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ON CONTRA n$\mathcal{I}_g$-CONTINUITY IN NANO IDEAL TOPOLOGICAL SPACES

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ABSTRACT. In this paper, $n\mathcal{I}_g$-closed sets and $n\mathcal{I}_g$-open sets are used to define and investigate a new class of maps called contra $n\mathcal{I}_g$-continuous maps in nano ideal topological spaces. We discuss the relationship with some other related maps.

1. INTRODUCTION

In 1990, Jankovic and Hamlett [[6], [7]] have considered the local function in ideal topological space any they have obtained a new topology. In 2016, Paimala et al [12] introduce a similar type with the local function in nano topological spaces. Before starting the discussion we shall consider the following concepts.

Let (U, $\mathcal{N}$) be a nano topological space, where $\mathcal{N}$ = $\tau_R(X)$.
A nano topological space (U, $\mathcal{N}$) with an ideal $\mathcal{I}$ on U is called a nano ideal topological space and is denoted by (U, $\mathcal{N}$, $\mathcal{I}$).
Let (U, $\mathcal{N}$) be a nano topological space and G_n(u) = $\{\mathrm{G}_n \mid \mathrm{u} \in \mathrm{G}_n, \mathrm{G}_n \in \mathcal{N}\}$ be the family of nano open sets which contain u.
Let (U, $\mathcal{N}$, $\mathcal{I}$) be an nano ideal topological space with an ideal $\mathcal{I}$ on U, where $\mathcal{N}$ = $\tau_R(X)$ and $(.)^*_n$: $\wp$(U) $\rightarrow$ $\wp$(U) ($\wp$(U) is the set of all subsets of U) [12, 13]. For a subset A $\subseteq$U, $\mathrm{A}^*_n(\mathcal{I}, \mathcal{N})$ = $\{\mathrm{u} \in \mathrm{U} : \mathrm{G}_n \cap \mathrm{A} \notin \mathcal{I}$, for every $\mathrm{G}_n \in \mathrm{G}_n(\mathrm{x})\}$, where G_n(u) = $\{\mathrm{G}_n \mid \mathrm{u} \in \mathrm{G}_n$, $\mathrm{G}_n \in \mathcal{N}\}$ is called the nano local function (brielfy n-local function) of A with repect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write A^*_n for A^*_n ($\mathcal{I}$, $\mathcal{N}$). Parimala et al [13] introduced the concept of nano ideal generalized closed sets in nano ideal topological spaces and investigated some of its basic properties. Recently, Ganesan [4] introduced and studied $n\mathcal{I}_g$-continuous map and $n\mathcal{I}_g$-irresolute map in nano ideal topological spaces. The main objective of this study is to introduce a new hybrid intelligent structure called contra $n\mathcal{I}_g$-continuity in nano ideal topological spaces. The significance of introducing hybrid structures is that the computational techniques, fuzzy and soft set topological spaces

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2. Preliminaries

Definition 2.1. [12, 13] A subset A of a nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is said to be n$\star$-closed if $A_n^* \subseteq$ A.

Lemma 2.1. [12, 13] *Let (U, $\mathcal{N}$, $\mathcal{I}$) be a nano topological space with an ideal $\mathcal{I}$ and A$\subseteq A_n^*$, then A_n^*=n-cl(A_n^*)=n-cl(A)*

Definition 2.2. [11] A subset M of a space (U, $\tau_R(X)$) is said to be nano pre-closed set if ncl(nint(M)) $\subseteq$ M. The complement of a nano pre-closed set is said to be nano pre-open.

Definition 2.3. [12, 13] A subset A of a nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is said to be

(1) nano-$\mathcal{I}$-generalized closed (briefly, $n\mathcal{I}$g-closed if $A_n^* \subseteq$ V whenever A$\subseteq$ V and V is n-open.
(2) $n\mathcal{I}$g-open if its complement is $n\mathcal{I}$g-closed.

Definition 2.4. [10] A nano topological space (U, $\tau_R(X)$) is said to nano locally indiscrete if every n-open set is n-closed.

