(X, \preceq)$ be a partially ordered set. Let $\{A, B\}$ be any partition of $\wedge_p = \{1, 2, \ldots, p\}$ and $\phi = (\sigma_1, \sigma_2, \ldots, \sigma_p)$ an p-tuple mappings from $\wedge_p$ into itself verifying that $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega^{'}_{A,B}$ if $i \in B$. Let $F : X^p \to X$ and $g : X \to X$ be two mappings such that

(3.8.1) $F(X^p) \subseteq g(X)$.

(3.8.2) F has the mixed g-monotone property on X.

(3.8.3) Assume that there exists a function $\phi \in \Phi_w$ such that

$$\begin{aligned}
&V\Big(F\left(y_1, y_2, \ldots, y_p\right), \ldots, F\left(y_1, y_2, \ldots, y_p\right), \\
&\qquad\qquad F(z_1, z_2, \ldots, z_p), \phi\left(t\right)\Big) \\
&\geq \gamma\Big(*_{i=1}^{p} V(gy_i, gy_i, \ldots, gy_i, gz_i, t)\Big)
\end{aligned}$$

and

$$W\Big(F(y_1, y_2, \ldots, y_p), \ldots, F(y_1, y_2, \ldots, y_p), F(z_1, z_2, \ldots, z_p), \phi(t)\Big) \leq \gamma\Big(\diamond_{i=1}^{p} W(gy_i, gy_i, \ldots, gy_i, gz_i, t)\Big)$$

for all $t > 0, y_1, y_2, \ldots, y_p, z_1, z_2, \ldots, z_p$ in X and $gy_i \preceq_i gz_i$ for all $i \in \{1, 2, \ldots, p\}$, where $\gamma : [0,1] \to [0,1]$ be such that $*_{i=1}^{p}\gamma(u) \geq u$ and $\diamond_{i=1}^{p}\gamma(u) \leq u$ for all $u \in [0,1]$. Suppose that

$$\gamma\Big(*_{i=1}^{p} V\left(gy_{\sigma_j(i)}, gy_{\sigma_j(i)}, \ldots, gy_{\sigma_j(i)}, gz_{\sigma_j(i)}, t\right)\Big) \geq \gamma\Big(*_{i=1}^{p} V\left(gy_i, gy_i, \ldots, gy_i, gz_i, t\right)\Big) \text{ and}$$

$$\gamma\Big(\diamond_{i=1}^{p} W\left(gy_{\sigma_j(i)}, gy_{\sigma_j(i)}, \ldots, gy_{\sigma_j(i)}, gz_{\sigma_j(i)}, t\right)\Big) \leq \gamma\Big(\diamond_{i=1}^{p} W\left(gy_i, gy_i, \ldots, gy_i, gz_i, t\right)\Big)$$

for $i, j \in \{1, 2, \ldots, p\}$,
$y_1, y_2, \ldots, y_p, z_1, z_2, \ldots, z_p \in X$ and
$gy_i \preceq_i gz_i$.

(3.8.4) F and g are continuous and $\phi-$compatible.

If there exists $y_{01}, y_{02}, \ldots, y_{0p} \in X$ satisfying $gy_{0i} \preceq_i F\left(y_{0\sigma_i(1)}, y_{0\sigma_i(2)}, \ldots, y_{0\sigma_i(p)}\right)$ for $i \in \{1, 2, \ldots, p\}$, then F and g have an $\phi-$coincidence point.

Proof: Let $(X,\ V,\ W,\ *,\ \diamond)$ be a GIFMS such that $*,\ \diamond$ are continuous t-norm, continuous t-conorm and $(X, \preceq)$ be a partially ordered set. By Lemma (3.6.), we get $(X^p, V^p, W^p,\ *,\ \diamond)$ is also a generalized intuitionistic fuzzy metric space. Define mappings $R : X^p \to X^p$ and $S : X^p \to X^p$ as

$$R(Y) = \Big(F\left(y_{\sigma_1(1)}, y_{\sigma_1(2)}, \ldots, y_{\sigma_1(p)}\right), \ldots, F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(p)}\right), \ldots, F\left(y_{\sigma_p(1)}, y_{\sigma_p(2)}, \ldots, y_{\sigma_p(p)}\right)\Big)$$

and $S(Y) = (gy_1, gy_2, \ldots, gy_p)$ (3.8.5)
for all $Y \in X^p$ and $y_i \in X$ for $i \in \{1, 2, \ldots, p\}$.

