ANNALS OF COMMUNICATIONS IN MATHEMATICS
Volume 7, Number 2 (2024), 100-107
DOI: 10.62072/acm.2024.070203
ISSN: 2582-0818
© http://www.technoskypub.com

ALMOST BI-IDEALS AND FUZZY ALMOST BI-IDEALS OF TERNARY SEMIGROUPS

VANITA R. DADDI

ABSTRACT. In this paper, we present the concepts of almost bi-ideals and fuzzy almost bi-ideals in ternary semigroups. The aim is to study their characterizations and establish the relation between different types of ideals in a ternary semigroup with various examples.

1. INTRODUCTION

Like any algebra theory, ideals play a significant role in the theory of ternary semigroups. F.M.Sioson [2] developed the theory of ideals in a ternary semigroup. Generalizing the notion of bi-ideals introduced by R. A. Good and D. R. Hughes [6], a detailed study of quasi-ideals and bi-ideals in a ternary semigroup is carried out by V.N. Dixit and S. Diwan [9,10]. The concept of a fuzzy set was introduced by L. A. Zadeh [4]. Fuzzy algebraic structures have been developed in many fields. S. Kar and P. Sarkar [7] applied the concepts of L. A. Zadeh to define fuzzy ideals of Ternary Semigroups.

The notion of almost ideals of semigroups was introduced and studied by O. Grosek and L.Satko [5] in 1980. Also studied the notions of minimal almost-ideals, maximal almost-ideals. Almost ideals and fuzzy almost ideals of ternary semigroups were studied by S. Suebsung, K. Wattanatripop, R. Chinram [8]. Moreover, they introduced the notion of minimal fuzzy almost ideals of ternary semigroups and studied properties of them. Subsequently, K. Wattanatripop, R. Chinramb, T. Changphas [3] introduced the notion of almost bi-ideals and fuzzy almost bi-ideals in semigroup. Recently, many researchers extended the idea of almost ideals to n-ary semigroups.

In this paper, an attempt is made to define the notions of almost bi-ideal and fuzzy almost bi-ideal of ternary semigroups. The main purpose is to study their characterizations and establish the relation between ideals, bi-ideals, quasi-ideals, almost ideals, almost bi-ideals and fuzzy ideals, fuzzy bi-ideals, fuzzy quasi-ideals, fuzzy almost ideals, fuzzy almost bi-ideals in a ternary Semigroup.

2020 *Mathematics Subject Classification.* 17A40, 20M17, 20M99.

Key words and phrases. ideals; bi-ideals; almost ideals; almost bi-ideals; fuzzy ideals; fuzzy bi-ideals; fuzzy almost bi-ideals.

Received: May 02, 2024. Accepted: June 15, 2024. Published: June 30, 2024.

2. PRELIMINARIES

In this section, we recall some definitions and results which will be used throughout this paper.

Definition 2.1. A non-empty set T together with a ternary operation [ ] defined on T is called a ternary semigroup if [ ] satisfies the associative law. i.e.
$[x_1x_2x_3x_4x_5] = [[x_1x_2x_3]x_4x_5] = [x_1[x_2x_3x_4]x_5] = [x_1x_2[x_3x_4x_5]]$ for all $x_i \in T$, $1 \leq i \leq 5$

For non-empty subsets A, B and C of T, Define
$[ABC] = \{[abc] : a \in A, b \in B, c \in C\}$.
We write $[\{a\}BC] = [aBC], [A\{b\}C] = [AbC], [AB\{c\}] = [ABc]$ and $[AAA] = A^3$.
Due to associative law in T, for non-empty subsets A, B, C, D, E of T. we get $[ABCDE] = [[ABC]DE] = [A[BCD]E] = [AB[CDE]]$

Throughout this paper, T stands for a ternary semigroup with respect to ternary operation [ ] unless otherwise stated.