Definition 2.5. [2] A nano topological space (U, $\tau_R(X)$) is said to be nano-regular space, if for each nano closed set F and each point x $\notin$ F , there exists disjoint nano open sets G and H such that x $\in$ G and F $\subset$ H

Definition 2.6. [9] A nano topological space (U, $\tau_R(X)$) is said to be nano-connected if (U, $\tau_R(X)$) cannot be expressed as a disjoint union of two non-empty nano-open sets. A subset of (U, $\tau_R(X)$) is nano-connected as a subspace. A subset is said to be nano disconnected if and only if it is not nano-connected

Definition 2.7. [14] A function f: (O, $\mathcal{N}$) $\rightarrow$ (P, $\mathcal{N}'$) is said to be nano contra g-continuous if f^{-1}(V) is a ng-closed set of (O, $\mathcal{N}$) for every n-open set V of (P, $\mathcal{N}'$).

Definition 2.8. A map f: (K, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (L, $\mathcal{N}'$) is said to be n$\star$-continuous [8] (resp.$n\mathcal{I}_g$-continuous [3, 4]) if f^{-1}(A) is n$\star$-closed (resp. $n\mathcal{I}_g$-closed) in (K, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set A of (L, $\mathcal{N}'$).

Definition 2.9. [4] A function f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is called $n\mathcal{I}_g$-irresolute if f^{-1}(V) is a $n\mathcal{I}_g$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every $n\mathcal{I}_g$-closed set V of (P, $\mathcal{N}'$, $\mathcal{J}$).

Theorem 2.2. *(1) Every n-closed is n$\star$-closed set but not conversely* [1, 5].
(2) Every n$\star$-closed set is $n\mathcal{I}_g$-closed but not conversely [13]
(3) Every ng-closed set is $n\mathcal{I}_g$-closed but not conversely [13]

3. Contra $n\mathcal{I}_g$-continuity

Let (O, $\mathcal{N}$, $\mathcal{I}$) (or O) represent nano ideal topological spaces on which no separation axioms are assumed unless otherwise mentioned.
Let (P, $\mathcal{N}'$) (or P) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned.
We introduce the following definitions

Definition 3.1. A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is said to be contra n$\star$-continuous if f^{-1}(V) is a n$\star$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every n-open set V of (P, $\mathcal{N}'$, $\mathcal{J}$).

Definition 3.2. A map f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is said to be contra $n\mathcal{I}_g$-continuous if f^{-1}(V) is a $n\mathcal{I}_g$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every n-open set V of (P, $\mathcal{N}'$, $\mathcal{J}$).

Proposition 3.1. *Every contra n⋆-continuous map is contra $n\mathcal{I}_g$-continuous.*

Proof. Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a contra n⋆-continuous map and let G be any n-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then, f^{-1}(G) is n⋆-closed in O. Since every n⋆-closed set is $n\mathcal{I}_g$-closed, f^{-1}(G) is $n\mathcal{I}_g$-closed in O. Therefore f is contra $n\mathcal{I}_g$-continuous. □

Example 3.3. Let O = $\{$5, 6, 7$\}$, with O/ R= $\{\{5\}, \{6, 7\}\}$ and X= $\{5\}$. Then the Nano topology $\mathcal{N}$ = $\{\phi$, $\{5\}$, O$\}$ and $\mathcal{I}$=$\{\emptyset, \{5\}\}$. Let P = $\{$5, 6, 7$\}$, with P/ R= $\{\{7\}, \{5, 6\}\}$ and X= $\{5, 7\}$. Then the Nano topology $\mathcal{N}'$ = $\{\phi$, $\{7\}$, $\{5, 6\}$, P$\}$ and $\mathcal{J}$=$\{\emptyset, \{7\}\}$. Then n⋆-closed sets are ϕ, O, $\{5\}$, $\{6, 7\}$ and $n\mathcal{I}_g$-closed sets are ϕ, O, $\{5\}$, $\{6\}$, $\{7\}$, $\{5, 6\}$, $\{5, 7\}$, $\{6, 7\}$. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be the identity map. Then f is contra $n\mathcal{I}_g$-continuous but not contra n⋆-continuous, since f^{-1}($\{5, 6\}$)= $\{5, 6\}$ is not n⋆-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