Since $F(X^p) \subseteq g(X)$ which implies that $R(X^p) \subseteq S(X^p)$.

Suppose $gy_{0i} \preceq_i F\left(y_{0\sigma_i(1)}, y_{0\sigma_i(2)}, \ldots, y_{0\sigma_i(p)}\right)$,
then there exists $Y_0 \in X^p$ such that $S(Y_0) \preceq_p R(Y_0)$.
Now to prove that R is a $S-$ isotone. Let $Y, Z \in X^p$ be such that $S(Y) \preceq_p S(Z)$ which implies that $gy_j \preceq gz_j$ when $j \in A$ and $gy_j \succeq gz_j$ when $j \in B$. So, we have
$\sigma_i \in \Omega_{A,B} = \{\sigma : \wedge_p \to \wedge_p :\ \sigma(A) \subseteq A \textit{ and } \sigma(B) \subseteq B\}$ when $i \in A$ and
$\sigma_i \in \Omega'_{A,B} = \{\sigma : \wedge_p \to \wedge_p :\ \sigma(A) \subseteq B \textit{ and } \sigma(B) \subseteq A\}$ when $i \in B$.

Thus, $gy_{\sigma_i(j)} \preceq gz_{\sigma_i(j)}$ when $j \in A$ and $gy_{\sigma_i(j)} \succeq gz_{\sigma_i(j)}$ when $j \in B$, for fixed $i \in A$. Since F is mixed g-monotone, when $j \in A$ and fixed $i \in A$, we have

$F\left(y_{\sigma_i(1)},\ldots,y_{\sigma_i(j-1)},y_{\sigma_i(j)},y_{\sigma_i(j+1)},\ldots,y_{\sigma_i(p)}\right)$
$\preceq F(y_{\sigma_i(1)},\ldots,y_{\sigma_i(j-1)},z_{\sigma_i(j)},y_{\sigma_i(j+1)},\ldots,y_{\sigma_i(p)})$.

Similarly, when $j\in B$ and fixed $i\in A$ the above inequality hold. Hence for fixed $i\in A$ and for all j the above inequality also hold. Thus we get,
$F\left(y_{\sigma_i(1)},y_{\sigma_i(2)},\ldots,y_{\sigma_i(p)}\right)\preceq$
$F\left(u_{\sigma_i(1)},u_{\sigma_i(2)},\ldots,u_{\sigma_i(p)}\right)$ for $i\in A$ and
$F\left(y_{\sigma_i(1)},y_{\sigma_i(2)},\ldots,y_{\sigma_i(p)}\right)\succeq$
$F\left(u_{\sigma_i(1)},u_{\sigma_i(2)},\ldots,u_{\sigma_i(p)}\right)$ for $i\in B$
Thus, $R(Y)\preceq_p R(Z)$ for $Y,Z\in X^p$. Hence R is an $S-$ isotone mapping.

Now, $Y,Z\in X^p$ we have $S(Y)\preceq_p S(Z)$. From Partially ordered set $(X,\preceq)$, we get $(y_{\sigma(1)},y_{\sigma(2)},\ldots,y_{\sigma(p)})$ and $(u_{\sigma(1)},u_{\sigma(2)},\ldots,u_{\sigma(p)})$ are comparable.
In the view of these points and using (3.8.5) , then for all $t>0$, we have
$V^p\left(R(Y),R(Y),\ldots,R(Y),R(Z),\phi(t)\right)$