Definition 2.2. 1) A non-empty subset S of T is a ternary sub-semigroup of T, if $S^3 \subseteq S$.
2) A left (right, lateral) ideal of T is a non-empty subset $L(R, M)$ of T such that $[TTL] \subseteq L$ ($[RTT] \subseteq R$, $[TMT] \subseteq M$).
3) A non-empty subset I of T is a two-sided ideal of T, if it is a left and a right ideal of T.
4) A non-empty subset I of T is an ideal of T, if it is a left, a right and a lateral ideal of T.
5) An ideal I of T is proper, if $I \neq T$.

Definition 2.3. A non-empty subset Q of T is a quasi-ideal of T, if
1) $[QTT] \cap [TQT] \cap [TTQ] \subseteq Q$ and
2) $[QTT] \cap [TTQTT] \cap [TTQ] \subseteq Q$.

Definition 2.4. A ternary sub-semigroup B of T is a bi-ideal of T, if $[BTBTB] \subseteq B$.

Definition 2.5. 1) A non-empty subset L of T is an almost left ideal of T, if $[ttL] \cap L \neq \emptyset$, for all $t \in T$.
2) A non-empty subset M of T is an almost lateral ideal of T, if $[tMt] \cap M \neq \emptyset$, for all $t \in T$.
3) A non-empty subset R of T is an almost right ideal of T, if $[Rtt] \cap R \neq \emptyset$, for all $t \in T$.
4) A non-empty subset I of T is an almost ideal of T, if it is an almost left, right and lateral ideal of T.

Theorem 2.1. *1) Let L be an almost left ideal of T. If A is a subset of T such that $L \subseteq A$, then A is an almost left ideal of T.*
2) Let R be an almost right ideal of T. If A is a subset of T such that $R \subseteq A$, then A is an almost right ideal of T.
3) Let M be an almost lateral ideal of T. If A is a subset of T such that $M \subseteq A$, then A is an almost lateral ideal of T.
4) Let I be an almost ideal of T. If A is a subset of T such that $I \subseteq A$, then A is an almost ideal of T.

Corollary 2.2. *1) If L_1 and L_2 are almost left ideals of T, then $L_1 \cup L_2$ is an almost left ideal of T.*
2) If R_1 and R_2 are almost right ideals of T, then $R_1 \cup R_2$ is an almost right ideal of T.
3) If M_1 and M_2 are almost lateral ideals of T, then $M_1 \cup M_2$ is an almost lateral ideal

of T.
4) If I_1 and I_2 are almost ideals of T, then $I_1 \cup I_2$ is an almost ideal of T.

Definition 2.6. A non-empty subset Q of T is an almost quasi-ideal of T, if
$[Qtt] \cap ([tQt] \cup [ttQtt]) \cap [ttQ] \cap Q \neq \emptyset$, for all $t \in T$.

Definition 2.7. A fuzzy subset of T is a function $f : T \to [0, 1]$

Definition 2.8. Let f and g be two fuzzy subsets of T . Then the union and the intersection of f and g, denoted by $f \cup g$ and $f \cap g$ are fuzzy subsets of T , defined as
$(f \cup g)(x) = max\{f(x), g(x)\}$,
$(f \cap g)(x) = min\{f(x), g(x)\}$ and
$f \subseteq g$, if $f(x) \leq g(x)$ for any $x \in T$.

Definition 2.9. Let f, g and h be fuzzy subsets of T. The product of f, g, h is denoted by $f \circ g \circ h$, is defined as, for any $x \in T$

$$[f \circ g \circ h](x) = \begin{cases} \bigvee\limits_{x=[pqr]} \{f(p) \wedge g(q) \wedge h(r)\}, & x = [pqr], p, q, r \in T \\ 0, & otherwise \end{cases}$$

Definition 2.10. Let f be a fuzzy subset of T, the support of f is defined by
$suppf = \{x \in T : f(x) \neq 0\}$.