Proposition 3.2. *Every nano contra g-continuous map is contra $n\mathcal{I}_g$-continuous.*

Proof. Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a nano contra g-continuous map and let G be any n-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then, f^{-1}(G) is ng-closed in O. Since every ng-closed set is $n\mathcal{I}_g$-closed, f^{-1}(G) is $n\mathcal{I}_g$-closed in O. Therefore f is contra $n\mathcal{I}_g$-continuous. □

Example 3.4. Let O = $\{$5, 6, 7$\}$, with O/ R= $\{\{7\}, \{5, 6\}, \{6, 5\}$ and X= $\{5, 6\}$. Then the Nano topology $\mathcal{N}$ = $\{\phi$, $\{5, 6\}$, O$\}$ and $\mathcal{I}$=$\{\emptyset, \{5\}\}$. Let P = $\{$5, 6, 7$\}$, with P/ R= $\{\{5\}$, $\{6, 7\}\}$ and X= $\{5\}$. Then the Nano topology $\mathcal{N}'$ = $\{\phi$, $\{5\}$, P$\}$ and $\mathcal{J}$=$\{\emptyset, \{5\}\}$. Then $n\mathcal{I}_g$-closed sets are ϕ, O, $\{5\}$, $\{7\}$, $\{5, 7\}$, $\{6, 7\}$, ng-closed sets are ϕ, O, $\{7\}$, $\{5, 7\}$, $\{$6, 7$\}$. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be the identity map. Then f is contra $n\mathcal{I}_g$-continuous but not nano contra g-continuous, since f^{-1}($\{5\}$)= $\{5\}$ is not ng-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

Remark. *The following example shows that $n\mathcal{I}_g$-continuity and contra $n\mathcal{I}_g$-continuity are independent.*

Example 3.5. Let O = $\{$5, 6, 7$\}$, with O/ R= $\{\{5\}, \{6, 7\}\}$ and X= $\{5\}$. Then the Nano topology $\mathcal{N}$ = $\{\phi$, $\{5\}$, O$\}$ and $\mathcal{I}$=$\{\emptyset\}$. Let P = $\{$5, 6, 7$\}$, with P/ R= $\{\{5\}$, $\{6, 7\}$. $\{$6, 7$\}\}$ and X= $\{6, 7\}$. Then the Nano topology $\mathcal{N}'$ = $\{\phi$, $\{6, 7\}$, P$\}$ and $\mathcal{J}$=$\{\emptyset, \{6\}\}$. Then $n\mathcal{I}_g$-closed sets are ϕ, O, $\{6\}$, $\{7\}$, $\{5, 6\}$, $\{5, 7\}$, $\{6, 7\}$. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be the identity map. Then f is contra $n\mathcal{I}_g$-continuous but not $n\mathcal{I}_g$-continuous, since f^{-1}($\{5\}$)= $\{5\}$ is not $n\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

Example 3.6. Let O = $\{$5, 6, 7, 8$\}$, with O/ R= $\{\{5\}, \{7\}, \{6, 8\}\}$ and X= $\{5, 6\}$. Then the Nano topology $\mathcal{N}$ = $\{\phi$, $\{5\}$, $\{6, 8\}$, $\{5, 6, 8\}$, O$\}$ and $\mathcal{I}$ = $\{\emptyset, \{5\}\}$. Let P = $\{$5, 6, 7, 8$\}$, with P/ R= $\{\{5\}, \{6\}, \{7, 8\}\}$ and X= $\{7\}$. Then the Nano topology $\mathcal{N}'$ = $\{\phi$, $\{$7, 8$\}$, P$\}$ and $\mathcal{J}$ = $\{\emptyset, \{6\}, \{5, 6\}\}$. Then $n\mathcal{I}_g$-closed sets are ϕ, O, $\{5\}$, $\{7\}$, $\{5, 7\}$, $\{6, 7\}$, $\{7, 8\}$, $\{5, 6, 7\}$, $\{5, 7, 8\}$, $\{6, 7, 8\}$. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) by f(5) = 7, f(6) = 8, f(7) = 5 and f(8) = 6. Then f is $n\mathcal{I}_g$-continuous but not contra $n\mathcal{I}_g$-continuous, since f^{-1}($\{7, 8\}$)= $\{5, 6\}$ is not $n\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