$$\begin{aligned}
&= *_{i=1}^{p}V(F\left(y_{\sigma_i(1)},y_{\sigma_i(2)},\ldots,y_{\sigma_i(p)}\right),\\
&\qquad \ldots F\left(y_{\sigma_i(1)},y_{\sigma_i(2)},\ldots,y_{\sigma_i(p)}\right),\\
&\qquad F\left(z_{\sigma_i(1)},z_{\sigma_i(2)},\ldots,z_{\sigma_i(p)}\right),\ \phi(t))\\
&\geq *_{i=1}^{p}\gamma\left(*_{j=1}^{p}V\left(gy_{\sigma_i(j)},\ldots,gy_{\sigma_i(j)},gz_{\sigma_i(j)},t\right)\right)\\
&\geq *_{i=1}^{p}\gamma\left(*_{j=1}^{p}V\left(gy_j,gy_j,\ldots,gy_j,gz_j,t\right)\right)\\
&\geq *_{i=1}^{p}\gamma\left(V^p\left(S(Y),S(Y),\ldots,S(Y),S(Z),t\right)\right)\\
&\geq V^p\left(S(Y),S(Y),\ldots,S(Y),S(Z),t\right)
\end{aligned}$$

and
$W^p\left(R(Y),R(Y),\ldots,R(Y),R(Z),\phi(t)\right)$

$$\begin{aligned}
&= \diamond_{i=1}^{p}W(F\left(y_{\sigma_i(1)},y_{\sigma_i(2)},\ldots,y_{\sigma_i(p)}\right),\\
&\qquad \ldots F\left(y_{\sigma_i(1)},y_{\sigma_i(2)},\ldots,y_{\sigma_i(p)}\right),\\
&\qquad F\left(z_{\sigma_i(1)},z_{\sigma_i(2)},\ldots,z_{\sigma_i(p)}\right),\ \phi(t))\\
&\leq \diamond_{i=1}^{p}\gamma\left(\diamond_{j=1}^{p}W\left(gy_{\sigma_i(j)},\ldots,gy_{\sigma_i(j)},gz_{\sigma_i(j)},t\right)\right)\\
&\leq \diamond_{i=1}^{p}\gamma\left(\diamond_{j=1}^{p}W\left(gy_j,gy_j,\ldots,gy_j,gz_j,t\right)\right)\\
&\leq \diamond_{i=1}^{p}\gamma\left(W^p\left(S(Y),S(Y),\ldots,S(Y),S(Z),t\right)\right)\\
&\leq W^p\left(S(Y),S(Y),\ldots,S(Y),S(Z),t\right).
\end{aligned}$$

Hence, condition (3.8.3) of Theorem (3.8) implies the condition (3.2.3) of Theorem (3.2.) with respect to $(X^p,V^p,W^p,\ *,\ \diamond)$.

Also, since F,g are continuous and $\phi-$compatible, then by (3.8.5) we get R and S are continuous. Also, by Lemma (3.7), we deduce that R and S are compatible. Hence condition (3.2.4) of Theorem (3.2) hold with respect to $(X^p,V^p,W^p,\ *,\ \diamond)$.

Hence all the conditions of Theorem (3.2) hold w.r.t. $(X^p,V^p,W^p,\ *,\ \diamond)$. Therefore, R and S have a coincidence point. From (3.8.5), we deduce the coincidence point will be the $\phi-$coincidence point of F and g.

Example 3.9

Let $X=\{0,\frac{1}{2},1,\frac{3}{2},2\}$. Consider X has the following partial order :
$x,y\in X, x\preceq y\Leftrightarrow x=y$ or $(x,y)=(0,1)$. Consider the function,
$F:X^p\to X$ and $g:X\to X$ defined by,

$$F(x_1,x_2,\ldots,x_p)=\begin{cases}0 & \text{if } x_1,\ldots,x_p\in\{0,\frac{1}{2},1\}\\ 1 & \text{if } otherwise.\end{cases}$$

and $g(x) = \begin{cases} 0 & \text{if } x = 0 \\ \frac{1}{2} & \text{if } x \in \{\frac{1}{2}, 1\} \\ 1 & \text{if } x \in \{\frac{3}{2}, 2\} \end{cases}$