Definition 2.11. 1) Let A be a non-empty subset of T, the characteristic mapping of A is a fuzzy subset of T is defined by
$$C_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$
2) Let $t \in T$, the characteristic mapping of $\{t\}$ is a fuzzy subset of T is denoted by $C_{\{t\}} = C_t$ and is defined by
$$C_t(x) = \begin{cases} 1, & x = t \\ 0, & x \notin t \end{cases}$$
3) Let $t \in T$ and $\alpha \in (0, 1]$, the fuzzy point t_α of T is a fuzzy subset of T and is defined by
$$t_\alpha(x) = \begin{cases} \alpha, & x = t \\ 0, & otherwise \end{cases}$$

Proposition 2.3. *Let A, B, D be three non-empty subset of T. Then*
(i) $C_A \cap C_B \cap C_D = C_{A \cap B \cap D}$
(ii) $C_A \circ C_B \circ C_D = C_{ABD}$

Definition 2.12. Let f be a fuzzy subset of T, then for all $x, y, z \in T$
1) f is fuzzy left ideal of T if $f([xyz]) \geq f(z)$
2) f is fuzzy right ideal of T if $f([xyz]) \geq f(x)$
3) f is fuzzy lateral ideal of T if $f([xyz]) \geq f(y)$
4) f is fuzzy ideal of T if it is fuzzy left ideal, fuzzy right ideal and fuzzy lateral ideal of T

Definition 2.13. A fuzzy ternary subsemigroup f of a ternary semigroup T is called a fuzzy bi-ideal of T if $f([uvwxy]) \geq f(u) \wedge f(w) \wedge f(y)$ for all $u, v, w, x, y \in T$.

Theorem 2.4. *Let f be a fuzzy subset of T, then*
1) f is a fuzzy subsemigroup of T if and only if $f \circ f \circ f \subseteq f$,
2) f is a fuzzy left ideal of T if and only if $T \circ T \circ f \subseteq f$,

3)f is a fuzzy right ideal of T if and only if $f \circ T \circ T \subseteq f$,
4) f is a fuzzy lateral ideal of T if and only if $T \circ f \circ T \subseteq f$,
5) f is a fuzzy ideal of T if and only if $T \circ T \circ f \subseteq f$, $f \circ T \circ T \subseteq f$ and $T \circ f \circ T \subseteq f$,
6) f is a fuzzy bi-ideal of T if and only if $f \circ T \circ f \circ T \circ f \subseteq f$

3. Almost bi-ideals in ternary semigroup

In this section, we introduce the notion of almost bi-ideal in ternary semigroup and study some of their properties.

Definition 3.1. A non-empty subset B of T is called an almost bi-ideal of T, if $[BtBtB] \cap B \neq \emptyset$, for all $t \in T$.

Proposition 3.1. *Every bi-ideal of T is an almost bi-ideal of T.*

Proof. Let B be a bi-ideal of T. Then $[BtBtB] \neq \emptyset$ and
$[BtBtB] \subseteq [BTBTB] \subseteq B$ for all $t \in T$.
Hence $[BtBtB] \cap B = [BtBtB] \neq \emptyset$, for all $t \in T$.Therefore B is an almost bi-ideal of T. □

Remark. *As every bi-ideal of T is an almost bi-ideal of T . But every almost bi-ideal of T need not be bi-ideal of T . We establish this in the following example.*

Example 3.1. Consider a ternary semigroup $T = \{e, a, b, c\}$ with respect to the ternary operation [ ], where [ ] is defined by $[xyz] = ((xy)z) = (x(yz))$ for all $x, y, z \in T$, and ( ) is defined by the table:

( )	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

Let $B = \{a, b, c\}$ be a subset of T.
For $t = e$, we have $[BtBtB] \cap B = \{e, a, b, c\} \cap B = B \neq \emptyset$,
For $t = a$, we have $[BtBtB] \cap B = \{e, a, b, c\} \cap B = B \neq \emptyset$,
For $t = b$, we have $[BtBtB] \cap B = \{e, a, b, c\} \cap B = B \neq \emptyset$,
For $t = c$, we have $[BtBtB] \cap B = \{e, a, b, c\} \cap B = B \neq \emptyset$,
Hence for each $t \in T$, we have $[BtBtB] \cap B \neq \emptyset$. Therefore B is an almost bi-ideal of T.
As $[BtBtB] \nsubseteq B$, it follows that B is not a bi-ideal of T.