Remark. *The composition of two contra $n\mathcal{I}g$-continuous maps need not be contra $n\mathcal{I}g$-continuous and this is shown from the following example.*

Example 3.7. Let O = $\{$5, 6, 7$\}$, with O/ R= $\{\{7\}, \{5, 6\}, \{6, 5\}\}$ and X= $\{5, 6\}$. Then the Nano topology $\mathcal{N}$ = $\{\phi$, $\{5, 6\}$, O$\}$ and $\mathcal{I}$ = $\{\emptyset\}$. Then $n\mathcal{I}$g-closed sets are ϕ, O, $\{7\}$, $\{5, 7\}$, $\{6, 7\}$. Let P = $\{$5, 6, 7$\}$, with P/ R= $\{\{5\}, \{6, 7\}\}$ and X= $\{5\}$. Then the Nano topology $\mathcal{N}'$ = $\{\phi$, $\{5\}$, P$\}$ and $\mathcal{J}$=$\{\emptyset, \{5\}\}$. Then $n\mathcal{I}$g-closed sets are ϕ, P, $\{5\}$, $\{6\}$, $\{7\}$, $\{5, 6\}$, $\{5, 7\}$, $\{6, 7\}$. Let Q = $\{$5, 6, 7$\}$ with Q/ R= $\{\{7\}, \{5, 6\}\}$ and X= $\{6, 7\}$. Then the Nano topology $\mathcal{N}'_*$= $\{\phi$, $\{7\}$, $\{5, 6\}$, Q$\}$ and $\mathcal{K}$ = $\{\phi, \{7\}\}$. Define f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P,

$\mathcal{N}'$, $\mathcal{J}$) and g : (P, $\mathcal{N}'$, $\mathcal{J}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) be the identity maps. Clearly f and g are contra $n\mathcal{I}$g-continuous but their g $\circ$ f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is not contra $n\mathcal{I}$g-continuous, because V = $\{5, 6\}$ is n-open in (Q, $\mathcal{N}'_*$) but (g $\circ$ f $^{-1}$($\{5, 6\}$)= f^{-1}(g^{-1}($\{5, 6\}$)) = f^{-1}($\{5, 6\}$) = $\{5, 6\}$, which is not $n\mathcal{I}$g-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

Theorem 3.3. *Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a map. Then the following conditions are equivalent*

1. *f is contra $n\mathcal{I}_g$-continuous.*
2. *The inverse image of each n-open set in P is $n\mathcal{I}_g$-closed in O.*
3. *The inverse image of each n-closed set in P is $n\mathcal{I}_g$-open in O.*
4. *For each point o in O and each n-closed set G in P with f(o) $\in$ G, there is an $n\mathcal{I}_g$-open set U in O containing o such that f(U) $\subset$ G.*

Proof. (1) $\Rightarrow$ (2). Let G be n-open in P. Then P $-$ G is n-closed in P. By definition of contra $n\mathcal{I}_g$-continuous, f^{-1}(P $-$ G) is $n\mathcal{I}_g$-open in O. But f^{-1}(P $-$ G) = O - f^{-1}(G). This implies f^{-1}(G) is $n\mathcal{I}_g$-closed in O.
(2) $\Rightarrow$ (3) Let G be any n-closed set in P. Then P$-$G is n-open set in P. By the assumption of (2), f^{-1}(P $-$ G) is $n\mathcal{I}_g$-closed in O. But f^{-1}(P $-$ G) = O - f^{-1}(G). This implies f^{-1}(G) is $n\mathcal{I}_g$-open in O.
(3) $\Rightarrow$ (4). Let o $\in$ O and G be any n-closed set in P with f(o) $\in$ G. By (3), f^{-1}(G) is $n\mathcal{I}_g$-open in O. Set U= f^{-1}(G). Then there is an $n\mathcal{I}_g$-open set U in O containing o such that f(U) $\subset$ G.
(4) $\Rightarrow$ (1). Let o $\in$ O and G be any n-closed set in P with f(o) $\in$ G. Then P $-$ G is n-open in P with f(o) $\in$ G. By (4), there is an $n\mathcal{I}_g$-open set U in O containing o such that f(U) $\subset$ G. This implies U= f^{-1}(G) . Therefore, O $-$ U = O $-$ f^{-1}(G) = f^{-1}(P$-$G) which is $n\mathcal{I}_g$-closed in O. □

