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SOMEWHAT FUZZY COMPLETELY e-IRRESOLUTE MAPPINGS

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ABSTRACT. The aim of this paper is to introduce and study the concept of somewhat fuzzy completely e-irresolute mapping and somewhat fuzzy irresolute e-open mapping. Further, some interesting properties of those mappings are given and some comparative results discussed.

1. INTRODUCTION

The introduction of fuzzy sets by Zadeh[10] in 1965 motivated Chang [3] to study the concept of fuzzy topology in 1968. The concept of fuzzy δ-open sets, fuzzy δ-closed sets and the notion of fuzzy δ-continuous functions in fuzzy topological spaces introduced by Supriti Saha [6]. The concept of fuzzy e-open set[5] studied by Seenivasan and Kamala. Recently, the notions of of somewhat fuzzy δ-irresolute continuous mapping and somewhat fuzzy e-irresolute mapping on a fuzzy topological space are respectively introduced and investigated in [7] and [9].

In section 3 of this article, we introduce and study the concepts of somewhat fuzzy completely e-irresolute mapping and some comparative results on a fuzzy topological space are investigated. Besides, some interesting properties of those mappings are also given. In section 4, the idea of somewhat fuzzy completely e-open mapping are studied. Finally in section 5 some preservative results under these mappings are investigated.

Now let X and Y be fuzzy topological spaces. We denote Int(μ) and Cl(μ) with the interior and with the closure of the fuzzy set μ on a fuzzy topological space X respectively.

A fuzzy subset λ of a space X is called fuzzy regular open [2] (resp. fuzzy regular closed) if λ =Int(Cl(λ)) (resp.λ =Cl(Int(λ))). Now Cl(λ) and Int(λ) are defined as follows $\text{Cl}(\lambda) = \bigwedge \{\mu : \mu \geq \lambda, \mu \text{ is fuzzy closed in } X\}$ and $\text{Int}(\lambda) = \bigvee \{\mu \leq \lambda, \mu \text{ is fuzzy open in } X\}$. The fuzzy δ-interior of a fuzzy subset λ of X is the union of all fuzzy regular open sets contained in λ. A fuzzy subset λ is called fuzzy δ-open [6] if $\lambda = \text{Int}_\delta(\lambda)$. The complement of fuzzy δ-open set is called fuzzy δ-closed (i.e, $\lambda = \text{Cl}_\delta(\lambda)$).

A fuzzy subset λ of a space X is called fuzzy e-open [5] if $\lambda \leq$cl(int$_\delta\lambda$)$\vee$int(cl$_\delta\lambda$) and fuzzy e-closed set if $\lambda \geq$cl(int$_\delta\lambda$)$\wedge$int(cl$_\delta\lambda$).

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Definition 1.1. *A fuzzy set μ on a fuzzy topological space X is called fuzzy completely dense*[8] *if there exists no fuzzy regular closed set ν in X such that $\mu < \nu < 1$. That is $rCl(\lambda) = 1$.*

Definition 1.2. *A fuzzy set μ on a fuzzy topological space X is called fuzzy e-dense*[9] *if there exists no fuzzy e-closed set ν in X such that $\mu < \nu < 1$. That is $Cl_e(\lambda) = 1$.*

2. Somewhat fuzzy completely e-irresolute mappings

In this section, we introduce fuzzy completely e-continuous, fuzzy completely e-irresolute, somewhat fuzzy completely e-continuous and somewhat fuzzy completely e-irresolute mappings.

Definition 2.1. *A mapping $f : X \to Y$ is called fuzzy completely e-irresolute if $f^{-1}(\nu)$ is a fuzzy regular open set on X for any fuzzy e-open set ν on Y.*

Definition 2.2. *A mapping $f : X \to Y$ is called somewhat fuzzy completely e-irresolute if there exists a fuzzy regular open set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy e-open set $\nu \neq 0_Y$ on Y .*

Every fuzzy completely e-irresolute mapping is a somewhat fuzzy completely e-irresolute mapping but not conversely.