Remark. *As every quasi-ideal of T is a bi-ideal of T and every bi-ideal of T is an almost bi-ideal of T . Hence every quasi-ideal of T is an almost bi-ideal of T. But every almost bi-ideal of T need not be quasi-ideal of T (Example 3.1).*

Theorem 3.2. *If B is an almost bi-ideal of T and M is a subset of T and suct that $B \subseteq M \subseteq T$, then M is an almost bi-ideal of T.*

Proof. Let B be an almost bi-ideal of T and M ibe a subset of T and suct that $B \subseteq M \subseteq T$. Then $\emptyset \neq [BtBtB] \cap B \subseteq [MtMtM] \cap M$, for all $t \in T$.
Thus M is an almost bi-ideal of T. □

Theorem 3.3. *The Union of any two almost bi-ideals of T is an almost bi-ideal of T.*

Proof. Let A and B be two almost bi-ideals of T.
Then $[AtAtA] \cap A \neq \emptyset$ and $[BtBtB] \cap B \neq \emptyset$, for all $t \in T$.
Therefore $\emptyset \neq [BtBtB] \cap B \subseteq [(A \cup B)t(A \cup B)t(A \cup B)] \cap (A \cup B)$, for all $t \in T$.
Thus $A \cup B$ is an almost bi-ideal of T. □

Corollary 3.4. *Arbitrary Union of almost bi-ideals of T is an almost bi-ideal of T.*

Remark. *As Union of any two almost bi-ideals of T is an almost bi-ideal of T. But intersection of two almost bi-ideals of T need not be an almost bi-ideal of T . We illustrate this in the following example.*

Example 3.2. Consider a ternary semigroup (Z_5, [ ]) defined by $[abc] = a + (b + c) = (a + b) + c$ for all $a, b, c \in Z_5$. We have $B_1 = \{\bar{1}, \bar{3}, \bar{4}\}$ and $B_2 = \{\bar{1}, \bar{2}, \bar{4}\}$ are almost bi-ideals of Z_5, but $B = B_1 \cap B_2 = \{\bar{1}, \bar{4}\}$ is not an almost bi-ideal of Z_5, because for $\bar{0} \in Z_5$, $[B + \bar{0} + B + \bar{0} + B] = \{\bar{0}, \bar{2}, \bar{3}\}$ and $[B + \bar{0} + B + \bar{0} + B] \cap B \neq \emptyset$.

Proposition 3.5. *If B is a bi-ideal of T. Then $[xBy]$ is an almost bi-ideal of T, for any $x, y \in T$.*

Proof. As B is an bi-ideal of T, then $[xBy]$ is non-empty subset of T, for any $x, y \in T$.
Let $t \in T$, then we have $[[xBy]t[xBy]t[xBy]] = [xB[ytx]B[ytx]By] = [x[Bt_1Bt_1B]y] \subseteq [xBy]$ where $t_1 = [ytx] \in T$. This impiles that $[[xBy]t[xBy]t[xBy]] \cap [xBy] = [xBy] \neq \emptyset$, for any $t \in T$. Thus $[xBy]$ is an almost bi-ideal of T. □

Proposition 3.6. *Let X and Y be non-empty subsets of T. Then $B = [XTY]$ is an almost bi-ideal of T.*

Proof. As X and Y are non-empty subsets of T, then $B = [XTY]$ is a non-empty subset of T.
Let $t \in T$, then we have
$[BtBtB] = [[XTY]t[XTY]t[XTY]] = [X[TYt][XTY][tXT]Y] \subseteq [X[TTT][TTT][TTT]Y] \subseteq [X[TTT]Y] \subseteq [XTY] = B$. Thus $[BtBtB] \cap B = B \neq \emptyset$, for any $t \in T$. Therefore $B = [XTY]$ is an almost bi-ideal of T. □

Proposition 3.7. *T has a proper almost bi-ideal if and only if there exists an element $a \in T$ such that $[(T - \{a\})x(T - \{a\})x(T - \{a\})] \cap (T - \{a\}) \neq \emptyset$, for every $x \in T$.*