Theorem 3.4. *Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) and g: (P, $\mathcal{N}'$, $\mathcal{J}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$). Then the following properties hold:*

1. *If f is contra $n\mathcal{I}_g$-continuous and g is n$\star$-continuous then g $\circ$ f is contra $n\mathcal{I}_g$-continuous.*
2. *If f is contra $n\mathcal{I}_g$-continuous and g is contra n$\star$-continuous then g $\circ$ f is $n\mathcal{I}_g$-continuous.*
3. *If f is $n\mathcal{I}_g$-continuous and g is contra n$\star$-continuous then g $\circ$ f is contra $n\mathcal{I}_g$-continuous.*

Proof. (1) Let G be n-closed set in Q. Since g is n$\star$-continuous, g^{-1}(G) is n-closed in P. Since f is contra $n\mathcal{I}_g$-continuous, $(g\circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $n\mathcal{I}_g$-open in O. Therefore g $\circ$ f is contra $n\mathcal{I}_g$-continuous.
(2) Let G be any n-closed set in Q. Since g is contra n$\star$-continuous, g^{-1}(G) is n-open in P. Since f is contra $n\mathcal{I}_g$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $n\mathcal{I}_g$-closed in O. Therefore g $\circ$ f is $n\mathcal{I}_g$-continuous.
(3) Let G be any n-closed set in Q. Since g is contra n$\star$-continuous, g^{-1}(G) is n-open in P. Since f is $n\mathcal{I}_g$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $n\mathcal{I}_g$-open in O. Therefore g $\circ$ f is contra $n\mathcal{I}_g$-continuous. □

Theorem 3.5. *Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is $n\mathcal{I}_g$-irresolute map and g : (P, $\mathcal{N}'$, $\mathcal{J}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is contra n$\star$-continuous map, then g $\circ$ f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is contra $n\mathcal{I}_g$-continuous map.*

Proof. Since g is contra n⋆-continuous from (P, $\mathcal{N}'$, $\mathcal{J}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$), for any n-open set in q as a subset of Q, we get, g^{-1}(q) = G is a n-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$). By Theorem 2.2 (1) and (2), it implies that g^{-1}(q) = G is $n\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). As f is $n\mathcal{I}_g$-irresolute map. We get (g ∘ f)$^{-1}$(q)= f^{-1}(g^{-1}(q)) = f^{-1}(G) = S and S is a $n\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Hence g ∘ f is a contra $n\mathcal{I}_g$-continuous map. □

Theorem 3.6. *Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is $n\mathcal{I}_g$-irresolute map and g : (P, $\mathcal{N}'$, $\mathcal{J}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is contra $n\mathcal{I}_g$-continuous map, then g ∘ f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$) is contra $n\mathcal{I}_g$-continuous map.*

Proof. Since g is contra $n\mathcal{I}_g$-continuous from (P, $\mathcal{N}'$, $\mathcal{J}$) $\rightarrow$ (Q, $\mathcal{N}'_*$, $\mathcal{K}$), for any n-open set in q as a subset of Q, we get, g^{-1}(q) = G is a $n\mathcal{I}_g$-closed set in (P, $\mathcal{N}'$, $\mathcal{J}$). As f is $n\mathcal{I}_g$-irresolute map. We get (g ∘ f)$^{-1}$(q)= f^{-1}(g^{-1}(q)) = f^{-1}(G) = S and S is a $n\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Hence g ∘ f is a contra $n\mathcal{I}_g$-continuous map. □

Theorem 3.7. *Let f: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a map and g: (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ ((O, $\mathcal{N}$, $\mathcal{I}$) × (P, $\mathcal{N}'$, $\mathcal{J}$)) the graph map of f, defined by g(o) = (o, f(o)) for every o ∈ O. If g is contra $n\mathcal{I}_g$-continuous, then f is contra $n\mathcal{I}_g$-continuous.*