Then we can easily get condition, $F(X^p) \subseteq g(X)$.
If $y, z \in X$ verify $gy \preceq gz$ then either $y, z \in \{0, \frac{1}{2}, 1\}$ or $y, z \in \{\frac{3}{2}, 2\}$. Then F has the mixed g-monotone property on X. Let $\phi(t) = \frac{t}{2}$ for all $t > 0$. Define V, W on $X^p \times (0, \infty)$ by
$V(x_1, x_2, \ldots, x_p, t) = \frac{t}{t + A(x_1, x_2, \ldots, x_p)}$
and $W(x_1, x_2, \ldots, x_p, t) = \frac{A(x_1, x_2, \ldots, x_p)}{t + A(x_1, x_2, \ldots, x_p)}$
where $A(x_1, x_2, \ldots, x_p) = \sum_{i=1}^{p} \sum_{i<j} |x_i - y_j|$ for all $x_1,\ x_2,\ x_3,\ \ldots,\ x_p \in X$ and $t > 0$. Let $x * y = \min\{x, y\}$ and $x \diamond y = \max\{x, y\}$ for all $x, y \in X$. Then $(X, V, W, *, \diamond)$ is complete GIFMS, also $(X^p, V^p, W^p, *, \diamond)$ is complete GIFMS.
If $y_1, y_2, \ldots, y_p, z_1, z_2, \ldots, z_p \in X$verify $gy_i \preceq_i gz_i$ for all i, then
$F(y_1, y_2, \ldots, y_p) = F(z_1, z_2, \ldots, z_p)$. Suppose, that p is even and let A be the set of odd numbers in $\wedge_p$ and B be the set of even numbers in $\wedge_p$. Consider $\sigma_i = (i, i+1, \ldots, p-1, p, 1, 2, \ldots, i-1)$ for all i.
Then $\sigma_i \in \Omega_{A,B}$ if i is odd $\sigma_i \in \Omega'_{A,B}$ if i is even. Let $\phi = (\sigma_1.\sigma_2, \ldots, \sigma_p)$ then condition (3.8.3) of Theorem (3.8) holds whatever γ(for instance taking $\gamma(a) = a$ for all $a \in [0, 1]$) with respect to $(X^p, V^p, W^p, *, \diamond)$.
Also F and g are continuous and $\phi-$ compatible. Take $y_{0i} = 0$ if i is odd and $y_{0i} = 1$ if i is even. Then $gy_{0i} \preceq_i F(y_{0\sigma_i(1)}, y_{0\sigma_i(2)}, \ldots, y_{0\sigma_i(p)})$ for all i.
Hence all the conditions of Theorem.3.8 are satisfied. Therefore F and g have ϕ-coincidence point.

Theorem 3.10

Let $(X,\ V,\ W,\ *,\ \diamond)$ be a GIFMS and $(X, \preceq)$ be a partially ordered set. Let $\{A, B\}$ be any partition of $\wedge_p = \{1, 2, \ldots, p\}$ and $\phi = (\sigma_1, \sigma_2, \ldots, \sigma_p)$ an p-tuple mappings from $\wedge_p$ into itself verifying that $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Let $F : X^p \to X$ and $g : X \to X$ be two mappings such that

(1) (3.10.1) $F(X^p) \subseteq g(X)$.
(2) (3.10.2) F has the mixed g-monotone property on X.
(3) (3.10.3) Assume that there exists a function $\phi \in \Phi_w$ such that

$$V\Big(F(y_1, y_2, \ldots, y_p), \ldots, F(y_1, y_2, \ldots, y_p), F(z_1, z_2, \ldots, z_p), \phi(t)\Big) \geq \gamma\Big(*_{i=1}^{p} V(gy_i, gy_i, \ldots, gy_i, gz_i, t)\Big)$$

and

$$W\Big(F(y_1, y_2, \ldots, y_p), \ldots, F(y_1, y_2, \ldots, y_p), F(z_1, z_2, \ldots, z_p), \phi(t)\Big) \leq \gamma\Big(\diamond_{i=1}^{p} W(gy_i, gy_i, \ldots, gy_i, gz_i, t)\Big)$$

for all $t > 0$, $y_1, y_2, \ldots, y_p$, $z_1, z_2, \ldots, z_p$ in X and $gy_i \preceq_i gz_i$ for all $i \in \{1, 2, \ldots, p\}$, where $\gamma : [0, 1] \to [0, 1]$ be such that $*_{i=1}^{p} \gamma(u) \geq u$ and $\diamond_{i=1}^{p} \gamma(u) \leq$