Proof. Assume that B is a proper almost bi-ideal of T. Let $a \notin B$, then $B \subset (T - \{a\})$ and $T - \{a\}$ is a proper almost bi-ideal of T. Hence $[(T - \{a\})x(T - \{a\})x(T - \{a\})] \cap (T - \{a\}) \neq \emptyset$, for every $x \in T$. Converse is obvious. □

Proposition 3.8. *For any $a \in T$, $T - \{a\}$ is not an almost bi-ideal of T if and only if there exists an element $x \in T$ such that $[(T - \{a\})x(T - \{a\})x(T - \{a\})] = \{a\}$.*

Proof. Let $a \in T$. Assume that $T - \{a\}$ is not an almost bi-ideal of T. Then there exists an element $x \in T$ such that $[(T - \{a\})x(T - \{a\})x(T - \{a\})] = \emptyset$. So we have $[(T - \{a\})x(T - \{a\})x(T - \{a\})] = \{a\}$.
Conversely assume that there exists an element $x \in T$ such that $[(T - \{a\})x(T - \{a\})x(T - \{a\})] = \{a\}$. Then $[(T - \{a\})x(T - \{a\})x(T - \{a\})] = \emptyset$. Thus $T - \{a\}$ is not an almost bi-ideal of T. □

4. FUZZY ALMOST BI-IDEALS IN TERNARY SEMIGROUP

In this section, we define the notion fuzzy almost bi-ideals in ternary semigroup and establish the relation between almost bi-ideals, fuzzy bi-ideal and fuzzy almost bi-ideals of ternary semigroups.

Definition 4.1. A non-zero fuzzy subset f of T is called a fuzzy almost bi-ideal of T, if $(f \circ t_\alpha \circ f \circ t_\alpha \circ f) \cap f \neq 0$, for all $t \in T$, $\alpha \in (0, 1]$.

Example 4.1 Consider a ternary semigroup $T = \{e, a, b, c, d\}$ with respect to the ternary operation [ ], where [ ] is defined by $[xyz] = ((xy)z) = (x(yz))$ for all $x, y, z \in T$, and ( ) is defined by the table:

( )	e	a	b	c	d
e	e	a	e	e	e
a	e	a	e	e	e
b	e	a	b	e	e
c	e	a	e	e	c
d	e	a	e	e	d

Let $f : T \to [0, 1]$ be defined by $f(e) = 0, f(a) = 0.3, f(b) = 0, f(c) = 0.1, f(d) = 0.2$. Then we have f is a fuzzy almost bi-ideal of T.

Theorem 4.1. *Every non-zero fuzzy bi-ideal of T is a fuzzy almost bi-ideal of T.*

Proof. Let f be a non-zero fuzzy bi-ideal of T, then there exists an element $a \in T$ such that $f(a) \neq 0$. Let $x, t \in T$ and $\alpha \in (0, 1]$ such that $x = [atata]$, then we have
$(f \circ t_\alpha \circ f \circ t_\alpha \circ f)(x)$
$= \bigvee_{x=[atata]=[puqvr]} \{f(p) \wedge t_\alpha(u) \wedge f(q) \wedge t_\alpha(v) \wedge f(r)\}$, where $x = [puqvr], p, q, r, u, v \in T$

$\geqslant f(a) \wedge t_\alpha(t) \wedge f(a) \wedge t_\alpha(t) \wedge f(a)$
$\geqslant f(a) \wedge \alpha \neq 0$.
Thus $(f \circ t_\alpha \circ f \circ t_\alpha \circ f)(x) \neq 0$, it follows that $f(x) = f([atata]) \geqslant f(a) \wedge f(a) \wedge f(a) = f(a) \neq 0$. Hence $((f \circ t_\alpha \circ f \circ t_\alpha \circ f) \cap f)(x) = (f \circ t_\alpha \circ f \circ t_\alpha \circ f)(x) \wedge f(x) \neq 0$. This implies that $(f \circ t_\alpha \circ f \circ t_\alpha \circ f) \cap f \neq 0$. Therefore f is a fuzzy almost bi-ideal of T. □

Remark. *As non-zero fuzzy bi-ideal of T is a fuzzy almost bi-ideal of T But every almost bi-ideal of T need not be fuzzy bi-ideal of T. From Example 3.3, we can see that f is a fuzzy almost bi-ideal of T but it is not a fuzzy bi-ideal of T. Because for $a, e, c, b, d \in T$, we have*
$[aecbd] = e$, $f([aecbd]) = f(e) = 0$ *and* $f(a) \cap f(c) \cap f(d) = 0.3 \wedge 0.1 \wedge 0.2 = 0.1$. $f([aecbd]) \ngeqslant f(a) \cap f(c) \cap f(d)$.