Proof. Let G be an n-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Then ((O, $\mathcal{N}$, $\mathcal{I}$) × G) is an n-open set in ((O, $\mathcal{N}$, $\mathcal{I}$) × (P, $\mathcal{N}'$, $\mathcal{J}$)). It follows from Theorem 3.3, that f^{-1}(G) = g^{-1}((O, $\mathcal{N}$, $\mathcal{I}$) × G) is $n\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Thus, f is contra $n\mathcal{I}_g$-continuous. □

Theorem 3.8. *If a map f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is contra $n\mathcal{I}_g$-continuous and P is nano regular, then f is $n\mathcal{I}_g$-continuous.*

Proof. Let o be an arbitrary point of O and G be any n-open set of P containiing f(o). Since P is nano regular, there exists an n-open set W in P containing f(o) such that A_n^*(W) ⊂ G. Since f is contra $n\mathcal{I}_g$-continuous, by Theorem 3.3, there exists an $n\mathcal{I}_g$-open set U containing o such that f(U) ⊂ A_n^*(W). Thus f(U) ⊂ A_n^*(W) ⊂ G. Hence f is $n\mathcal{I}_g$-continuous. □

Definition 3.8. A space (O, $\mathcal{N}$, $\mathcal{I}$) is said to be an $n\mathcal{I}_g$-space if every $n\mathcal{I}_g$-open set is n-open in (O, $\mathcal{N}$, $\mathcal{I}$).

Theorem 3.9. *A map f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is contra $n\mathcal{I}_g$-continuous and O is $n\mathcal{I}_g$-space, then f is contra n⋆-continuous.*

Proof. Let G be n-closed set in P. Since f is contra $n\mathcal{I}_g$-continuous, f^{-1}(G) is $n\mathcal{I}_g$-open in O. Since O is an $n\mathcal{I}_g$-space, f^{-1}(G) is n-open in O. Therefore f is contra n⋆-continuous. □

Definition 3.9. An nano ideal topological space (O, $\mathcal{N}$, $\mathcal{I}$) is said to be $n\mathcal{I}_g$-connected if (O, $\mathcal{N}$, $\mathcal{I}$) cannot be expressed as the union of two disjoint non empty $n\mathcal{I}_g$-open subsets of (O, $\mathcal{N}$, $\mathcal{I}$).

Theorem 3.10. *A contra $n\mathcal{I}_g$-continuous image of a $n\mathcal{I}_g$-connected space is nano connected.*

Proof. Let f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be a contra $n\mathcal{I}_g$-continuous map of an $n\mathcal{I}_g$-connected space (O, $\mathcal{N}$, $\mathcal{I}$) onto a nano topological space (P, $\mathcal{N}'$). If possible, let P be nano disconnected. Let G and S form a nano disconnection of P. Then G and S are nano clopen and P = G ∪ S where G ∩ S = ϕ. Since f is contra $n\mathcal{I}_g$-continuous, O = f^{-1}(P) = f^{-1}(G ∪ S) = f^{-1}(G) ∪ f^{-1}(S), where f^{-1}(G) and f^{-1}(S) are non empty $n\mathcal{I}_g$-open sets in O. Also f^{-1}(G) ∩ f^{-1}(S) = ϕ. Hence O is not $n\mathcal{I}_g$-connected. This is a contradiction. Therefore P is nano connected. □

Lemma 3.11. *For an nano ideal topological space (O, $\mathcal{N}$, $\mathcal{I}$), the following are equivalent.*

(1) O is $n\mathcal{I}_g$-connected.

(2) The only subset of O which are both $n\mathcal{I}_g$-open and $n\mathcal{I}_g$-closed are the empty set ϕ and O.