Example 2.3. (Swaminathan [9]) *Let λ_1,${\lambda_1}^c$,λ_2,λ_3,λ_4 and λ_5 be fuzzy sets on $X = \{a, b, c\}$ with*
$\lambda_1 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.5}{c}$, ${\lambda_1}^c = \frac{0.7}{a} + \frac{0.6}{b} + \frac{0.5}{c}$, $\lambda_2 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.5}{c}$, $\lambda_3 = \frac{0.7}{a} + \frac{0.6}{b} + \frac{0.4}{c}$,
$\lambda_4 = \frac{0.3}{a} + \frac{0.4}{b} + \frac{0.4}{c}$, $\lambda_5 = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.4}{c}$ *and* $\lambda_6 = \frac{0.5}{a} + \frac{0.5}{b} + \frac{0.5}{c}$
And let,$\tau_1 = \{0_X, \lambda_1, \lambda_2, \lambda_4, \lambda_5, 1_X\}$, $\tau_2 = \{0_X, \lambda_1, {\lambda_1}^c, \lambda_2, \lambda_3, \lambda_4, \lambda_5, 1_X\}$ be fuzzy topologies on X. Consider the fuzzy identity mapping f:$(X, \tau_1) \to (X, \tau_2)$. For fuzzy e-open sets λ_1,${\lambda_1}^c$,λ_2,λ_3,λ_4 and λ_5 on (X, τ_2), we have $f^{-1}(\lambda_1) = \lambda_1$, $\lambda_1 \leq f^{-1}({\lambda_1}^c) = {\lambda_1}^c$, $\lambda_1 \leq f^{-1}(\lambda_2) = \lambda_2$,$\lambda_4 \leq f^{-1}(\lambda_3) = \lambda_3$,$\lambda_4 \leq f^{-1}(\lambda_4) = \lambda_4$ and $\lambda_3 \leq f^{-1}(\lambda_5) = \lambda_5$. Since λ_1 and λ_4 are a fuzzy e-open sets on (X, τ_1), f is somewhat fuzzy completely e-irresolute mapping. But for a fuzzy e-open set λ_3 on (X, τ_2), $f^{-1}(\lambda_3) = \lambda_3$ is not a fuzzy regular open set on (X, τ_1). Hence f is not a fuzzy completely e-irresolute mapping.

Theorem 2.1. If $f : X \to Y$ be a mapping. Then the following are equivalent:
(1) f is somewhat fuzzy completely e-irresolute.
(2) If ν is a fuzzy e-closed set of Y such that $f^{-1}(\nu) \neq 1_X$, then there exists a fuzzy regular closed set $\mu \neq 1_X$ of X such that $f^{-1}(\nu) \leq \mu$.
(3) If μ is a fuzzy completely dense set on X, then $f(\mu)$ is a fuzzy e-dense set on Y.

Proof. (1)⇒(2) :Let ν be a fuzzy e-closed set on Y such that $f^{-1}(\nu) \neq 1_X$. Then ν^c is a fuzzy e-open set on Y and $f^{-1}(\nu^c) = (f^{-1}(\nu))^c \neq 0_X$. Since f is somewhat fuzzy completely e-irresolute, there exists a fuzzy regular open set $\lambda \neq 0_X$ on X such that $\lambda \leq f^{-1}(\nu)^c$. Let $\mu = \lambda^c$. Then $\mu \neq 1_X$ is fuzzy regular closed such that $f^{-1}(\nu) = 1 - f^{-1}(\nu^c) \leq 1 - \mu^c = \mu$.

(2)⇒(3): Let μ be a fuzzy completely dense set on X and suppose $f(\mu)$ is not fuzzy e-dense on Y . Then there exists a fuzzy e-closed set ν on Y such that $f(\mu) < \nu < 1$. Since $\nu < 1$ and ,$f^{-1}(\nu) \neq 1_X$, there exists a fuzzy regular closed set $\delta \neq 1_X$ such that $\mu \leq f^{-1}(f(\mu)) < f^{-1}(\nu) \leq \delta$. This contradicts to the assumption that μ is a fuzzy completely dense set on X. Hence $f(\mu)$ is a fuzzy e-dense set on Y .

(3)⇒(1): Let $\nu \neq 0_Y$ be a fuzzy e-open set on Y and $f^{-1}(\nu) \neq 0_X$. Suppose there exists no fuzzy regular open $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu)$. Then $(f^{-1}(\nu)^c)$ is a fuzzy set on X such that there is no fuzzy regular closed set δ on X with $(f^{-1}(\nu)^c) < \delta < 1$. Suppose there exists a fuzzy e-open set δ^c such that $\delta^c \leq f^{-1}(\nu)$, then it is a contradiction. Therefore $(f^{-1}(\nu))^c$ is a fuzzy completely dense set on X. Then $f((f^{-1}(\nu))^c)$ is a fuzzy e-dense set on Y . But $f((f^{-1}(\nu))^c) = f((f^{-1}(\nu))^c)) \neq \nu^c < 1$. This contradicts to the fact that $f((f^{-1}(\nu))^c)$ is fuzzy e-dense on Y . Hence there exists a fuzzy regular open set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu)$. Hence f is somewhat fuzzy completely e-irresolute. □

Theorem 2.2. Let X_1 be product related to X_2 and Y_1 be product related to Y_2. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of somewhat fuzzy completely e-irresolute mappings $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is also somewhat fuzzy completely e-irresolute.