Theorem 4.2. *A non empty subset B of T is an almost bi-ideal of T if and only if C_B is a fuzzy almost bi-ideal of T.*

Proof. Assume that a non empty subset B of T is an almost bi-ideal of T. Let $t \in T$, $\alpha \in (0, 1]$. Then $[BtBtB] \cap B \neq \emptyset$ and hence there exists $x \in [BtBtB] \cap B$, so $C_B(x) = 1 \neq 0$ and $x = [b_1tb_2tb_3]$, where $b_1, b_2, b_3 \in B$.
$(C_B \circ t_\alpha \circ C_B \circ t_\alpha \circ C_B)(x)$
$= \bigvee_{x=[b_1tb_2tb_3]=[puqvr]} \{C_B(p) \wedge t_\alpha(u) \wedge C_B(q) \wedge t_\alpha(v) \wedge C_B(r)\}$

$\geqslant C_B(b_1) \wedge t_\alpha(t) \wedge C_B(b_2) \wedge t_\alpha(t) \wedge C_B(b_3)$
$\geqslant 1 \wedge \alpha \wedge \alpha \wedge 1$
$= \alpha \neq 0.$
$((C_B \circ t_\alpha \circ C_B \circ t_\alpha \circ C_B) \cap C_B)(x) = (C_B \circ t_\alpha \circ C_B \circ t_\alpha \circ C_B)(x) \wedge (C_B)(x) = \alpha \cap 1 = \alpha \neq 0,$
for all $t, x \in T$. Therefore C_B is a fuzzy almost bi-ideal of T.
Conversely assume that for a non empty subset B of T, C_B is a fuzzy almost bi-ideal of T. Let $t \in T$, then $(C_B \circ t_\alpha \circ C_B \circ t_\alpha \circ C_B) \cap C_B \neq 0$. Hence there exists $x \in T$ such that $[(C_B \circ t_\alpha \circ C_B \circ t_\alpha \circ C_B) \cap C_B](x) \neq 0$. $((C_B \circ t_\alpha \circ C_B \circ t_\alpha \circ C_B) \cap C_B)(x) = ((C_{[BtBtB] \cap B})(x) \neq 0$. Therefore $x \in [BtBtB] \cap B$. So $[BtBtB] \cap B \neq \emptyset$, for all $t \in T$. Hence B is an almost bi-ideal of T. □

Theorem 4.3. *If g is any non- zero fuzzy subset of T and f is a fuzzy almost bi-ideal of T suct that $f \subseteq g \subseteq T$, then g is a fuzzy almost bi-ideal of T.*

Proof. Let g be any non- zero fuzzy subset of a ternary semigroup T and let $t \in T$, $\alpha \in (0, 1]$. Assume that f be a fuzzy almost bi-ideal of T suct that $f \subseteq g \subseteq T$. This implies that $(f \circ t_\alpha \circ f \circ t_\alpha \circ f) \cap f \neq 0$ and $(f \circ t_\alpha \circ f \circ t_\alpha \circ f) \cap f \subseteq (g \circ t_\alpha \circ g \circ t_\alpha \circ g) \cap g$. Then we have $0 \neq (f \circ t_\alpha \circ f \circ t_\alpha \circ f) \cap f \subseteq (g \circ t_\alpha \circ g \circ t_\alpha \circ g) \cap g$, so $(g \circ t_\alpha \circ g \circ t_\alpha \circ g) \cap g \neq 0$. Hence g is a fuzzy almost bi-ideal of T.