Proof. (1) ⇒ (2) Let G be an $n\mathcal{I}_g$-open and $n\mathcal{I}_g$-closed subset of O. Then O − G is both $n\mathcal{I}_g$- open and $n\mathcal{I}_g$-closed. Since O is $n\mathcal{I}_g$-connected, O can be expressed as union of two disjoint non empty $n\mathcal{I}_g$-open sets O and O − G, which implies O − G is empty.
(2) ⇒ (1) Suppose O = G ∪ S where G and S are disjoint non empty $n\mathcal{I}_g$-open subsets of O. Then G is both $n\mathcal{I}_g$-open and $n\mathcal{I}_g$-closed. By assumption either G = ϕ or O which contradicts the assumption G and S are disjoint non empty $n\mathcal{I}_g$-open subsets of O. Therefore O is $n\mathcal{I}_g$-connected. □

Definition 3.10. A map f: (O, $\mathcal{N}$) → (P, $\mathcal{N}'$)is called nano preclosed if the image of every nano closed subset of O is nano preclosed in P.

Theorem 3.12. *Let f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) be a surjective nano preclosed contra $n\mathcal{I}_g$-continuous map. If O is an $n\mathcal{I}_g$-space, then P is nano locally indiscrete.*

Proof. Suppose that G is n-open in P. By hypothesis f is contra $n\mathcal{I}_g$-continuous and therefore f^{-1}(G) = U is $n\mathcal{I}_g$-closed in O. Since O is an $n\mathcal{I}_g$-space, U is n-closed in O. Since f is nano preclosed, then G is also nano preclosed in P. Now we have ncl(G) = ncl(nint(G)) ⊂ G. This means that G is n-closed and hence P is nano locally indiscrete. □

4. Conclusions

The Contra $n\mathcal{I}_g$-continuity in nano ideal topological spaces and discuss their properties and give various characterizations. In future, we have extended this work in various nano ideal topological spaces. The results of this study may be help in many researches.

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References

[1] R. Asokan, O. Nethaji and I. Rajasekaran, On nano generalized ⋆-closed sets in an ideal nano topological space, Asia Mathematika, 2(3), (2018), 50-58.
[2] M. Bhuvaneswari and N. Nagaveni, On Nwg-Normal and Nwg-Regular spaces, International Journal of Mathematics Trends and Technology (IJMTT)-special issue ICRMIT March (2018), 1-5.
[3] S. Ganesan, $n\mathcal{I}_{g\mu}$-closed sets in nano ideal topological spaces, Palestine Journal of Mathematics, 10(1), (2021), 340-348.
[4] S. Ganesan, $n\mathcal{I}$g-continuous maps in nano ideal topological spaces, MathLab Journal, 4(2019), 182-190.
[5] S. Ganesan, C. Alexander, S. Jeyashri and S. M. Sandhya, $n\mathcal{I}_{\ddot{g}}$-Closed Sets, MathLab Journal, 5(2020), 23-34.
[6] T. R. Hamlett and D. Jankovic, Ideals in topological space and the set operator ψ, Bull. U. M. I., (7)(4-B)(1990), 863-874.
[7] D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 97(4)(1990), 295-310.
[8] J. Jayasudha and T. Rekhapriyadharsini, On some decompositions of nano ⋆-continuity, International Journal of Mathematics and Statistics Invention, 7(1), (2019), 01-06.

[9] S. Krishnaprakash, R. Ramesh and R. Suresh, Nano-compactness and nano-connectedness in nano topological spaces, International Journal of Pure and Applied Mathematics, 119(13), (2018), 107-115.
[10] Mohammed M. Khalaf and Kamal N. Nimer, Nano P_S-Open Sets and Nano P_S-Continuity, International Journal of Contemporary Mathematical Sciences, 10(1), (2015), 1 - 11.
[11] M. Lellis Thivagar and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1(1)(2013), 31-37.
[12] M. Parimala, T. Noiri and S. Jafari, New types of nano topological spaces via nano ideals, ResearchGate, August (2016), 1-10.
[13] M. Parimala, S. Jafari and S. Murali, Nano ideal generalized closed sets in nano ideal topological spaces, Annales Univ.Sci. Budapest, 60(2017), 3-11.
[14] S. Visalakshi, A. Pushpalatha, Nano contra semi c(s) generalized continuity in nano topological spaces, International Journal of Mathematics Trends and Technology (IJMTT), 65(7), July (2019), 6-14.

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