u for all $u \in [0,1]$. Suppose that

$$\gamma\Big(*_{i=1}^{p} V\left(gy_{\sigma_j(i)}, gy_{\sigma_j(i)}, \ldots, gy_{\sigma_j(i)}, gz_{\sigma_j(i)}, t\right) \Big)$$

$$\geq \gamma\Big(*_{i=1}^{p} V\left(gy_i, gy_i, \ldots, gy_i, gz_i, t\right) \Big) \text{ and}$$

$$\gamma\Big(\diamond_{i=1}^{p} W\left(gy_{\sigma_j(i)}, gy_{\sigma_j(i)}, \ldots, gy_{\sigma_j(i)}, gz_{\sigma_j(i)}, t\right) \Big)$$

$$\leq \gamma\Big(\diamond_{i=1}^{p} W\left(gy_i, gy_i, \ldots, gy_i, gz_i, t\right) \Big)$$

for $i, j \in \{1, 2, \ldots, p\}$,

$y_1, y_2, \ldots, y_p, z_1, z_2, \ldots, z_p \in X$ and $gy_i \preceq_i gz_i$.

(4) (3.10.4) X has the following properties:
   (a) If $\{x_p\}$ is a non-decreasing sequence such that $x_p \to x$, then $x_p \leq x$ for all $p \in N$.
   (b) If $\{x_p\}$ is a non-increasing sequence such that $x_p \to x$, then $x_p \geq x$ for all $p \in N$.
(5) (3.10.5) $g(X)$ is closed.

If there exists $y_{01}, y_{02}, \ldots, y_{0p} \in X$ satisfying
$gy_{0i} \preceq_i F\left(y_{0\sigma_i(1)}, y_{0\sigma_i(2)}, \ldots, y_{0\sigma_i(p)}\right)$ for
$i \in \{1, 2, \ldots, p\}$, then F and g have $\phi-$coincidence point.

Proof. Let $(X,\ V,\ W,\ *,\ \diamond)$ be a GIFMS and $(X, \preceq)$ be a partially ordered set. By Lemma (3.6), we get $(X^p, V^p, W^p,\ *,\ \diamond)$ is also a generalized intuitionistic fuzzy metric space. Define the mappings $R: X^p \to X^p$ and $S: X^p \to X^p$ by

$$R(Y) = \Big(F\left(y_{\sigma_1(1)}, y_{\sigma_1(2)}, \ldots, y_{\sigma_1(p)}\right), \ldots,$$
$$F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(p)}\right), \ldots,$$
$$F\left(y_{\sigma_p(1)}, y_{\sigma_p(2)}, \ldots, y_{\sigma_p(p)}\right) \Big)$$

and $S(Y) = (gy_1, gy_2, \ldots, gy_p)$ (3.10.6)
for all $Y \in X^p$ and $y_i \in X$ for $i \in \{1, 2, \ldots, p\}$.

Since $F(X^p) \subseteq g(X)$, we have $R(X^p) \subseteq S(X^p)$.

Suppose $gy_{0i} \preceq_i F\left(y_{0\sigma_i(1)}, y_{0\sigma_i(2)}, \ldots, y_{0\sigma_i(p)}\right)$. Then there exists $Y_0 \in X^p$ such that $S(Y_0) \preceq_p R(Y_0)$.

Now to prove that R is a $S-$ isotone, we let $Y, Z \in X^p$ be such that $S(Y) \preceq_p S(Z)$ which implies that $gy_j \preceq gz_j$ when $j \in A$ and $gy_j \succeq gz_j$ when $j \in B$.Thus
$\sigma_i \in \Omega_{A,B} = \{\sigma : \wedge_p \to \wedge_p :\ \sigma(A) \subseteq A \ and\ \sigma(B) \subseteq B\}$ when $i \in A$ and
$\sigma_i \in \Omega'_{A,B} = \{\sigma : \wedge_p \to \wedge_p :\ \sigma(A) \subseteq B \ and\ \sigma(B) \subseteq A\}$ when $i \in B$. Thus, we have $gy_{\sigma_i(j)} \preceq gz_{\sigma_i(j)}$ when $j \in A$ and $gy_{\sigma_i(j)} \succeq gz_{\sigma_i(j)}$ when $j \in B$, for fixed $i \in A$. Since F is mixed g-monotone, when $j \in A$ and fixed $i \in A$, we have
$F\left(y_{\sigma_i(1)}, \ldots, y_{\sigma_i(j-1)}, y_{\sigma_i(j)}, y_{\sigma_i(j+1)}, \ldots, y_{\sigma_i(p)}\right)$
$\preceq F(y_{\sigma_i(1)}, \ldots, y_{\sigma_i(j-1)}, z_{\sigma_i(j)}, y_{\sigma_i(j+1)}, \ldots, y_{\sigma_i(p)})$.