Proof. Let $\lambda = \bigvee_{i,j}(\mu_i \times \nu_j)$ be a fuzzy e-open set on $Y_1 \times Y_2$ where $\mu_i \neq 0_{Y_1}$ and $\nu_j \neq 0_{Y_2}$ are fuzzy e-open sets on Y_1 and Y_2 respectively. Then $(f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j))$. Since f_1 is somewhat fuzzy completely e-irresolute, there exists a fuzzy regular open set $\delta_i \neq 0_{X_1}$ such that $\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{X_1}$. Also f_2 is somewhat fuzzy completely e-irresolute, there exists a fuzzy regular open set $\eta_j \neq 0_{X_2}$ such that $\eta_j \leq f_2^{-1}(\nu_j) \neq 0_{X_2}$. Now $\delta_i \times \eta_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)$ and $\delta_i \times \eta_j \neq 0_{X_1 \times X_2}$ is a fuzzy regular open set on $X_1 \times X_2$. Hence $\bigvee_{i,j}(\delta_i \times \eta_j) \neq 0_{X_1 \times X_2}$ is a fuzzy regular open set on $X_1 \times X_2$ such that $\bigvee_{i,j}(\delta_i \times \eta_j) \leq \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\mu_i)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\mu_i \times \nu_j)) = (f_1 \times f_2)^{-1}(\lambda) \neq 0_{X_1 \times X_2}$. Therefore, $f_1 \times f_2$ is somewhat fuzzy completely e-irresolute. □

Theorem 2.3. Let $f : X \to Y$ be a mapping. If the graph $g : X \to X \times Y$ of f is a somewhat fuzzy completely e-irresolute mapping, then f is also somewhat fuzzy completely e-irresolute.

Proof. Let ν be a fuzzy e-open set on Y. Then $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$. As g is somewhat fuzzy completely e-irresolute and $(1 \times \nu)$ is a fuzzy e-open set on $X \times Y$, there exists a fuzzy regular open set $\mu \neq 0_X$ on X such that $\mu \leq g^{-1}(1 \times \nu) = f^{-1}(\nu) \neq 0_X$. Therefore, f is somewhat fuzzy e-irresolute continuous. □

3. SOMEWHAT FUZZY IRRESOLUTE e-OPEN MAPPINGS

In this section, we introduce fuzzy completely irresolute e-open and somewhat fuzzy completely irresolute e-open mapping. Also we discuss some comparative results on somewhat fuzzy completely irresolute e-open mapping.

Definition 3.1. *A mapping $f : X \to Y$ is called fuzzy completely irresolute e-open if $f(\mu)$ is a fuzzy e-open set on Y for any fuzzy regular open set μ on X.*

Definition 3.2. *A mapping $f : X \to Y$ is called somewhat fuzzy completely irresolute e-open if there exists a fuzzy e-open set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu) \neq 0_Y$ for any fuzzy regular open set $\mu \neq 0_X$ on X.*

Theorem 3.1. If $f : X \to Y$ be a bijection. Then the following are equivalent.
(1) f is somewhat fuzzy completely irresolute e-open.
(2) If μ is a fuzzy regular closed set on X such that $f(\mu) \neq 1_Y$, then there exists a fuzzy e-closed set $\nu \neq 1_Y$ on Y such that $f(\mu) < \nu$.

Proof. (1)⇒(2): Let μ be a fuzzy regular closed set on X such that $f(\mu) \neq 1_Y$. Since f is bijective and μ^c is a fuzzy regular open set on X, $f(\mu^c) = (f(\mu))^c \neq 0_Y$. And, since f is somewhat fuzzy completely irresolute e-open, there exists a fuzzy e-open set $\delta \neq 0_Y$ on

Y such that $\delta < f(\mu^c) = (f(\mu))^c$. Consequently, $f(\mu) < \delta^c = \nu \neq 1_Y$ and ν is a fuzzy e-closed set on Y .

(2)⇒(1): Let μ be a fuzzy regular open set on X such that $f(\mu) \neq 0_Y$.Then μ^c is a fuzzy e-closed set on X and $f(\mu^c) \neq 1_Y$. Hence there exists a fuzzy e-closed set $\nu \neq 1_Y$ on Y such that $f(\mu^c) < \nu$. Since f is bijective, $f(\mu^c) = (f(\mu))^c < \nu$. Hence $\nu^c < f(\mu)$ and $\nu^c \neq 0_X$ is a fuzzy e-open set on Y . Therefore, f is somewhat fuzzy completely irresolute e-open. □

Theorem 3.2. If $f : X \to Y$ be a surjection. Then the following are equivalent.
(1) f is somewhat fuzzy irresolute e-open.
(2) If ν is a fuzzy e-dense set on Y , then $f^{-1}(\nu)$ is a fuzzy completely dense set on X.