□

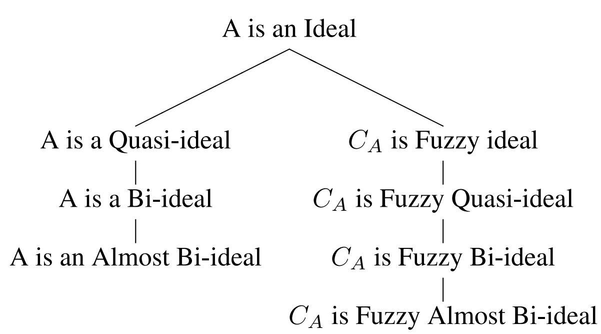
Corollary 4.4. *Let f and g be a fuzzy almost bi-ideal of a ternary semigroup T. Then $f \cup g$ is a fuzzy almost bi-ideal of T.*

Proof. Since $f \subseteq f \cup g$, by Theorem 4.2, $f \cup g$ is a fuzzy almost bi-ideal of T. □

Remark. *As Union of any two fuzzy almost bi-ideals of a ternary semigroup T is a fuzzy almost bi-ideal of T. But intersection of two fuzzy almost bi-ideals of T need not be a fuzzy almost bi-ideal of T . We establish this in the following example.*

Example 4.2. Consider a ternary semigroup (Z_5, [ ]) defined by $[abc] = a + (b + c) = (a + b) + c$ for all $a, b, c \in Z_5$. Let $f : Z_5 \to [0, 1]$ be defined by $f(0) = 0, f(1) = 0.5, f(2) = 0, f(3) = 0.1, f(4) = 0.1$ and $g : Z_5 \to [0, 1]$ be defined by $g(0) = 0, g(1) = 0.2, g(2) = 0.1, g(3) = 0g(4) = 0.2$. We have f and g are fuzzy almost bi-ideals of Z_5 but $f \cap g$ is not a fuzzy almost bi-ideal of Z_5.

The relationships between different ideals in a ternary semigroup T is given below.



5. Conclusions and/or Discussions

In this paper, we introduced the notions almost bi-ideals and fuzzy almost bi-ideals in ternary semigroups and studied their properties. We established the relation between different types of ideals in a ternary semigroup with various examples. One can extend this work by studying the other algebraic structures like ordered ternary semigroups, ternary semirings, etc

6. Acknowledgements

The author would like to thank the referees for their valuable comments and suggestions for improving the manuscript.

References

[1] D. H. Lehmer. A ternary analogue of abelian groups, Amer. J. Math., 54 (1932), 329–338.
[2] F. M. Sioson. Ideal Theory in ternary semigroups, Math. Japan., 10 (1965), 63-84.
[3] K. Wattanatripop, R. Chinramb and T. Changphas. Fuzzy almost Bi-ideals in semigroups, International Journal of Mathematics and Computer Science, 13 (2018), no. 1, 51–58.
[4] L. A. Zadeh. Fuzzy sets, Information and Control (1965),8, 338-353.
[5] O. Grosek and L. Satko. A new notion in the theory of semigroup, Semigroup Forum, 20 (1980), 233–240.
[6] R. A. Good and D. R. Hughes. Associated groups for a semigroup, Bull.Amer. Math. Soc., 58(1952), 624-625.
[7] S. Kar and P. Sarkar. Fuzzy Ideals of Ternary Semigroups. Fuzzy Information and Engineering, 4:2,(2012) 181-193.
[8] S. Suebsung, K. Wattanatripop and R. Chinram. A-ideals and fuzzy A-ideals of ternary semi-groups, Songklanakarin J. Sci. Tech., 41 (2019), 299–304.
[9] V. N. Dixit and S. Dewan. A note on quasi and bi-ideals in ternary semigroups , International Journal of Mathematics and Mathematical Science, 18 (1995), 501-508.
[10] V. N. Dixit and S. Dewan. Minimal quasi-ideals in ternary semigroups, Indian Journal Pure and Applied Mathematics, 28 (5) (1997), 625-632.

Vanita R. Daddi
Department of First Year Engineering,
Dr. D. Y. Patil Institute of Engineering,
Management and Research,
Akurdi, Pune, India
Email address: vanitadaddi8@gmail.com