Similarly, when $j \in B$ and fixed $i \in A$ the above inequality hold. Hence for fixed $i \in A$ and for all j the above inequality also hold. Thus we get,
$F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(p)}\right) \preceq$
$F\left(u_{\sigma_i(1)}, u_{\sigma_i(2)}, \ldots, u_{\sigma_i(p)}\right)$ for $i \in A$ and
$F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(p)}\right) \succeq$
$F\left(u_{\sigma_i(1)}, u_{\sigma_i(2)}, \ldots, u_{\sigma_i(p)}\right)$ for $i \in B$.
Thus, $R(Y) \preceq_p R(Z)$ for $Y, Z \in X^p$. Hence R is a $S-$ isotone mapping.

For $Y, Z \in X^p$, we have $S(Y) \preceq_p S(Z)$. From partially ordered se $(X, \preceq)$, we deduce that
$(y_{\sigma(1)}, y_{\sigma(2)}, \ldots, y_{\sigma(p)})$ and $(u_{\sigma(1)}, u_{\sigma(2)}, \ldots, u_{\sigma(p)})$ are comparable.
In the view of these points and using (3.10.6), we hace for all $t > 0$,

$$\begin{aligned}
&V^p\left(R(Y), R(Y), \ldots, R(Y), R(Z), \phi(t)\right)\\
&\quad= *_{i=1}^{p} V(F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(p)}\right),\\
&\qquad\qquad \ldots F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(p)}\right),\\
&\qquad\qquad F\left(z_{\sigma_i(1)}, z_{\sigma_i(2)}, \ldots, z_{\sigma_i(p)}\right),\ \phi(t))\\
&\quad\geq *_{i=1}^{p} \gamma\left(*_{j=1}^{p} V\left(gy_{\sigma_i(j)}, gy_{\sigma_i(j)}, \ldots, gy_{\sigma_i(j)}, gz_{\sigma_i(j)}, t\right)\right)\\
&\quad\geq *_{i=1}^{p} \gamma\left(*_{j=1}^{p} V\left(gy_j, gy_j, \ldots, gy_j, gz_j, t\right)\right)\\
&\quad\geq *_{i=1}^{p} \gamma\left(V^p\left(S(Y), S(Y), \ldots, S(Y), S(Z), t\right)\right)\\
&\quad\geq V^p\left(S(Y), S(Y), \ldots, S(Y), S(Z), t\right)
\end{aligned}$$

and

$$\begin{aligned}
&W^p\left(R(Y), R(Y), \ldots, R(Y), R(Z), \phi(t)\right)\\
&\quad= \diamond_{i=1}^{p} W(F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(p)}\right),\\
&\qquad \ldots F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(p)}\right),\\
&\qquad F\left(z_{\sigma_i(1)}, z_{\sigma_i(2)}, \ldots, z_{\sigma_i(p)}\right),\ \phi(t))\\
&\quad\leq \diamond_{i=1}^{p} \gamma\left(\diamond_{j=1}^{p} W\left(gy_{\sigma_i(j)}, gy_{\sigma_i(j)}, \ldots, gy_{\sigma_i(j)}, gz_{\sigma_i(j)}, t\right)\right)\\
&\quad\leq \diamond_{i=1}^{p} \gamma\left(\diamond_{j=1}^{p} W\left(gy_j, gy_j, \ldots, gy_j, gz_j, t\right)\right)\\
&\quad\leq \diamond_{i=1}^{p} \gamma\left(W^p\left(S(Y), S(Y), \ldots, S(Y), S(Z), t\right)\right)\\
&\quad\leq W^p\left(S(Y), S(Y), \ldots, S(Y), S(Z), t\right).
\end{aligned}$$

Hence, condition (3.10.3) of Theorem (3.10.)implies the condition (3.3.3) of Theorem (3.3) with respect to $\left(X^p, V^p, W^p, *, \diamond\right)$.