Proof. (1)⇒(2): Let ν be a fuzzy e-dense set on Y . Suppose $f^{-1}(\nu)$ is not fuzzy completely dense on X. Then there exists a fuzzy regular closed set μ on X such that $f^{-1}(\nu) < \mu < 1$. Since f is somewhat fuzzy irresolute e-open and μ^c is a fuzzy regular open set on X, there exists a fuzzy e-open set $\delta \neq 0_Y$ on Y such that $\delta \leq f(\text{Int}\mu^c) \leq f(\mu^c)$. Since f is surjective, $\delta \leq f(\mu^c) < f(f^{-1}(\nu^c)) = \nu^c$. Thus there exists a fuzzy e-closed set δ^c on Y such that $\nu < \delta^c < 1$ which is a contradiction. Hence $f^{-1}(\nu)$ is fuzzy completely dense on X.

(2)⇒(1): Let μ be a fuzzy regular open set on X and $f(\mu) \neq 0_Y$. Suppose there exists no fuzzy e-open $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu)$. Then $(f(\mu))^c$ is a fuzzy set on Y such that there exists no fuzzy e-closed set δ on Y with $(f(\mu))^c < \delta < 1$. It means that $(f(\mu))^c$ is fuzzy e-dense on Y . Thus $f^{-1}((f(\mu))^c)$ is fuzzy e-dense on X. But $f^{-1}((f(\mu))^c) = (f^{-1}(f(\mu)))^c \leq \mu^c < 1$. This contradicts to the fact that $f^{-1}((f(\nu))^c$ is fuzzy completely dense on X. Hence there exists a fuzzy e-open set $\nu \neq 0_Y$ on Y such that $\nu \leq f(\mu)$. Therefore, f is somewhat fuzzy irresolute e-open. □

4. SOME PRESERVATION RESULTS

In this section by means of fuzzy e-irresolute and fuzzy completely e-irresolute mapping preservation of some fuzzy topological structures are discussed.

Definition 4.1. *A fuzzy topological space $(X, \mathcal{F})$ is called*
(i)fuzzy nearly compact [4] *if every fuzzy regular open cover has a finite subcover.*
(ii) fuzzy e-compact [1] *if every fuzzy e-open cover has a finite subcover.*

Theorem 4.1. Every surjective fuzzy e-irresolute image of a fuzzy e-closed space is fuzzy e-compact.

Proof. Let $f : X \to Y$ be a fuzzy completely e-continuous mapping of a fuzzy e-closed space $(X, \mathcal{F}_1)$ onto a fuzzy space $(Y, \mathcal{F}_2)$. Let $\{\beta_a : a \in A\}$ be any fuzzy e-open cover of Y. Since f is fuzzy e-irresolute , $\left\{f^{-1}(\beta_a) : a \in A\right\}$ is a fuzzy e-open cover of X. Since X is a fuzzy e-closed space, then there exists a finite subfamily $\left\{f^{-1}(\beta_{a_i}) : i = 1, ..., n\right\}$ of $\left\{f^{-1}(\beta)\right\}$ which covers X. It implies that $\{\beta_{a_i} : i = 1, ..., n\}$ is a finite subcover of $\{\beta_a : a \in A\}$ which covers Y. Hence $f(X) = Y$ is fuzzy e-compact. □

Theorem 4.2. Every surjective fuzzy completely e-irresolute image of a fuzzy regular closed space is fuzzy e-compact.

Proof. Let $f : X \to Y$ be a fuzzy completely e-irresolute mapping of a fuzzy regular closed space $(X, \mathcal{F}_1)$ onto a fuzzy space $(Y, \mathcal{F}_2)$. Let $\{\beta_a : a \in A\}$ be any fuzzy e-open cover of Y. Since f is fuzzy completely e-irresolute , $\left\{f^{-1}(\beta_a) : a \in A\right\}$ is a fuzzy regular open cover of X. Since X is a fuzzy regular closed space, then there exists a finite subfamily $\left\{f^{-1}(\beta_{a_i}) : i = 1, ..., n\right\}$ of $\left\{f^{-1}(\beta)\right\}$ which covers X. It implies that $\{\beta_{a_i} : i = 1, ..., n\}$ is a finite subcover of $\{\beta_a : a \in A\}$ which covers Y. Hence $f(X) = Y$ is fuzzy e-compact. □

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