Let $\{Y_r\} \in X^p$ be a non-decreasing sequence such that $Y_r \to Y$ as $r \to \infty$ for some $Y \in X^p$. Lemma (3.6) implies that $y_{ri} \to y_i$ as $r \to \infty$ for all $i \in \{1, 2, \ldots, p\}$. Since $Y_r \preceq_p Y_{r+1}$ for all $r \in N \cup \{0\}$, then $\{y_{ri}\}$ is non-decreasing sequence when $i \in A$ and $\{y_{ri}\}$ is non-increasing sequence when $i \in B$ for all $r \in N \cup \{0\}$. Condition (3.10.4) of Theorem (3.10) implies that $y_{ri} \preceq y_i$ when $i \in A$ and $y_{ri} \succeq y_i$ when $i \in B$ for all $r \in N \cup \{0\}$. So, we have $Y_r \preccurlyeq_p Y$ for all $r \in N \cup \{0\}$. Similarly, by assuming $\{Y_r\} \in X^p$ as a non-increasing sequence, we get that $Y_r \succeq_p Y$ for all $r \in N \cup \{0\}$.

Hence all the conditions of Theorem (3.3) hold with respect to $\left(X^p, V^p, W^p, *, \diamond\right)$. Therefore, R and S have a coincidence point. Condition (3.10.6) implies that this coincidence point is a $\phi-$coincidence point of F and g.

Example 3.11 Let $X = \{0\} \cup [\frac{1}{4}, 4]$. Consider X has the following partial order :
$x, y \in X, x \preceq y \Leftrightarrow x = y$ or $(x, y) = (0, 1)$. Consider the function,
$F : X^p \to X$ and $g : X \to X$ defined by,

$$F(x_1, x_2, \ldots, x_p) = \begin{cases} 0 & \text{if } x_1, \ldots, x_p \in \{0\} \cup [\frac{1}{4}, 2] \\ 1 & \text{if } otherwise. \end{cases}$$

and $$g(x) = \begin{cases} 0 & \text{if } x = 0 \\ \frac{3}{2} & \text{if } \frac{1}{4} \leq x < \frac{3}{2} \\ 4 - x & \text{if } \frac{3}{2} \leq x < \frac{11}{4} \\ \frac{5}{4} & \text{if } \frac{11}{4} \leq x \leq 4. \end{cases}$$

Let $\phi(t) = \frac{t}{2}$ for all $t > 0$. Define V, W on $X^p \times (0, \infty)$ by
$V(x_1, x_2, \ldots, x_p, t) = \frac{t}{t + A(x_1, x_2, \ldots, x_p)}$
and $W(x_1, x_2, \ldots, x_p, t) = \frac{A(x_1, x_2, \ldots, x_p)}{t + A(x_1, x_2, \ldots, x_p)}$
where $A(x_1, x_2, \ldots, x_p) = \sum_{i=1}^{p} \sum_{i<j} |x_i - y_j|$ for all $x_1,\ x_2,\ x_3,\ \ldots,\ x_p \in X$ and $t > 0$. Let $x*y = min\{x, y\}$ and $x \diamond y = max\{x, y\}$ for all $x, y \in X$. Then $(X, V, W, *, \diamond)$ is complete GIFMS, also $(X^p, V^p, W^p, *, \diamond)$ is complete GIFMS. Hence all the conditions of Theorem 3.10 are satisfied, F and g have atleast one ϕ-coincidence point.

4. CONCLUSION

In this article, we proved some results on multidimensional coincidence point and fixed point theorems for ϕ - compatible in partially ordered generalized intuitionistic fuzzy metric spaces. We have presented some suitable examples that support our main results. There is a scope for extending and generalizing various fixed point theorems in the setting of neutrosophic metric space and bipolar metric space. Also, these results can be applied to a non-linear integral equation to obtain the existence and uniqueness solution.

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